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# AN ESTIMATE OF THE UNCERTAINTY OF POLICY EFFECTS IN A MACROECONOMETRIC MODEL

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# AN ESTIMATE OF THE UNCERTAINTY OF POLICY EFFECTS IN A MACROECONOMETRIC MODEL*

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## I. Introduction

Although macroeconometric models are widely used to analyze the effects of alternative government actions on the economy, estimates of the uncertainty of these effects are rarely, if ever, presented.[1] In this paper estimates of the uncertainty of the effects of eight policy actions are presented for my model [2]. The estimates are computed by means of stochastic simulation. The general procedure that was followed to obtain the estimates is discussed in Section II, and the particular policy experiments that were performed are described in Section III. The results are presented and discussed in Section IV. Section V contains a brief summary and conclusion.

Policy effects in a model are subject to two main sources of uncertainty, the coefficient estimates and the possible misspecification of the model, and it is important to note at the outset that only the first

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[1] I am unaware of any previous study in which estimates similar to those presented in this paper have been obtained. The closest study in this respect is probably that of Haitovsky and Wallace [8], where estimates of the total forecasting uncertainty of the FRB-MIT and Michigan models are obtained for different policy rules.

source is accounted for here. In a previous study [6] I have proposed a method for estimating the total forecasting uncertainty of a model that does account for the possible misspecification of the model, but there appears to be no way to carry over the idea behind this method to the present case.[2] The estimates presented in this paper are thus lower bounds of the true uncertainty of policy effects in the model. Even given this limitation, however, the following results should provide some indication of the likely uncertainty of policy effects in econometric models.

## II. The General Procedure

The procedure can be applied to a model that is nonlinear in both variables and coefficients. Let $G$ denote the total number of equations in the model, $M$ the number of stochastic equations, and $N$ the total number of predetermined (both exogenous and lagged endogenous) variables. Assume (for expositional convenience only) that the model is quarterly, and let the $i^{th}$ equation of the model for quarter $t$ be written:

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[2] The method proposed in [6] accounts for the four main sources of uncertainty of a forecast: (1) the error terms, (2) the coefficient estimates, (3) the exogenous-variable forecasts, and (4) the possible misspecification of the model. Estimating the uncertainty from the fourth source is based on a comparison of estimated variances computed by means of stochastic simulation with estimated variances computed from outside-sample forecast errors. With respect to the present case, it is important to note that uncertainty from the error terms and the exogenous-variable forecasts does not, with one minor exception, contribute to the uncertainty of policy effects. When running two simulations, one before and one after the policy change, any mistakes in forecasting the values of the error terms and the exogenous variables will affect the two simulations in the same way and thus be "cancelled out" in the computation of the effects of the policy change. The minor exception to this concerns nonlinear models, where policy effects are a function of the size of the error terms and the exogenous variables. For these models the mistakes will not exactly cancel out, but quantitatively this is not likely to be an important issue. It has been ignored for purposes of the computationsin this study.

$$(1) \qquad \phi_i(y_{1t}, \ldots, y_{Gt}, x_{1t}, \ldots, x_{Nt}, \beta_i) = e_{it} \ , \quad i = 1, \ldots, G \ ,$$

where the $y_{it}$ are the endogenous variables, the $x_{it}$ are the predetermined variables, $\beta_i$ is the vector of unknown coefficients in equation $i$ , and $e_{it}$ is the error term corresponding to equation $i$ . For identities, $e_{it}$ is zero for all $t$ . Also, let $\beta$ denote the vector of all the unknown coefficients in the model.

Assume that one is interested in the period $t = 1, \ldots, T$ . Given a vector of coefficient estimates (say, $\hat{\beta}$ ), given values of the exogenous variables for the period, and given some assumption about the error terms for the period, one can solve numerically for the values of the endogenous variables (provided a solution exists). If generated values of the lagged endogenous variables are used for the solution, which will be the case considered here, then this is a dynamic simulation of the model. Let $\hat{y}_{it}$ denote the predicted value of $y_{it}$ from this simulation. Consider now a second dynamic simulation in which everything is the same as before except that the values of one exogenous variable have been changed. Let $\hat{\hat{y}}_{it}$ denote the predicted value of $y_{it}$ from this simulation. Then $d_{it} = \hat{\hat{y}}_{it} - \hat{y}_{it}$ is the estimated effect of the change in the exogenous variable on $y_{it}$ .

Uncertainty regarding $d_{it}$ arises from the fact that only an estimate of $\beta$ is available and from the fact that the model may be misspecified. The concern of this paper is estimating the uncertainty from the first source. This can be done by means of stochastic simulation as follows. Let $\hat{V}$ denote the estimated variance-covariance matrix of $\hat{\beta}$ , and let $\beta^*$ denote the particular value of $\beta$ used in a given stochastic-simulation trial. If, say, $\hat{\beta}$ is assumed to be normally distributed, then values of $\beta^*$ can be drawn from a multivariate normal distribution with mean $\hat{\beta}$ and variance-covariance matrix $\hat{V}$ . For each trial (i.e., for each draw of $\beta^*$ ), one can compute $\hat{y}_{it}$ , $\hat{\hat{y}}_{it}$ , and thus $d_{it}$ . Let $d_{it}^{(j)}$ denote the value of $d_{it}$ computed on the $j^{th}$ trial. Given, say, N trials, one can compute the mean $(\hat{\bar{d}}_{it})$ and variance $(\hat{\sigma}_{it}^2)$ of $d_{it}$ :

$$(2) \qquad \hat{\bar{d}}_{it} = \frac{1}{N}\sum_{j=1}^{N} d_{it}^{(j)} ,$$

$$(3) \qquad \hat{\sigma}_{it}^2 = \frac{1}{N}\sum_{j=1}^{N} (d_{it}^{(j)} - \hat{\bar{d}}_{it})^2 .$$

$\hat{\sigma}_{it}^2$ is then an estimate of the uncertainty of the effect of the change in the exogenous variable on $y_{it}$ .

With respect to the stochastic simulation, note that the same values of the error terms and the exogenous variables are used for all the trials. As noted in footnote 2, uncertainty from the error terms and the exogenous-variable forecasts does not, with one minor exception, contribute to the uncertainty of policy effects, and the above simulation procedure is consistent with this fact. Note also that for each trial both $\hat{y}_{it}$ and $\hat{\hat{y}}_{it}$ are computed, which means that there are two dynamic simulations of the model per trial. Finally, note that more than one exogenous variable can be changed for the policy experiment. All that is required in computing $\hat{\sigma}_{it}^2$ is that whatever changes in the exogenous variables are made between the first and second dynamic simulations, they be the same from trial to trial.

## III. The Model and the Policy Experiments

My model has been updated since [2] was published, and the version that has been used in this study is presented in [7]. This version consists of 97 equations, 29 of which are stochastic, and has 182 unknown coefficients to estimate (including 12 serial correlation coefficients). The equations were estimated for the 1954I-1978II period (98 observations) by two-stage least squares, with account taken, when necessary, of first order serial correlation of the error terms. The technique that was used for this purpose is described in [1], and the variables that were used as regressors in the first stage regressions for each equation are listed in [7]. This technique yields an estimate of the first-order serial correlation coefficient for each equation in addition to the estimates of the structural coefficients.

By treating the serial correlation coefficient as a structural coefficient, it is possible to transform an equation with a serially correlated error into an equation without one. This introduces nonlinear restrictions on the coefficients, but otherwise the equation is like any other equation with a non-serially correlated error.[3] Therefore, even though some of the equations of the model have been estimated under the assumption of first order serial correlation, the model should be thought of as one with nonlinear coefficient restrictions and no serially correlated errors. All references to the variance-covariance matrices of the coefficient estimates in the following discussion, for example, are for the coefficient estimates inclusive of the estimates of the serial correlation coefficients.

[3] See, for example, the discussion in Chapter 3 in [2].

On November 7, 1978, I made a forecast with the model for the 1978IV-1982IV period.[4] This was an *ex ante* forecast, and so guessed values of the exogenous variables were used. This forecast was used as the basis for the present policy experiments. Eight experiments were performed. Each experiment corresponded to changing the values of one fiscal-policy variable from the values used for the base forecast. The changes that were made are presented in Table 1. The eight fiscal-policy variables include two expenditure variables ( XG and JOBGC ) and six tax variables ( $d_1$ , $d_3$ , $d_4$ , $d_5$ , $d_6$ , and YG ).

The amounts indicated in Table 1 by which each policy variable was changed require some explanation. The variables were changed so as to make the eight experiments roughly comparable. This need not have been done, since the uncertainty estimates are for one experiment at a time, but it seemed useful to provide along with the uncertainty estimates some indication of the different policy effects in the model. Note first in Table 1 that XG was changed by 2.5 billion dollars (10.0 billion dollars at an annual rate). XG is in units of 1972 dollars, and so the nominal increase in government expenditures on goods each quarter is PG·XG , where PG is the price deflator for government expenditures on goods. Given the experiment for XG , the other experiments were designed to roughly match it. Consider, for example, the change in YG , transfer payments, for the fifth experiment. YG is in nominal terms, and so the equivalent change in YG to the 2.5 for XG is 2.5·PG . PG is, however, an endogenous variable in the model, and so it is affected by policy changes.

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[4] The actual forecast that I made at the time ended in 1981IV, but for purposes of this paper I have extended it through 1982IV.

TABLE 1. The Eight Policy Experiments

| No. | Variable Changed | Amount Changed Each Quarter |
|---|---|---|
| 1 | $XG_t$ | 2.5 |
| 2 | $JOBGC_t$ | $2.5 \cdot PG_t/(WGC_t HPGC_t)$ |
| 3 | $d_{3t}$ | $-2.5 \cdot PG_t/YH_t$ |
| 4 | $d_{6t}$ | $-2.5 \cdot PG_t/WAGE_t$ |
| 5 | $YG_t$ | $2.5 \cdot PG_t$ |
| 6 | $d_{1t}$ | $-2.5 \cdot PG_t/\Pi F_t$ |
| 7 | $d_{5t}$ | $-2.5 \cdot PG_t/WAGE_t$ |
| 8 | $d_{4t}$ | $-2.5 \cdot PG_t/(CON_t + 2.5PG_t)$ |

Notes:
1. Simulation period is 1978IV-1982IV.
2. The values used for the endogenous variables in the above expressions were the predicted values from the base forecast.
3. All flow variables are at quarterly rates.

Variables ( † denotes exogenous variable):

$CON_t$ = personal consumption expenditures on durables, nondurables, and services in billions of current dollars. ( $= PCD_t CD_t + PCN_t CN_t + PCS_t CS_t$ in [2].)

† $d_{1t}$ = profit tax rate.

† $d_{3t}$ = personal income tax rate.

† $d_{4t}$ = indirect business tax rate.

† $d_{5t}$ = employer social security tax rate.

† $d_{6t}$ = employee social security tax rate.

† $JOBGC_t$ = number of government civilian jobs in thousands.

$PG_t$ = price deflator for government expenditures on goods.

$WAGE_t$ = wage bill of the firm sector in billions of current dollars. ( $= WFF_t(HPFN_t + 1.5 \cdot HPFO_t)JOBF_t$ in [2].)

† $XG_t$ = government expenditures on goods in billions of 1972 dollars.

† $YG_t$ = transfer payments from the government to households, not counting unemployment insurance benefits, in billions of current dollars.

$YH_t$ = taxable income of households in billions of current dollars.

$\Pi F_t$ = before-tax profits in billions of current dollars.

What was done in this case was to take for the values of PG the predicted values from the base forecast. These changes in YG then remained the same from trial to trial, even though the predicted values of PG varied from trial to trial (and from the predicted values for the base forecast). A similar procedure was followed for the other policy variables. JOBGC was changed to correspond roughly to a nominal change in government expenditures on labor of $2.5 \cdot PG$ each quarter. Each tax rate was changed to correspond roughly to a nominal change in taxes from the particular tax source of $2.5 \cdot PG$ each quarter.

In the version of the model used for these experiments, monetary policy is endogenous. An equation explaining the behavior of the Federal Reserve is included in the model. This equation, which is explained in [3], is an equation in which the Fed "leans against the wind." As the economy expands or as inflation increases, the Fed is estimated to cause the bill rate to rise.

The exact procedure that was followed for the stochastic simulations is as follows. For each of the 29 stochastic equations, an estimate of the variance-covariance matrix $(\hat{V}_i)$ of the 2SLS coefficient estimates $(\hat{\beta}_i)$ is available. Let $\beta_i^*$ denote the vector of coefficient values for equation $i$ used in a given trial. For each equation, values of $\beta_i^*$ were drawn from a multivariate normal distribution with mean $\hat{\beta}_i$ and variance-covariance matrix $\hat{V}_i$.[5] There are 182 unknown coefficients in

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[5] The draws for the $\beta_i^*$ vectors were performed as follows. First, for each $\hat{V}_i$, a matrix $P_i$ was computed such that $P_i P_i' = \hat{V}_i$. This was done using the LUDECP subroutine in the IMSL library. Then for each $i$, $n_i$ values of a standard normal random variable with mean 0 and variance 1 were drawn, where $n_i$ is the number of coefficients in equation $i$. (cont.)

all, and so each trial corresponds to drawing 182 numbers. It is important to note that this procedure does not take into account the correlation between the coefficient estimates in one equation and those in another. In other words, the $\hat{V}$ matrix in Section II is taken for the present application to be block diagonal. Ideally, one would like to have full information estimates of the model and use the estimated variance-covariance matrix of these estimates for the stochastic simulation, but these estimates are not available.

The number of trials for the experiments was 250. The same 250 draws were used for each of the eight experiments, which meant that for any given trial the first simulation for each experiment was the same. A total of only nine simulations was thus required per trial for the eight experiments. Each simulation (of 17 quarters) takes about 1.8 seconds of computer time on the IBM 370-158 at Yale, and so the total time for the 250 x 9 simulations was about 68 minutes.

It should finally be noted that although the simulation period used here is outside of the estimation period, this need not be the case. There is nothing inherent in the procedure discussed in Section II that precludes within-sample experiments.

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This was done using the function RNOR, which is part of the SUPER DUPER random number generator package at Yale. Let $u_i$ denote the $n_i \times 1$ vector of these draws. Then $\beta_i^*$ was computed as $\hat{\beta}_i + P_i u_i$. Since $Eu_i u_i' = I$, then $E(\beta_i^* - \hat{\beta}_i)(\beta_i^* - \hat{\beta}_i)' = EP_i u_i u_i' P_i' = \hat{V}_i$, which is as desired for the distribution of $\beta_i^*$.

One other point about these calculations should be noted. There is in the model a restriction imposed on the coefficients of the wage-rate equation, and so the number of freely estimated coefficients is one less than the actual number of coefficients in this equation. This restriction, however, clearly has some uncertainty attached to it, and this uncertainty was accounted for in this study by using for $\hat{V}_i$ the estimated variance-covariance matrix of the unrestricted estimates.

IV. The Results

Results for six selected variables are presented in Table 2. Results for all eight experiments are presented for real GNP and the GNP deflator, and results for the first experiment only are presented for the unemployment rate, the bill rate, the money supply, and the wage rate. The a rows contain for each quarter the average (over the 250 trials) of the difference between the predicted values of the variable from the two simulations (i.e., the $\bar{\hat{d}}_{it}$ values) and the b rows contain for each quarter the square root of the estimated variance of the difference (i.e., the $\hat{\sigma}_{it}$ values).[6]

The general properties of the model are discussed elsewhere,[7] and so no attempt will be made here to provide a complete explanation of the a-row values in Table 2. A few of the main differences in these values across experiments will, however, be noted. One of the important features of the model that should be kept in mind in the following discussion is its "microeconomic" basis. Included as explanatory variables in the consumption and labor supply equations, for example, are variables that one expects from microeconomic theory to affect the consumption and labor decisions of a utility maximizing household: the wage rate, the price level, interest rates, tax rates, nonlabor income, and the initial value of wealth.

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[6] For the GNP deflator and the wage rate, the a- and b-row values in Table 2 are expressed as a percentage of the level of the variable. To be more precise, let $\bar{\hat{y}}_{it}$ denote the mean of $\hat{y}_{it}$ (over the 250 trials). The numbers in the a rows are then $100(\bar{\hat{d}}_{it}/\bar{\hat{y}}_{it})$ , and the numbers in the b rows are $100(\hat{\sigma}_{it}/\bar{\hat{y}}_{it})$ .

[7] For the original discussion of the model's properties, see Chapter 9 in [2]. For more recent discussion, see [3], [4], and [5]. Finally, see Section VI in [7] for a discussion of the addition of the profit tax rate and the employer social security tax rate to the main price equation of the model.

TABLE 2

The Results of the Stochastic Simulations

a = average (over the 250 trials) of the difference between the predicted values of the variable from the two simulations (the $\hat{d}_{it}$ values).
b = estimated standard error of the difference (the $\hat{c}_{it}$ values).

Real GNP (billions of 1972 dollars at an annual rate)

| Policy Variable Changed | | 1978 IV | 1979 I | 1979 II | 1979 III | 1979 IV | 1980 I | 1980 II | 1980 III | 1980 IV | 1981 I | 1981 II | 1981 III | 1981 IV | 1982 I | 1982 II | 1982 III | 1982 IV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| XG | a | 9.4 | 12.1 | 12.9 | 13.0 | 12.4 | 11.4 | 10.3 | 9.3 | 8.3 | 7.4 | 6.6 | 5.9 | 5.4 | 5.0 | 4.6 | 4.3 | 4.1 |
| | b | 1.0 | 0.7 | 0.9 | 1.2 | 1.4 | 1.6 | 1.7 | 1.7 | 1.8 | 1.8 | 1.8 | 1.7 | 1.7 | 1.7 | 1.8 | 1.8 | 1.8 |
| JOBGC | a | 11.5 | 10.9 | 9.7 | 8.4 | 6.7 | 5.2 | 3.8 | 2.5 | 1.2 | 0.3 | -0.4 | -1.1 | -1.6 | -2.0 | -2.2 | -2.4 | -2.6 |
| | b | 0.3 | 0.8 | 1.3 | 1.7 | 2.1 | 2.4 | 2.6 | 2.7 | 2.8 | 2.8 | 2.9 | 2.9 | 2.9 | 2.9 | 3.0 | 3.0 | 3.0 |
| $d_3$ | a | 0.0 | 2.6 | 4.1 | 5.0 | 5.6 | 5.9 | 6.0 | 6.0 | 6.0 | 5.9 | 5.7 | 5.6 | 5.5 | 5.4 | 5.3 | 5.3 | 5.2 |
| | b | 0.0 | 1.4 | 1.9 | 2.1 | 2.2 | 2.3 | 2.2 | 2.2 | 2.1 | 2.1 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.1 |
| $d_6$ | a | 0.0 | 3.4 | 5.3 | 6.4 | 7.1 | 7.4 | 7.5 | 7.4 | 7.3 | 7.1 | 6.9 | 6.7 | 6.5 | 6.4 | 6.2 | 6.1 | 6.0 |
| | b | 0.0 | 1.9 | 2.6 | 3.0 | 3.1 | 3.2 | 3.1 | 3.0 | 2.9 | 2.8 | 2.8 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 |
| YG | a | 0.3 | 1.1 | 1.9 | 2.6 | 3.2 | 3.7 | 4.0 | 4.3 | 4.5 | 4.6 | 4.6 | 4.7 | 4.7 | 4.7 | 4.7 | 4.6 | 4.6 |
| | b | 0.3 | 0.4 | 0.7 | 0.9 | 1.2 | 1.4 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.0 | 2.1 | 2.1 | 2.1 | 2.1 | 2.1 |
| $d_1$ | a | 0.0 | 0.2 | 0.6 | 1.1 | 1.9 | 2.9 | 4.0 | 5.2 | 6.4 | 7.4 | 8.2 | 8.8 | 9.2 | 9.5 | 9.6 | 9.6 | 9.6 |
| | b | 0.0 | 0.1 | 0.3 | 0.6 | 0.9 | 1.3 | 1.9 | 2.6 | 3.2 | 3.8 | 4.2 | 4.6 | 4.8 | 5.0 | 5.1 | 5.2 | 5.2 |
| $d_5$ | a | 0.0 | 0.1 | 0.4 | 0.8 | 1.4 | 2.1 | 3.1 | 4.1 | 5.0 | 5.8 | 6.4 | 6.9 | 7.2 | 7.4 | 7.4 | 7.4 | 7.3 |
| | b | 0.0 | 0.1 | 0.2 | 0.4 | 0.6 | 1.0 | 1.5 | 2.0 | 2.6 | 3.0 | 3.4 | 3.7 | 3.9 | 4.0 | 4.1 | 4.1 | 4.0 |
| $d_4$ | a | 1.4 | 2.5 | 3.2 | 3.6 | 3.9 | 4.1 | 4.2 | 4.2 | 4.1 | 4.1 | 4.0 | 4.0 | 4.0 | 3.9 | 3.9 | 3.9 | 3.8 |
| | b | 0.5 | 0.6 | 0.7 | 0.7 | 0.7 | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 1.0 | 1.0 | 1.0 | 1.0 | 1.1 | 1.1 | 1.1 |

TABLE 2 (continued)

GNP Deflator (percent of the level of the deflator in percentage points)

| Policy Variable Changed | | 1978 IV | 1979 I | 1979 II | 1979 III | 1979 IV | 1980 I | 1980 II | 1980 III | 1980 IV | 1981 I | 1981 II | 1981 III | 1981 IV | 1982 I | 1982 II | 1982 III | 1982 IV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| XG | a | .069 | .130 | .185 | .235 | .276 | .311 | .338 | .358 | .372 | .381 | .388 | .391 | .392 | .391 | .390 | .389 | .387 |
| | b | .010 | .019 | .032 | .046 | .059 | .072 | .083 | .093 | .102 | .109 | .116 | .121 | .126 | .130 | .134 | .138 | .141 |
| JOBGC | a | .197 | .359 | .477 | .566 | .637 | .682 | .712 | .731 | .745 | .745 | .741 | .733 | .729 | .717 | .705 | .694 | .689 |
| | b | .039 | .074 | .107 | .135 | .160 | .182 | .200 | .215 | .227 | .236 | .243 | .249 | .254 | .258 | .263 | .268 | .273 |
| $d_3$ | a | .000 | -.017 | -.013 | -.008 | -.002 | .005 | .012 | .019 | .026 | .034 | .042 | .050 | .059 | .069 | .079 | .090 | .101 |
| | b | .000 | .010 | .010 | .017 | .026 | .035 | .043 | .051 | .057 | .063 | .069 | .074 | .079 | .084 | .089 | .095 | .101 |
| $d_6$ | a | .000 | -.024 | -.020 | -.016 | -.012 | -.007 | -.000 | .000 | .001 | .014 | .020 | .026 | .032 | .040 | .048 | .057 | .067 |
| | b | .000 | .014 | .014 | .024 | .036 | .049 | .061 | .071 | .080 | .089 | .096 | .103 | .109 | .115 | .120 | .126 | .132 |
| YG | a | .000 | .004 | .013 | .025 | .038 | .054 | .071 | .089 | .107 | .126 | .144 | .163 | .181 | .199 | .218 | .236 | .253 |
| | b | .000 | .003 | .005 | .010 | .015 | .021 | .029 | .037 | .046 | .055 | .065 | .075 | .084 | .094 | .103 | .113 | .123 |
| $d_1$ | a | .000 | -.051 | -.144 | -.274 | -.436 | -.624 | -.782 | -.915 | -1.024 | -1.114 | -1.188 | -1.248 | -1.297 | -1.337 | -1.369 | -1.396 | -1.418 |
| | b | .000 | .027 | .077 | .148 | .235 | .336 | .423 | .497 | .559 | .612 | .656 | .694 | .727 | .754 | .779 | .800 | .819 |
| $d_5$ | a | .000 | -.041 | -.116 | -.222 | -.353 | -.505 | -.633 | -.740 | -.827 | -.897 | -.952 | -.996 | -1.029 | -1.054 | -1.072 | -1.084 | -1.091 |
| | b | .000 | .021 | .062 | .118 | .189 | .270 | .339 | .398 | .446 | .486 | .519 | .546 | .568 | .585 | .599 | .610 | .619 |
| $d_4$ | a | -.679 | -.673 | -.665 | -.656 | -.645 | -.634 | -.623 | -.613 | -.602 | -.591 | -.581 | -.570 | -.558 | -.546 | -.534 | -.522 | -.509 |
| | b | .004 | .005 | .008 | .012 | .015 | .018 | .021 | .023 | .026 | .028 | .030 | .033 | .036 | .040 | .044 | .048 | .053 |

TABLE 2 (continued)

| Policy Variable Changed | | 1978 IV | 1979 I | 1979 II | 1979 III | 1979 IV | 1980 I | 1980 II | 1980 III | 1980 IV | 1981 I | 1981 II | 1981 III | 1981 IV | 1982 I | 1982 II | 1982 III | 1982 IV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Unemployment Rate (percentage points) | | | | | | | | | | | | | | | | |
| XG | a | -.180 | -.368 | -.455 | -.477 | -.468 | -.437 | -.398 | -.356 | -.317 | -.281 | -.251 | -.226 | -.206 | -.191 | -.179 | -.171 | -.165 |
| | b | .042 | .056 | .065 | .069 | .071 | .072 | .074 | .075 | .077 | .080 | .082 | .084 | .086 | .088 | .089 | .091 | .092 |
| | | Bill Rate (percentage points) | | | | | | | | | | | | | | | | |
| XG | a | .175 | .302 | .375 | .424 | .451 | .460 | .457 | .447 | .432 | .414 | .396 | .378 | .360 | .344 | .329 | .314 | .301 |
| | b | .072 | .084 | .089 | .094 | .098 | .102 | .107 | .111 | .115 | .119 | .121 | .123 | .125 | .125 | .126 | .126 | .126 |
| | | Money Supply (billions of current dollars at an annual rate) | | | | | | | | | | | | | | | | |
| XG | a | 0.57 | 1.30 | 2.09 | 2.86 | 3.55 | 4.16 | 4.69 | 5.14 | 5.53 | 5.88 | 6.19 | 6.47 | 6.74 | 7.01 | 7.28 | 7.55 | 7.82 |
| | b | 0.39 | 0.93 | 1.51 | 2.07 | 2.58 | 3.02 | 3.41 | 3.75 | 4.04 | 4.31 | 4.55 | 4.78 | 5.00 | 5.22 | 5.43 | 5.65 | 5.86 |
| | | Wage Rate (percent of the level of the wage rate in percentage points) | | | | | | | | | | | | | | | | |
| XG | a | .018 | .057 | .100 | .141 | .178 | .208 | .233 | .252 | .265 | .275 | .281 | .284 | .286 | .286 | .285 | .284 | .282 |
| | b | .009 | .024 | .042 | .059 | .075 | .089 | .101 | .112 | .120 | .127 | .132 | .137 | .140 | .143 | .146 | .149 | .152 |

Note: XG was changed each quarter from its value for the base forecast by 10 billion dollars at an annual rate. The changes in the other policy variables were made to be roughly comparable to this. See the discussion of Table 1.

It should also be noted that although reference is sometimes made below to a change in one endogenous variable "leading" to a change in another endogenous variable, this discussion is loose. The model is simultaneous, and so in fact everything affects everything else.

Some of the important differences across the a-row values are the following. First, the two expenditure changes ( XG and JOBGC ) result in much faster initial responses than do the tax changes. This is a common property of macroeconometric models and requires no further discussion here. The JOBGC experiment is more inflationary than is the XG experiment, primarily because it led to a larger initial decrease in the unemployment rate (not shown in Table 2). The higher inflation rate in the JOBGC experiment led the Fed to raise the bill rate more than it did in the XG experiment (also not shown in Table 2), which led to a less expansionary economy in the JOBGC experiment after the first quarter. The decreases in the profit tax rate ($d_1$) and the employer social security tax rate ($d_5$) led to less inflation than otherwise, and this is one of the reasons for the increase in real output in these two experiments. (Prices have, other things being equal, a negative effect on demand in the model.) The decrease in the indirect business tax rate ($d_4$) also led to a lower GNP deflator than otherwise, in this case primarily because indirect business tax rates are included in the GNP deflator. With respect to the $d_1$ versus $d_5$ experiments, one of the reasons the $d_1$ experiment is more expansionary is that employer social security taxes are deductible from profits. The decrease in $d_5$ by the amount indicated in Table 1 thus led to a smaller reduction in overall taxes paid by firms than did the decrease in $d_1$ .

It is interesting to note that the decreases in the personal income tax rate ($d_3$) and the employee social security tax rate ($d_6$) led to an initial decrease in the rate of inflation. A decrease in either of these two rates has, other things being equal, a positive effect on the labor force (one of the "microeconomic" features of the model) and thus a positive effect on the unemployment rate. An increase in the unemployment rate in turn has a negative effect on the rate of inflation. This indirect negative effect of $d_3$ and $d_6$ on the rate of inflation was large enough to lead to lower initial rates of inflation in the two experiments. This effect is exactly reversed for the increase in transfer payments (YG) , which has, other things being equal, a negative effect on the labor force.

Consider now the uncertainty estimates in the b rows. In particular, consider first the values for real GNP for the last quarter of the period. The least uncertain of the cases is the $d_4$ experiment, with an estimated standard error of 1.1 billion dollars. This is followed by the XG experiment (1.8 billion dollars) and then by the $d_3$ and YG experiments (2.1 billion dollars each). The most uncertain are the $d_1$ and $d_5$ experiments (5.2 and 4.0 billion dollars, respectively). This general pattern also holds for the GNP deflator, although for this variable the standard error for the last quarter for the XG experiment of 0.141 percentage points is slightly larger than the standard errors for the $d_3$ , $d_6$ , and YG experiments (0.101, 0.132, and 0.123 percentage points, respectively).

For the most part the standard errors increase as the horizon lenghtens. The primary exception to this is in the XG experiment for real GNP, where the two- and three-quarter-ahead standard errors are less than the one-quarter-ahead error. The reason for this is hard to explain and not very interesting, but I am sure I will be asked. The reason is explained in the following footnote.[8]

The standard errors for the money supply and the wage rate (presented for the XG experiment only) are generally larger as a percentage of the a-row values than are the standard errors for the other variables. As reported elsewhere ([3] and [6]), the demand for money equations and the wage rate equation are on a number of statistical criteria some of the weakest equations of the model, and this reflects itself in the present case in fairly large b-row values for the money supply and the wage rate. The standard errors for the bill rate, on the other hand, which primarily reflect the statistical quality of the Fed behavioral equation, are fairly small. They are between about one-third and one-fourth of the corresponding a-row values.

The standard errors in Table 2 are in general smaller than I would have expected before I began stochastic simulation of the model, although others may obviously have different priors than I had. The results in this paper are consistent with the results in [6], where estimates of the total forecasting uncertainty of the model are presented. For these estimates

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[8] First, the XG experiment results in a large initial change in sales of goods, something which is not true of any of the other experiments. Second, the production equation in the model is in log form and has a serially correlated error. The log of sales is one of the explanatory variables in this equation. Therefore, the change in production in the first quarter is proportional, among other things, to the change in sales in the first quarter times $e^{\hat{\rho}\hat{U}_{-1}}$, where $\hat{\rho}$ is the estimate of the serial correlation coefficient and $\hat{U}_{-1}$ is the estimated error term from the previous quarter. The change in production in the second quarter is proportional to the change in sales in the second quarter times $e^{\hat{\rho}^2\hat{U}_{-1}}$. Since $\hat{\rho}$ is less than one, $\hat{U}_{-1}$ contributes less to the two-quarter-ahead forecast than it does to the one-quarter-ahead forecast. The uncertainty of $\hat{\rho}$ thus contributes more to the uncertainty of the one-quarter-ahead forecast than it does to the others, and this effect is large enough in the XG experiment (because of the multiplicative nature of the error term and the change in sales) to lead to an overall standard error for real GNP that is larger in the first quarter than it is in the second and third quarters.

the contribution of the uncertainty of the coefficient estimates to the total uncertainty is in general relatively small. To the extent that most of the forecasting uncertainty comes from the error terms and the exogenous-variable forecasts, this is encouraging for policy analysis, since the uncertainty from these two sources does not contribute to the uncertainty of policy effects. The situation is, of course, quite different if much of the forecasting uncertainty comes from the misspecification of the model.

## IV. Summary and Conclusion

A procedure for estimating the uncertainty of policy effects in an econometric model has been proposed and implemented in this study. The procedure, which is a relatively straightforward exercise in stochastic simulation, accounts for the uncertainty from the coefficient estimates but not from the possible misspecification of the model. The uncertainty estimates are thus lower bounds of the true uncertainty of policy effects.

The results of applying the procedure to my model for eight policy actions are presented in Table 2. The least uncertain of the effects are for the change in the indirect business tax rate, and the most uncertain of the effects are for the changes in the profit tax rate and the employer social security tax rate. In general the uncertainty estimates seem smaller than one might have expected, although this is obviously dependent on one's initial priors. The estimates are consistent with the estimates of the total forecasting uncertainty of the model presented in [6], where the contribution of the uncertainty of the coefficient estimates to the total uncertainty is in general relatively small. It is an open question whether this feature of my model is also true of other models. Given the obvious importance of knowing how much confidence to place on the results of any particular policy experiment from a model, it is hoped that this study will stimulate other model builders to obtain similar uncertainty estimates.

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