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A MODEL OF THE BALANCE OF PAYMENTS

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# A MODEL OF THE BALANCE OF PAYMENTS\*

by

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## 1. *Introduction*

It is common in the literature to distinguish among the elasticity, absorption, and monetary approaches to the balance of payments, with recent attention focusing on the monetary approach.<sup>1</sup> Although, as Mundell [13, pp. 150-151] has pointed out, these approaches all assert identical propositions in an accounting sense, they have provided a way of categorizing alternative theories or models of the balance of payments. An important question is whether this categorization provides a useful framework for further work. A model of the balance of payments is presented in this paper that indicates that it does not. The model does not fall naturally into any of the above categories, and furthermore it indicates that none of the three approaches provides a complete explanation of the balance of payments.

A useful way of distinguishing between the model developed in this paper and previous models is to consider the determination of the exchange rate. Recent studies that follow the monetary approach have stressed the stock market aspect of this determination. Dornbusch [4, p. 276], for

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<sup>1</sup>See, for example, Frenkel and Johnson [9], Dornbusch [4], Frenkel and Rodriguez [10], and Kouri [12]. See also the recent survey by Myhrman [14].

example, states that "the exchange rate is determined on the asset market," and Frenkel and Rodriguez [10, p. 686] state that "the equilibrium exchange rate is that relative price of monies at which the existing stocks are willingly held." This treatment is contrasted with the "characterization of exchange-rate determination as arising in the market for foreign exchange with an emphasis on the financing of current trade flows" (Dornbusch [4, p. 276]) and "the popular notion that the exchange rate is determined in the flow market so as to assure a balanced balance of payments" (Frenkel and Rodriguez [10, p. 686]). A key difference between the monetary approach and the other two approaches is thus this question of stock-market determination versus flow-market determination of the exchange rate.<sup>2</sup> In the model in this paper, on the other hand, there is no natural distinction between stock-market and flow-market determination. The exchange rate is not in any rigorous sense determined either in a stock market or in a flow market. The exchange rate has an effect on many of the decisions of the economic agents in the model, decisions regarding both stock and flow variables, and these decisions in turn affect a number of different markets. The exchange rate, like the price level, the wage rate, and the interest rate, is merely one endogenous variable out of many in the model, and in no rigorous sense can it be said to be "the" variable that clears a particular market.

The inspiration for the model in this paper came from my earlier work [5, 6] on developing a macroeconomic model for a single country.

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<sup>2</sup>There are, of course, numerous other studies in the literature on international monetary economics in which the stock-flow distinction is important. For recent examples, see Allen [1], Black [2], Branson [3], and Girton and Henderson [11]. See also again the recent survey by Myhrman [14].

The stress in this earlier work is on deriving the decisions of the individual agents in the economy from the assumption of maximizing behavior, on accounting for all flows of funds in the system, and on accounting in an explicit way for possible disequilibrium effects. The decisions of the agents are based on the solutions of multiperiod maximization problems. Firms and banks maximize the present discounted value of expected future profits, and households maximize the present discounted value of expected future utility. Accounting for all flows of funds means that any saving or dissaving of an agent in a period results in the change in at least one of its assets or liabilities, that any asset of one agent is a corresponding liability of some other agent, and that the government budget constraint is accounted for. Disequilibrium, which may arise because of expectation errors,<sup>3</sup> takes the form of banks constraining firms and households in how much they can borrow at the current loan rate and of firms constraining households in how much they can work at the current wage rate. The idea for the model in this paper came from considering how one would link the single-country model in [5] and [6] to a similar model for another country so as to form a closed two-country model of the world.

The basic model is presented in Section II, and then various extensions and alterations of it are discussed in Section III. The model as presented in Section II is sufficient for seeing the principal differences between the present approach and previous approaches to the balance of payments. The two primary issues discussed in Section III are the incor-

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<sup>3</sup>Agents do not know the complete model, and so even though there are no random shocks in the model their expectations may not always be correct. For a defense of this assumption versus the assumption of rational expectations, see Fair [7].

poration of disequilibrium effects into the model and the addition of a bank sector to the model.

The main points of this paper can be made without reference to the effects of policy actions in the model, and so an examination of these effects is left for another paper [8]. In order to examine policy effects in the model, it is necessary to specify the decision equations that are presented in the next section in more detail than is done in this paper. With some simplification, it might be possible to specify these equations in such a way as to allow the effects of policy actions to be analyzed analytically or graphically, but this is not the procedure that was followed in [8]. Instead, a version of the model was obtained by linking the 84-equation econometric model of the U.S. economy in [6] to the mirror image of itself. This resulted, after the addition of a few equations to close the model, in a 180-equation two-country model. The effects of various policy actions in the model were then examined by means of simulation techniques. This model, which will be called Model A, is briefly discussed at the end of Section III of this paper.

To summarize, a model of the balance of payments is presented in this paper in which stock and flow effects are completely integrated.<sup>4</sup> The exchange rate has an effect on both stock and flow variables, and it is simultaneously determined in the model along with the other endogenous variables. As should be clear in what follows, the two key features

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<sup>4</sup>Regarding the integration of stock and flow effects, Branson [3] at the end of his paper (p. 48) states: "But the introduction of capital movements, with the value of the exchange rate being determined by both continuing flows and discontinuous stock shifts, raises some analytical problems that are yet to be solved." As I understand this statement, these problems have been solved in this paper, since the present model does provide a complete integration of stock and flow effects.

of the model that allow this integration to take place are the explicit consideration of the decision problems of the individual agents in the economy and the accounting for all flows of funds in the system. This integration is true of both the model as presented in Section II, in which the labor, goods, and financial markets are in equilibrium, and the model as modified in Section III, in which these markets may at times be in disequilibrium.

## II. *The Basic Model*

### *The Equations*

The model is a two-country model. Capital letters will denote variables for country 1, lower case letters will denote variables for country 2, and an \* on a variable will denote the other country's holdings or purchase of the variable. There are three sectors per country: household, firm, and government. Subscripts  $h$ ,  $f$ , and  $g$  will be used to denote these sectors, respectively. Each country specializes in the production of one good  $(X, x)$ . Labor  $(L, l)$  is homogeneous within a country, and there is no labor mobility between countries. Each country has its own money  $(M, m)$ , which is issued by the government, and its own bond  $(B, b)$ . The bonds are one-period securities. If a sector is a debtor with respect to a bond (i.e., a supplier of the bond), then the value of  $B$  or  $b$  for this sector is negative. The interest rate on  $B$  is  $R$  and on  $b$  is  $r$ ; the wage rate for  $L$  is  $W$  and for  $l$  is  $w$ ; and the price of  $X$  is  $P$  and of  $x$  is  $p$ . The exchange rate, the price of country 2's currency in terms of country 1's currency, is  $e$ . The government of each country holds a positive amount of the international reserve  $(Q, q)$ , whose price is 1.0, and it taxes

its citizens using a vector  $(T, t)$  of tax parameters. For now,  $X$  and  $x$ ,  $M$  and  $m$ , and  $B$  and  $b$  are assumed not to be perfect substitutes.

Consider country 1. The household sector jointly determines its labor supply and its demands for the two goods, the two monies, and the two bonds. It takes as given the wage rate, the two prices, the two interest rates, the tax parameters, the exchange rate, and all lagged values. The vector of all relevant lagged values will be denoted at  $Z_h$ .<sup>5</sup> These decisions are assumed to be derived from a multiperiod maximization problem. Expectations of various future values, which are needed for such problems, are assumed to be a function of current and lagged values.<sup>6</sup> The equations representing the decisions for the current period will be written as:

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<sup>5</sup>Included in  $Z_h$  are such obvious things as the stocks of the two bonds and the two monies of the household sector at the end of the previous period. If the stock of government debt also affects household decisions, then this variable should likewise be considered to be in  $Z_h$  (i.e.,  $B_{g-1}$  should be considered to be in  $Z_h$ ).

<sup>6</sup>See Chapter 4 in [5] for an analysis of household multiperiod maximization problems in a single-country context. In the present two-country case, expectations of future values of the exchange rate are likely to have an important effect on current decisions. If these expectations are a function of current and lagged values, as is assumed to be true of all expectations in the model, then their effects can be assumed to be accounted for by the RHS variables in equations (1)-(7), especially by  $Z_h$ . One key expectation in this regard is the expected one-period change in the exchange rate, denoted as  $\rho$ . For present purposes, an explicit equation for  $\rho$  does not have to be specified, but one will be needed later.

It should also be noted that an important way in which  $e$  is likely to affect household decisions is through the term  $e \cdot p$ , the price of country 2's good in terms of country 1's currency.

- (1)  $L_h = f_1(W, P, p, R, r, T, e, Z_h)$  [supply of labor]
- (2)  $X_h = f_2( \quad " \quad )$  [demand for the good of country 1]
- (3)  $x_h^* = f_3( \quad " \quad )$  [demand for the good of country 2]
- (4)  $M_h = f_4( \quad " \quad )$  [demand for the money of country 1]
- (5)  $m_h^* = f_5( \quad " \quad )$  [demand for the money of country 2]
- (6)  $B_h = f_6( \quad " \quad )$  [supply of (-) or demand for the bond of country 1]
- (7)  $b_h^* = f_7( \quad " \quad )$  [supply of (-) or demand for the bond of country 2]

These seven equations are not independent, since they must satisfy a budget constraint. The taxable income of the household sector ( $Y_h$ ) is assumed to be

$$(8) \quad Y_h = W \cdot L_h + R \cdot B_h + e \cdot r \cdot b_h^* , \quad [\text{taxable income}]$$

where the first term on the RHS is wage income and the second and third terms are interest income or interest payments. Net taxes paid by the household sector ( $V_h$ ) is assumed to be a function of  $Y_h$  and  $T$  :

$$(9) \quad V_h = f_9(Y_h, T) . \quad [\text{net taxes paid}]$$

The financial saving of the household sector ( $S_h$ ) is then

$$(10) \quad S_h = Y_h - V_h - P \cdot X_h - e \cdot p \cdot x_h^* , \quad [\text{household saving}]$$

where the last two terms are expenditures on goods. Finally, the budget



constraint is

$$(11) \quad 0 = S_h - \Delta M_h - e \cdot \Delta m_h^* - \Delta B_h - e \cdot \Delta b_h^*, \quad [\text{household budget constraint}]$$

which says that any nonzero level of saving of the household sector must result in the change in at least one of its assets or liabilities.

For simplicity, it is assumed that the firm sector does not purchase the good of the foreign country, does not hold the bond of the foreign country, and holds no money. It jointly determines its demand for labor, its supply of the good net of the amount used for investment purposes, and its net demand for the bond. It takes as given  $W$ ,  $P$ ,  $R$ ,  $T$ , and all lagged values  $(Z_f)$ . These decisions are also assumed to be derived from a multiperiod maximization problem, with the equations representing the decisions for the current period written as:<sup>7</sup>

$$(12) \quad L_f = f_{12}(W, P, R, T, Z_f) \quad [\text{demand for labor}]$$

$$(13) \quad X_f = f_{13}(\quad \quad \quad) \quad [\text{net supply of the good}]$$

$$(14) \quad B_f = f_{14}(\quad \quad \quad) \quad [\text{supply of (-) or demand for the bond of country 1}]$$

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<sup>7</sup>See Chapter 3 in [5] for an analysis of firm multiperiod maximization problems in a single-country context, where a firm jointly determines its price, production, investment, and employment or wage rate. For the model in this section it is assumed that the firm sector takes the price level and the wage rate as given. This assumption will be relaxed in Section III when disequilibrium issues are discussed. In order for equations (12) and (13) to be determinant, there must be diminishing returns in the economy, and this is assumed here. For an example of the introduction of diminishing returns into a model, see Frenkel and Rodriguez [10], who assume for their model that gross capital function is subject to increasing marginal cost.

These three equations also must satisfy a budget constraint. The value of taxes paid by the firm sector ( $V_f$ ) is assumed to be a function of  $T$  and of variables that determine profits:

$$(15) \quad V_f = f_{15}(L_f, X_f, B_f, W, P, R, Z_f, T) . \quad [\text{taxes paid}]$$

The financial saving of the firm sector ( $S_f$ ) is

$$(16) \quad S_f = P \cdot X_f + R \cdot B_f - W \cdot L_f - V_f , \quad [\text{firm saving}]$$

and its budget constraint is

$$(17) \quad 0 = S_f - \Delta B_f . \quad [\text{firm budget constraint}]$$

The government purchases labor from its own citizens ( $L_g$ ) and both goods ( $X_g$  and  $x_g^*$ ). The amount of money that it issues is  $M_g$ , and its net holdings of the bond of country 1 is  $B_g$ . It also holds the money and the bond of the other country ( $m_g^*$  and  $b_g^*$ ), in addition to the international reserve ( $Q$ ). The financial saving of the government ( $S_g$ ) is

$$(18) \quad S_g = V_h + V_f + R \cdot B_g + e \cdot r \cdot b_g^* - W \cdot L_g - P \cdot X_g - e \cdot p \cdot x_g^* , \quad [\text{government saving}]$$

and its budget constraint is

$$(19) \quad 0 = S_g + \Delta M_g - e \cdot \Delta m_g^* - \Delta B_g - e \cdot \Delta b_g^* - \Delta Q . \quad [\text{government budget constraint}]$$

The first two terms on the RHS of (18) are tax revenue, the next two terms are interest income or interest payments, and the last three terms are purchases of labor and goods. Equation (19) states that any nonzero value of government saving must result in the change in at least one of the

government's assets or liabilities.

Equations (1)-(19) also hold for country 2, with capital and lower case letters reversed and with  $1/e$  replacing  $e$ . Call these equations (1)'-(19)'. The model is then closed by the following equations:

- (39)  $L_h = L_f + L_g$  [supply of labor equals demand for labor in country 1]
- (40)  $l_h = l_f + l_g$  [supply of labor equals demand for labor in country 2]
- (41)  $X_f = X_h + X_g + X_h^* + X_g^*$  [supply of the good of country 1 equals the demand for it]
- (42)  $x_f = x_h + x_g + x_h^* + x_g^*$  [supply of the good of country 2 equals the demand for it]
- (43)  $M_g = M_h + M_h^* + M_g^*$  [supply of the money of country 1 equals the demand for it]
- (44)  $m_g = m_h + m_h^* + m_g^*$  [supply of the money of country 2 equals the demand for it]
- (45)  $0 = B_h + B_f + B_g + B_h^* + B_g^*$  [supply of the bond of country 1 equals the demand for it]
- (46)  $0 = b_h + b_f + b_g + b_h^* + b_g^*$  [supply of the bond of country 2 equals the demand for it]
- (47)  $0 = \Delta Q + \Delta q$  . [no change in total world reserves]

Of the 47 equations, 5 are redundant. The redundant equations are: one from (1)-(11), one from (12)-(17); one from (1)'-(11)', one from (12)'-(17)', and one because the savings of all sectors sum to zero:

$S_h + S_f + S_g + e(s_h + s_f + s_g) = 0$ . It will be convenient to drop (6), (6)', (14), (14)', and (46), leaving 42 independent equations. If all

the government variables (i.e., all the variables with subscript  $g$ ) except  $M_g$ ,  $m_g$ ,  $S_g$ , and  $s_g$  are taken to be exogenous and if all the lagged values are taken to be predetermined, then there are 43 variables left. One further variable thus must be taken to be exogenous. In the fixed exchange rate regime this variable is  $e$ , and in the flexible exchange rate regime the variable is  $Q$ .

It may be helpful to consider the matching of variables to equations to see that all the variables are accounted for. Equations (1)-(5), (7)-(10), (12), (13), (15), (16), (18), and the corresponding equations for country 2 can be matched to the LHS variables in the equations. Of the other independent equations before (39),  $B_h$  can be matched to (11),  $B_f$  to (14),  $M_g$  to (19),  $b_h$  to (11)',  $b_f$  to (14)', and  $m_g$  to (19)'. Matching  $M_g$  to (19) and  $m_g$  to (19)' shows the nature of the government budget constraints. For the government of country 1, for example, given  $m_g^*$ ,  $B_g$ ,  $b_g^*$ ,  $e$ , and  $Q$ , any nonzero value of its saving must result in a change in the money supply. To continue the matching,  $W$  and  $w$  can be matched to (39) and (40), the equilibrium conditions for the labor markets;  $P$  and  $p$  can be matched to (41) and (42), the equilibrium conditions for the goods markets; and  $R$  and  $r$  can be matched to (43) and (44), the equilibrium conditions for the money markets.  $q$  can then be matched to (47), which leaves only equation (45), the equilibrium condition for country 1's bond market, unaccounted for. (Remember that (46) was one of the equations dropped.) The two variables unaccounted for are  $e$  and  $Q$ , and so one of these can be matched to (45), with the other one taken to be exogenous.

This completes the outline of the model. The point made in the Introduction about the determination of the exchange rate should now be

clear. The exchange rate affects the decisions of the household sector and also enters a number of the definitions in the model. Although in the previous paragraph  $e$  was matched to equation (45), it is not in any rigorous sense determined by this equation. Rather, it is simultaneously determined in the model, along with the other 41 endogenous variables. In this sense,  $e$ , like  $P$ ,  $p$ ,  $W$ ,  $w$ ,  $R$ , and  $r$ , affects both endogenous flow variables and endogenous stock variables. There is no natural distinction in the model between stock-market determination and flow-market determination. This feature is clearly seen once all the flows of funds are accounted for in the model.

*The Case Where  $B$  and  $b$  are Perfect Substitutes*

If  $B$  and  $b$  are perfect substitutes, then arbitrage will insure that  $R = r + \rho$ , where  $\rho$  is the expected one-period change in  $e$ . The model in this case is modified as follows. First, equations (7) and (7)' drop out, since the household sectors are indifferent between the two bonds. (Remember that equation (6) and (6)' have already been dropped.) Second, two equations are added: the equation  $R = r + \rho$ , and an equation explaining  $\rho$ .<sup>8</sup> Third and last, the model is underidentified with respect to  $B_h$ ,  $B_h^*$ ,  $b_h$ , and  $b_h^*$ , and so one of these variables must be taken to be exogenous.<sup>9</sup>

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<sup>8</sup>This is the place where an explicit equation for  $\rho$  is needed. (See footnote 6.) It is assumed for purposes of the present discussion that both household sectors have the same expectations regarding the one-period change in  $e$ .

<sup>9</sup>This indeterminacy is analogous to the indeterminacy that arises in, say, a two-consumer, two-firm model in which the two consumers are indifferent between the goods produced by the two firms. It is not possible in this model to determine the allocation of the two goods between the two consumers. To see this, let  $x_i$  denote the supply of the good of firm  $i$  ( $i = 1, 2$ ), let  $y_i$  denote the total demand for goods of consumer  $i$  ( $i = 1, 2$ ), let  $x_i^j$  denote the consumption of the good of

The model thus consists of the same number of equations as before, with one new endogenous variable added ( $\rho$ ) and one previous endogenous variable taken to be exogenous ( $B_h$ ,  $B_h^*$ ,  $b_h$ , or  $b_h^*$ ).

*The Case Where  $X$  and  $x$  are Perfect Substitutes*

In this case,  $P = e \cdot p$ , so one new equation is added to the model. Two equations drop out: either (2) and (2)' or (3) and (3)'. The model is underidentified with respect to  $X_h$ ,  $X_h^*$ ,  $x_h$ , and  $x_h^*$ , and so one of these variables must be taken to be exogenous. (See footnote 9.) The model thus consists of one less equation and one less endogenous variable.

firm  $i$  by consumer  $j$  ( $i = 1, 2$ ;  $j = 1, 2$ ), and let  $p$  denote the (one) price of the goods. Consider the following model:

$$(1) \text{ and } (2) \quad x_i = f_i(p, \dots), \quad (i = 1, 2),$$

$$(3) \text{ and } (4) \quad y_i = g_i(p, \dots), \quad (i = 1, 2),$$

$$(5) \quad y_1 = x_1^1 + x_2^1,$$

$$(6) \quad y_2 = x_1^2 + x_2^2,$$

$$(7) \quad x_1 = x_1^1 + x_1^2,$$

$$(8) \quad x_2 = x_2^1 + x_2^2.$$

Equations (1) and (2) are supply equations, equations (3) and (4) are demand equations, and equations (5)-(8) are definitions. This model consists of 8 equations and 9 unknowns:  $x_1$ ,  $x_2$ ,  $y_1$ ,  $y_2$ ,  $x_1^1$ ,  $x_2^1$ ,  $x_1^2$ ,  $x_2^2$ , and  $p$ . It is thus not possible to determine the allocation of the two goods between the two consumers. One of the  $x_i^j$  variables has to be taken as given.

*The Case Where  $M$  and  $m$  are Perfect Substitutes*

This case is probably unlikely, but it is of interest to consider. If  $M$  and  $m$  are perfect substitutes, then the expected one period change in  $e$ ,  $\rho$ , must be zero. The model is thus modified as follows. Two equations drop out: either (4) and (4)' or (5) and (5)'. Two equations are added: the equation  $\rho = 0$ , and an equation explaining  $\rho$ .<sup>10</sup> Finally, the model is underidentified with respect to  $M_h$ ,  $M_h^*$ ,  $m_h$ , and  $m_h^*$ , and so one of these variables must be taken to be exogenous. (Again, see footnote 9.) The model thus consists of the same number of equations as before, with one new endogenous variable added ( $\rho$ ) and one previous endogenous variable taken to be exogenous ( $M_h$ ,  $M_h^*$ ,  $m_h$ , or  $m_h^*$ ).

*Policy Variables of the Governments*

For the government of country 1, the key policy variables are  $L_g$ ,  $X_g$ ,  $B_g$ , and  $T$ . Its other policy variables are  $x_g^*$ ,  $b_g^*$ , and  $m_g^*$ . The same is true for the government of country 2, with capital and lower case letters reversed. The only other exogenous variable in the model is either  $Q$  or  $e$ . Given a set of initial conditions (i.e., a set of lagged values), policy effects can be examined by comparing alternative solution paths of the endogenous variables, where the alternative solution paths are based on different assumptions about the paths of the policy variables.

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<sup>10</sup> See footnote 8.

### III. Extensions and Alterations of the Basic Model

#### *Disequilibrium Effects*

Although the model in Section II is based on the assumption that the labor, goods, and financial markets are in equilibrium, it can be modified to incorporate disequilibrium effects. One possible modification with respect to the labor and goods markets, which is in the spirit of the model in [5], is as follows. Consider country 1. If  $W$  and  $P$  are not determined by the requirement that the labor and goods markets be in equilibrium, then some mechanism for their determination must be specified. Assume, therefore, that the firm sector jointly determines  $W$  and  $P$  along with its other decision variables, so that two new equations are added to the model:

$$(48) \quad W = f_{48}(\dots),$$

$$(49) \quad P = f_{49}(\dots).$$

If it is possible for firms to make expectation errors, as is assumed in [5], then the values of  $W$  and  $P$  may not be equilibrium values, so that equations (39) and (41) may not hold. Some modification of the model must thus be made to account for the case in which the values of  $W$  and  $P$  are not equilibrium values.

Consider first equation (12), which in the equilibrium case represents the firm sector's demand for labor,  $L_f$ . It will now be assumed that equation (12) represents the maximum amount of labor that the firm sector will employ in the period. The maximum amount that the household sector can work is thus  $L_f + L_g$ . If it is further assumed that the firm and government sectors make their decisions regarding  $L_f$  and  $L_g$  before



the household sector makes its decisions, then the household sector can be assumed to take this possible labor constraint into account in making its decisions. Equations (1)-(7) can thus be taken to represent the household sector's decisions that incorporate the possible labor constraint, so that  $L_h$  in (1) is always less than or equal to  $L_f + L_g$ .

Consider now how the firm sector adjusts to a disequilibrium situation. If  $L_h$  is strictly less than  $L_f + L_g$ , then the firm sector is assumed to get only the amount  $L_h - L_g$  of labor in the period. Call this amount  $L'_f$ . In the case in which  $L'_f < L_f$ , the firm sector is assumed to change its decision regarding the net supply of the good,  $X_f$ , (but not regarding  $W$  and  $P$ ) and so equation (13) should now be interpreted as reflecting this possible change. The firm sector is also assumed to hold an inventory of the good,  $I$ . By definition

$$(50) \quad I - I_{-1} = X_f - X'_f,$$

where  $X_f$  is, as before, production and where  $X'_f$  denotes sales. Any difference between production and sales in a period results in a change in inventories, and the firm sector is assumed to adjust over time to an undesired level of inventories by changing production relative to sales.

The model in the disequilibrium case is thus as follows. Three new equations are added, (48) - (50), and three new endogenous variables are introduced,  $L'_f$ ,  $X'_f$ , and  $I$ . Also,  $L'_f$  should replace  $L_f$  in (39), where the equation is now interpreted as determining the actual amount of labor that the firm sector gets in the period. This amount may be less than the maximum amount demanded,  $L_f$ . Likewise,  $X'_f$  should replace  $X_f$  in (41), where the equation is now interpreted as determining the actual sales of the firm sector. In addition,  $L'_f$  and

$X'_f$  should replace  $L_f$  and  $X_f$ , respectively, in (15) and (16). Finally, it should be noted that  $I_{-1}$  is now included among the lagged variables that affect the firm sector's decisions, that  $X_f$  in (13) reflects the possible constraint  $L'_f < L_f$ , and that the household sector's decisions as represented by equations (1)-(7) reflect the possible labor constraint on it. Similar modifications can be made for country 2.

After the introduction of a bank sector into the model, as will be done next, one could introduce the possibility of disequilibrium in the financial market. Banks may at times constrain firms and households in how much they can borrow at the current loan rate. This issue, however, will not be pursued here. The interested reader is referred to the model in [5], where possible disequilibrium effects in the labor, goods, and financial markets are considered together.

#### *A Bank Sector*

A bank sector is easy to add to the model. Assume for simplicity that the bank sector in each country hires no labor, buys no goods, pays no taxes, and holds no foreign bonds and money. Assume also that there is no currency in the system, and let  $M$  and  $m$  now denote demand deposits. Consider country 1. Let  $M_b$  denote the value of demand deposits of the bank sector, and let  $B_b$  denote the bank sector's net demand for the bond of country 1.  $M_b$  replaces  $M_g$  in equation (43). Also, the government is assumed to hold no demand deposits, so that  $M_g$  is dropped from the model. Let  $BR$  denote bank reserves,  $BO$  bank borrowing from the government,  $RD$  the discount rate, and  $RR$  the reserve requirement ratio. Bank borrowing is assumed to be a function of  $R$  and  $RD$  :

$$(51) \quad BO = f_{51}(R, RD) ,$$

and the bank sector is assumed to hold no excess reserves:

$$(52) \quad BR = RR \cdot M_b .$$

The financial saving of the bank sector ( $S_b$ ) is

$$(53) \quad S_b = R \cdot B_b - RD \cdot BO ,$$

and its budget constraint is

$$(54) \quad 0 = S_b - \Delta B_b + \Delta M_b - \Delta(BR - BO) .$$

The main characteristic of the bank sector is that it takes in deposits ( $M_b$ ) and makes loans ( $B_b$ ). Equation (53) states that its saving equals the difference between the interest revenue on its loans and the interest payments to the government on its borrowing. Equation (54) states that the change in bank loans plus unborrowed reserves ( $\Delta B_b + \Delta(BR - BO)$ ) equals saving plus the change in deposits ( $S_b + \Delta M_b$ ).

The model with a bank sector is thus as follows. Four new equations are added, (51)-(54); five new endogenous variables are introduced,  $M_b$ ,  $BO$ ,  $BR$ ,  $S_b$ , and  $B_b$ ; one endogenous variable is dropped,  $M_g$ ; and two new exogenous variables are added,  $RD$  and  $RR$ . In addition,  $M_b$  should replace  $M_g$  in (43);  $\Delta(BR - BO)$  should replace  $\Delta M_g$  in the government budget constraint (19);  $RD \cdot BO$  should be added to the government saving equation (18); and  $B_b$  should be added to (45). Similar modifications can be made for country 2.

The only two behavioral or decision equations for the bank sector are (51) and (52), although an equation explaining  $R$  could be added if

the possibility of disequilibrium in the financial market were considered. Otherwise, the introduction of a bank sector makes little difference in the model. With respect to the government, it now has control over two more policy variables,  $RD$  and  $RR$ , and in the government budget constraint the change in the money supply is replaced by the change in non-borrowed reserves.

#### *Other Possible Extensions*

There are a number of other additions that could be made to the model without changing its basic structure. Bonds with a maturity longer than one period could be introduced. This would require keeping track of the capital gains and losses on the bonds and of the possible effects of these gains and losses on behavior. The firm sector could be assumed to hold the bond of the other country and both monies without changing the model's basic structure. Likewise, a more detailed specification with respect to the firm sector's investment and production decisions could be made without a basic change in structure. Finally,  $Q$  and  $q$  (and equation (47)) could be dropped from the model if the currency of one of the countries were taken to be the international reserve currency.

#### *Model A*

As mentioned in the Introduction, Model A is a version of the model presented in this paper, a version in which the effects of policy actions can be analyzed by means of simulation techniques. It has a bank sector, and it accounts for possible disequilibrium effects in the labor, goods, and financial markets. It also has a more detailed specification of the firm sector than is outlined in this paper, where, among other things, the possibility that firms may at times hold excess labor and excess capital

is considered. Some capital gains and losses are also accounted for. The model is half empirical in the sense that half of it is an actual econometric model of the U.S. economy, but the complete model is not. The analysis in [8] of the effects of policy actions in the model is not an empirical analysis; it is merely an examination of policy effects in one version of the theoretical model outlined in this paper.

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