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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 27

BEYOND GENERAL EQUILIBRIUM

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 27

BEYOND GENERAL EQUILIBRIUM*

by

Martin Shubik

1. MICROECONOMICS AND THE ACHIEVEMENTS OF GENERAL EQUILIBRIUM THEORY

1.1. The Walrasian System

The great insights of Leon Walras[1] have led, during the course of the twentieth century to date, to the development of a precise and mathematically rigorous elegant formulation of the role of a price system in an ownership economy with production and exchange.

The pioneering work of Hicks[2] and Samuelson[3] further developed by Arrow and Debreu[4] and Debreu[5] brought about a new style of thinking

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1 L. WALRAS, Eléments d'économie politique pure, Lausanne, Corbaz, 1874. (Translated as Elements of Pure Economics by W. Jaffe, London, Allen and Unwin, 1954).

2 J. R. HICKS, Value and Capital, Oxford, 1939.

3 P. A. SAMUELSON, Foundations of Economic Analysis, Cambridge, 1948.

4 K. J. ARROW and G. DEBREU, "Existence of an Equilibrium for a Competitive Economy," Econometrica, 22, 1954, pp. 265-290.

5 G. DEBREU, Theory of Value, 1959, New York, John Wiley and Sons.

and greater levels of explicitness and care in the modelling of the economic phenomenon of production and exchange. The brief and concise book of Debreu provides an example of precision and clarity in theorizing unexcelled in any of the social sciences.

The rigorous examination of the conditions under which a price system exists and leads to a distribution of goods and resources which is Pareto optimal has given economists considerable insight into the organization of production and exchange.

There is still a great amount of work to be done using general equilibrium theory as the point of departure. The recent book of Arrow and Hahn serves as an example of progress.[6] The computational methods of Scarf and Hansen[7] provide a powerful method for calculating price systems. Paradoxically this work can be viewed as a signal contribution not only to mathematical economics, but to central planning as it offers a way in which a centrally controlled economy could go about setting prices which are not generated by a market mechanism.

The way is now clear to start to investigate the effect of transactions costs and indivisibilities in a closed static economic system.[8,9] Attempts have been made to include the role of taxation.[10]

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6 K. J. ARROW and F. H. HAHN, General Competitive Analysis, Holden-Day, 1971.

7 H. SCARF with T. HANSEN, The Computation of Economic Equilibrium, New Haven, Yale University Press, 1973.

8 F. H. HAHN, "Equilibrium with Transactions Costs," Econometrica, 39, 1971, pp. 417-440.

9 D. K. FOLEY, "Economic Equilibrium with Costly Marketing," Journal of Economic Theory, 2, 1970, pp. 276-291.

10 R. R. MANTEL, "General Equilibrium and Optimal Taxes," Journal of Mathematical Economics, 2, 1975, pp. 187-200.

1.2. The Core

A different approach to the emergence of the price system has come about through cooperative game theory. In particular in the last twenty years there has been considerable development of the theory of the core, a concept introduced into game theory by Gillies[11] and Shapley.[12] Shubik[13] observed the relationship between the work of Edgeworth[14] on recontracting and the core and using the method of market replication showed for a relatively limited case the linkage between the core and the competitive equilibrium for replicated economics. He conjectured that this relationship would hold generally for economies without sidepayments and suggested this to Scarf, Scarf[15] and Debreu and Scarf[16] proved the general result. Since then, Aumann[17, 18] extended the analysis to markets with a continuum of traders; Brown and Robinson extended the analysis of the core to nonstandard exchange economies.[19] Many of the further contributions

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11 D. B. GILLIES, "Solutions to General Nonzero Sum Games," Annals of Mathematics Study, 40, 1959, pp. 47-85.

12 L. S. SHAPLEY, Reported at the first Princeton conference on n-person Game Theory, 1953.

13 M. SHUBIK, "Edgeworth Market Games," in Contributions to the Theory of Games, IV, Annals of Mathematical Studies, 40, R. D. Luce and A. W. Tucker, eds., Princeton, Princeton University Press, pp. 267-278, 1959.

14 F. Y. EDGEWORTH, Mathematical Psychics, London, Kegan Paul, 1881.

15 H. SCARF, "An Analysis of Markets with a Large Number of Participants," in Recent Advances in Game Theory, Conference proceedings, 1962, Princeton pp. 127-155.

16 G. DEBREU and H. SCARF, "A Limit Theorem on the Core of an Economy," International Economic Review, 4, 1963, pp. 235-246.

17 R. J. AUMANN, "Markets with a Continuum of Traders," Econometrica, 32, 1964, pp. 39-50.

18 R. J. AUMANN, "Existence of Competitive Equilibria in Markets with a Continuum of Traders," Econometrica, 34, 1966, pp. 1-17.

19 D. J. BROWN and A. ROBINSON, "Non-Standard Exchange Economies," Econometrica, 43, 1, 1975, pp. 41-55.

have been excellently covered by Hildenbrand[20] in a recent book on the core and equilibria.

The work on the core and competitive equilibrium shows that the competitive equilibrium has a combinatoric or coalitional stability property that is different from the type of stability tested for in economic analogues to the stability of a dynamic physical system. The virtual displacement from the core must be examined in terms of its effect on all coalitions not on the behavior of isolated individuals.

The core approach to the competitive equilibrium had another important advantage over the development of the Walrasian based mathematical models. In those models, as can be seen by an examination of Debreu's book, the number of competitors in a market plays no role. The existence proofs for the price system are independent of the presence of two or two billion competitors. Yet the verbal discussions of competition and even certain mathematical models of open economic systems in oligopoly theory since the time of Cournot[21] have stressed the importance of the number of competitors as a necessary (but not sufficient) condition for the emergence of an efficient price system.

Starting from an essentially non-monetary barter model of economic exchange with complete information, costless communication and zero transactions costs the cooperative game models of the core led to an analysis in which an efficient price system emerged as a result of the interactions of coalitions among many traders.

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20 W. HILDENBRAND, Core and Equilibria of a Large Economy, Princeton, Princeton University Press, 1974.

21 A. A. COURNOT, Recherches sur les principes mathematiques de la theorie des richesses, Paris, Hachette, 1838. (Translated as Researches into the Mathematical Principles of the Theory of Wealth, New York, MacMillan, 1929.)

Even though the core analysis brought into the mathematical formulation the explicit role of numbers, it shares with the general equilibrium models many features which make it difficult to reconcile these developments in microeconomic theory with macroeconomics, money, financial and other institutions. In Section 2 these difficulties are discussed. In Section 3 it is argued that an alternative approach to the price system, formulating the economy as a game of strategy and applying the Cournot-Nash[22] concept of a noncooperative equilibrium solution offers a way to reconcile micro and macroeconomics and to develop a theory which includes money, markets and financial and other institutions. The approaches of general equilibrium theory and the core do not appear to be anywhere near as appropriate or fruitful in accomplishing this task as does this alternative approach.

2. A CRITIQUE OF GENERAL EQUILIBRIUM THEORY

2.1. On Macroeconomics

Since Keynes there has been a phenomenal growth in both the applications of macroeconomic thinking to the economics of control of national economies and in the development of macroeconomic theory.[23] These developments have come about in concert with important work on economic and financial statistics as is evinced by the efforts of Kuznets,[24] Friedman

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22 J. F. NASH, Jr., "Equilibrium Points in N-Person Games," Proceedings of the National Academy of Sciences of the U.S.A., 36, 1950, pp. 48-49.

23 See A. LEJONHUFVUD, On Keynesian Economics and the Economics of Keynes, New York, Oxford University Press, 1968, for a comprehensive discussion.

24 S. KUZNETS, National Income and Its Composition 1919-1938, New York, National Bureau of Economic Research, 1941.

and Schwartz,[25] Goldsmith[26] and many others.

The models used in macroeconomics are basically dynamic, with many different economic agents and institutions. Money plays a central role. Aggregates are important. Not only is much of macroeconomics institutionally oriented, the rules of motion postulated are frequently behavioral.

In spite of the successes of macroeconomics, there are many competing dynamic models, and the development of theory has not been as rigorous and satisfactory as with general equilibrium theory. Model construction and the selection of behavioral assumptions in macroeconomics appear to be extremely ad hoc, be they associated with the propensity to consume, the Phillip's curve, the reasons for private investment and so forth.

General equilibrium theory may be judged to be "too unrealistic," to be static and unsatisfactory from many other points of view. But at least it is a monument to parsimony and shows how much can be yielded by an extremely small set of well chosen assumptions concerning economic behavior.

An attempt to bridge the gap between the view of the economy presented by macroeconomics and the general equilibrium theory involves making an essentially static microeconomic theory potentially, if not actually dynamic (see 3). It involves maintaining parsimony in our selection of basic assumptions, yet it calls for the appearance of the multitude of differentiated institutions that exist in our economies. The step beyond

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25 M. FRIEDMAN and A. J. SCHWARTZ, A Monetary History of the United States, 1867-1960, Princeton, Princeton University Press, 1963.

26 R. W. GOLDSMITH, The Flow of Capital Funds in the Postwar Economy, New York, Columbia University Press, 1965.

general equilibrium must be a step towards dynamics and towards the understanding of economic process. The economic institutions of society are the carriers of process and they must appear in the theory. This point is developed further in Section 4.

2.2. What is Missing in General Equilibrium Models

The general equilibrium model is:

(1) basically noninstitutional.

(2) It makes use of few differentiated economic actors.

(3) It is essentially static. No explanation of price formation is given.

(4) There is no essential role for money.

(5) It is nonstrategic. Individuals are not even permitted to commit errors.

(6) It cannot accommodate an intermix of oligopolistic and competitive sectors. Equilibrium is independent of the number of traders.

(7) It is an inadequate model for the analysis of nonsymmetric information conditions. In general it is vague about information and communication conditions.

(1) The general equilibrium model does not need banks or other credit or financial institutions, as trust is implicitly perfect. Furthermore trade is so perfect that there is no float. All trade is balanced at the end of the market and each individual at the start of trade has available implicitly a credit line equal to his net worth "at market," i.e. the worth of his initial assets at the final market price.

Firms are not institutions but there are production correspondences available to all who have the resources. It is as though the only items necessary to bake a cake were a recipe (free to all), and the ingredients. The firm as an entity with an internal organization and a management with goals of its own is not included in this abstraction.

(2) In the Debreu model there is really only one major actor--the consumer, and one shadow actor--the producer. The consumer has his own preference ordering and tries to maximize his own welfare. The producer is a shadowy manager of a firm which may be owned by stockholders other than himself. Even though his interests may conflict with others he is modelled as a profit maximizing automaton who flows through the profits to a nonvoting[27] group of stockholders.

In macroeconomic models and in economic life we frequently distinguish investors, savers, speculators, brokers, bankers, consumers, manufacturers, retailers and others. It is important to ask at what level of abstraction do or should these distinctions appear.

(3) The general equilibrium model is essentially static in the sense that time is handled by merely enlarging the number of variables and renormalizing.[28] Recent work on sequences of economies[29] and the infinite horizon[30] have modified this. Even so in the work of Arrow and

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27 It is hard to give much meaning to stockholder voting in the general equilibrium model without specifying a great amount of corporate law and running the risk of losing the price system if the laws are not appropriate.

28 This stress on a static formulation has made the task of understanding causality in microeconomic models extremely difficult. A valuable discussion of this is given by H. O. WOLD, "Mergers of Economics and Philosophy of Science," Synthese, 20, 1969, pp. 427-482.

29 J. R. GREEN, "Temporary General Equilibrium in a Sequential Trading Model with Spot and Futures Transactions," Econometrica, 41, 1973.

30 J. J. M. EVERS, "Invariant Competitive Equilibrium in a Dynamic Economy with Negotiable Shares," Tilburg Research Memorandum 56, 1975.

Debreu[31] and Debreu[32] no explicit mechanism for the formation of price is given, i.e., no formal markets exist. It is not surprising that at this level of theorizing one cannot distinguish a competitive economy with prices arising from competitive behavior from a socialist economy with controlled prices. The difference between them is in the mechanism of price formation; but in this theory the mechanism is left out.

(4) It is clear that in any complex economy money plays a far more important and subtle role than merely acting as a unit of account. Yet there is no role for it in the general equilibrium model. In the past few years there have been considerable efforts made by Hahn,[33] Starr,[34] Granmont and Younes,[35] and many others, to introduce money into a modified general equilibrium model. It is my belief that although considerable progress has been made that this work is hampered by trying to stay too close to the original general equilibrium formulation.

(5) The general equilibrium theory is nonstrategic. Prices appear by magic and the individual consumer is constrained to maximize his welfare given these prices. Regardless of his strategic influence on the market he is constrained to ignore it. It is an assumption of the general equi-

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31 ARROW and DEBREU, op.cit.

32 DEBREU, op.cit.

33 F. H. HAHN, "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy," in The Theory of Interest Rates, F. H. Hahn and F. Brechling, eds., New York, MacMillan, 1965.

34 R. STARR, "The Price of Money in a Pure Exchange Economy with Taxation," Econometrica, 42, 1974.

35 J. M. GRANMONT and Y. YOUNES, "On the Role of Money and the Existence of a Monetary Equilibrium," Review of Economic Studies, 39, 1972.

librium theory that the individual maximizes his welfare as though prices were given. This is a deduction from the theory when the system is modelled as a game of strategy and solved for its noncooperative equilibria.

(6) Because the general equilibrium model is examined for the existence of an efficient price system that is independent of the number of traders, it does not offer a means for studying the effect of thin markets and few competitors. Such a distinction calls for showing how the presence of different numbers of competitors influences the formation of price.

(7) The general equilibrium model not being fully formalized in extensive form is somewhat vague as to the information and communication conditions present in the economy. One interpretation that is consistent with the theory is that all individuals trade simultaneously in ignorance of each others actions. They know only the following information:

(a) their own endowments for periods $t = 1, \ldots, T$;

(b) all prices from periods $t = 1, \ldots, T$;

and (c) the size of all dividends to be received in all periods.

Given a completely formulated n-person game, even with each individual constrained to making one move, the size of the partition of information sets becomes enormous. Each configuration represents a somewhat different economy. The general equilibrium model appears, at most to cover only symmetric information conditions.

Although Arrow[36] and Debreu[37] have suggested how to extend the

36 K. J. ARROW, "Généralisation des théories de l'équilibre économique général et du rendement social au cas du risque," Econometrie, Paris, CNRS, 1953, pp. 81-120.

37 DEBREU, op.cit.

general equilibrium analysis to handle the case of exogenous uncertainty (by expanding the number of the goods, by defining as different goods, each good in each contingency), this construct handles only symmetric information conditions.

It is important to note the difference between symmetric information concerning the moves of other economic agents and information concerning the moves of Nature. Figure 1a shows the extensive form of a two person game where the two agents P_1 and P_2 each move without

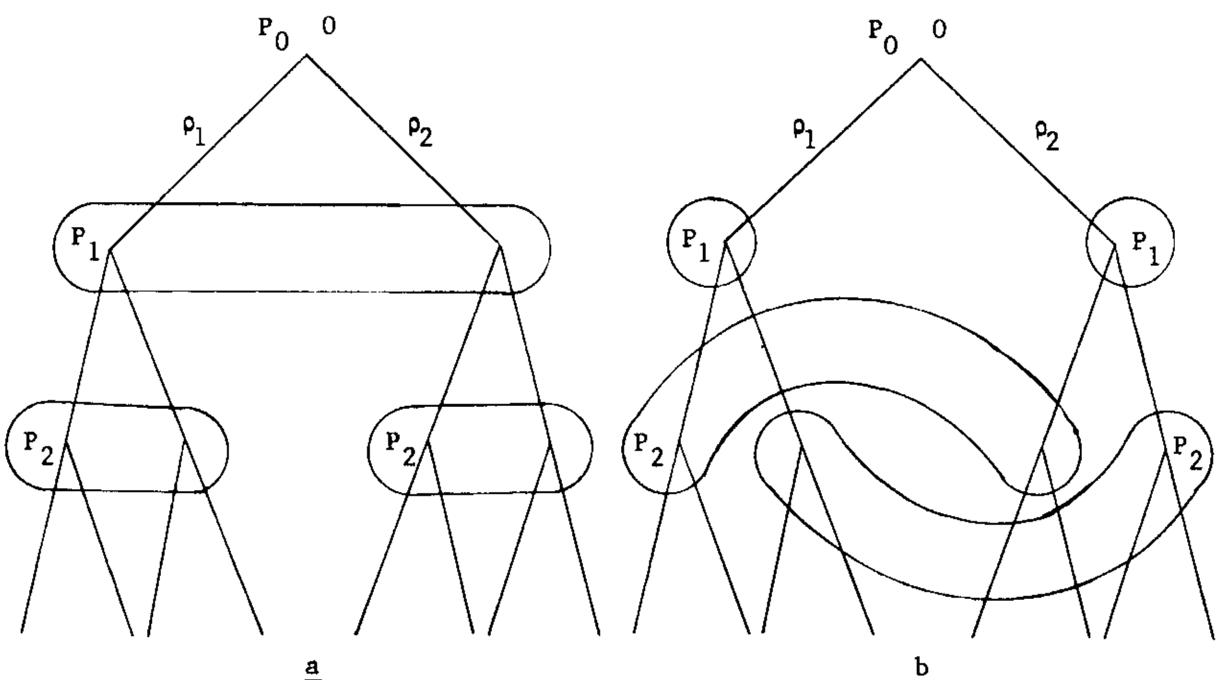

FIGURE 1

knowledge of what the other has done. However P_2 has been informed of Nature's move (Nature is P_0) at vertex 0 , whereas all that P_2 knows are the probabilities ρ_1 and ρ_2 of Nature's choice, not the actual choice.[38]

38 The closed curves encircling the various vertices are information sets. A better title might be "lack of information" sets because any vertex that lies in an information set cannot be distinguished by the player who "owns" that set, from any other vertex in the same set.

In Figure 1b P_1 is informed of Nature's choice; P_2 is informed of P_1's choice, but not of Nature's choice. Full symmetry of information is shown in Figures 2a and 2b. In 2a both P_1 and P_2 know Nature's move in advance, but not each other's move. In 2b neither P_1 nor P_2 know of Nature's move nor each other's move.

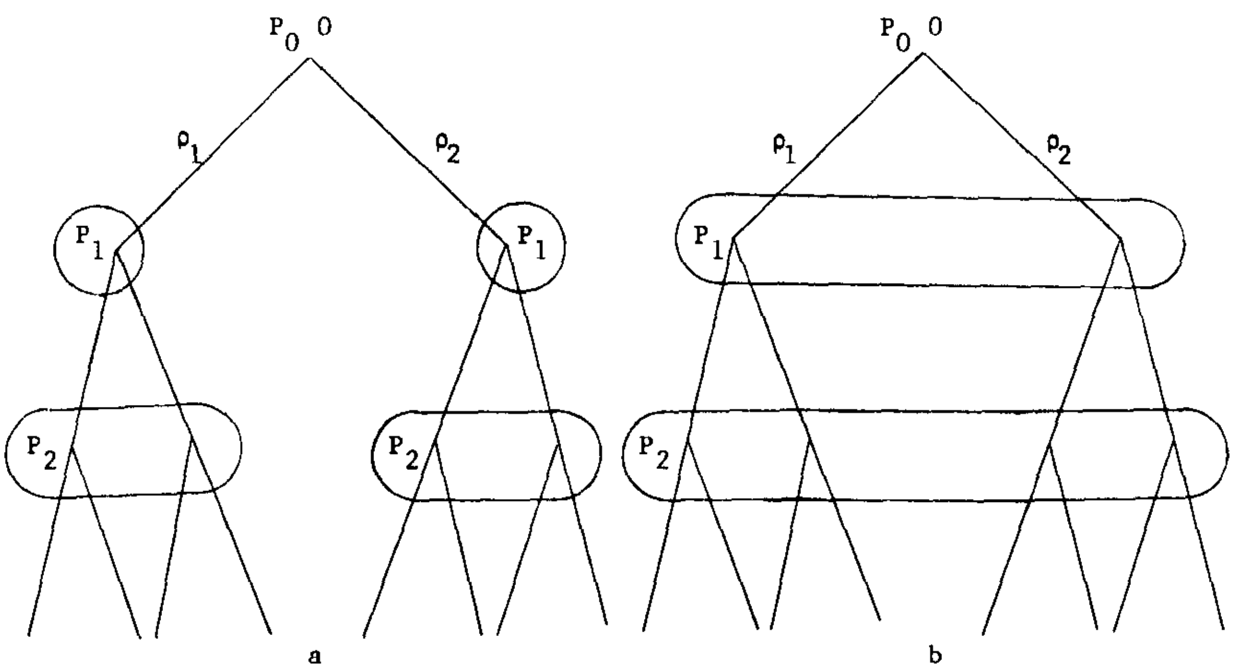

FIGURE 2

Radner[39] has discussed the difficulties in handling information of the type shown in Figure 1a. The Arrow, Debreu approach covers the cases shown in Figures 2a and 2b. Dubey and Shubik[40] have constructed

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39 R. RADNER, "Competitive Equilibrium under Uncertainty," Econometrica, 36, 1968, pp. 31-58.

40 P. DUBEY and M. SHUBIK, "Trade and Prices in a Closed Economy with Exogenous Uncertainty, Different Levels of Information, Money and Special Futures Markets," CFDP 410(Revised), January 1976.

and analyzed a noncooperative game model of a closed trading economy for all cases.

2.3. What Can Be Said Constructively?

In Section 1 an indication was given of the great power of the general equilibrium theory. As a tool for studying the efficiency, distribution and production properties of an economy using a price system it is of key importance. When added features such as the indivisibility of economic units are considered it is clear that much remains to be done. However it is argued in the remainder of this paper that a somewhat different approach is called for if we wish to take steps towards the development of a theory of economic dynamics with money and financial institutions.

There appears to be three stages of work required in the development of a microeconomic dynamics with money and financial institutions. The first is the construction of fully defined process models. The second is the analysis of these models for the appropriate static or stationary solutions to see if these are consistent with general equilibrium theory, and if they are not, to examine why not. The third task is to develop satisfactory dynamic solution concepts.

We are in a position to do the first two. They are preliminary to our being able to develop a satisfactory dynamics. A premature attempt to pass over detailed process modelling is an invitation to failing to understand how and why economic institutions and information processing play a critical role in the shaping of economic dynamics.

3. A DYNAMIC MODEL AND STATIC SOLUTION

3.1. Questions, Models and Solutions

The type of question being asked will influence the model that is built. The solution concept selected will depend on the question to be answered. The different relationships among questions, models and solutions is illustrated in the Theory of Games where there are three fundamentally different types of model associated with many different solution concepts. These are illustrated in Table 1.

Model Type	Solutions	Questions
Extensive form	Behavioral, Bayesian, perfect equilibrium	Information conditions, details on moves, dynamics
Strategic form	Maxmin, noncooperative equilibrium	Existence of equilibria and individually optimal strategies under conditions of no direct communication
Coalitional form*	Cooperative solutions: core, value, bargaining set, stable set, kernel, nucleolus	Distribution given the employment of jointly optimal strategies; fair division, power

*As the remainder of this paper does not involve the coalitional form it is not discussed further. References are given in R. D. LUCE and H. RAIFFA, Games and Decisions, New York, Wiley, 1957 and L. S. SHAPLEY and M. SHUBIK, "Competition, Welfare and Theory of Games" (in process), RAND R-904-NSF/6, 1973, for the reader desiring more information.

TABLE 1

The extensive form (already illustrated in Figures 1 and 2) is a natural model for the consideration of the details of process. The eventual development of satisfactory dynamic solution theories will depend on it.

The strategic form (most frequently exemplified by matrix games) supresses much detail, but nevertheless can be regarded as a highly coded process model. This is because it contains instructions as to how to calculate every feasible outcome in terms of the strategies of traders. When a solution concept such as the noncooperative equilibrium is applied to a game in strategic form, attention is given to a static outcome although the model is implicitly dynamic.

We may regard the noncooperative equilibrium solution to a game in strategic form as a static solution applied to a model which is nevertheless a process model, because it contains specific information on the mechanism which attains all states in equilibrium or disequilibrium.

3.2. The General Equilibrium Model is not a Process Model

Suppose we assume that in an economy with trade, a set of prices appears exogenously. Furthermore assume that each individual is constrained to maximize his welfare on the assumption that he has no influence on prices whatsoever and that his budget constraint is given by the worth of his initial assets at the prices announced.

If we wish to answer the question: "Does there exist a set of prices which, if announced would balance all supply and demand arising from the constrained maximization of the traders?" we do not need a fully defined process model.

Given a set of prices for which there exists excess supply or demand we can say immediately that those are not the prices we were looking for, and go on to examine another set. The question being asked does not make it necessary to describe who is rationed and how they are rationed if the wrong set of prices are announced.[41]

3.3. The Noncooperative Equilibrium Solution Requires a Process Model

In order to model a closed economic system as a well defined game of strategy all rules concerning price formation, credit, market clearance and so forth must be made explicit.

It is clear that there are many ways in which detailed market process models can be constructed. In particular we could model a price controlled economy where a distinguished individual or mechanism controls all prices. In contrast we could assume that market prices emerge as a result of the actions of all individuals.

It is in the details of this process modelling that the institutions of an economy appear. The specification of price formation results in mathematical constructs which can be identified with market institutions. The specification of the nature of the bids of the traders leads naturally into considering the role of money and credit. The specification of credit conditions leads to a consideration of banking, insurance, bankruptcy, bank failure and insurance reserves. In short, markets, money and financial institutions emerge as necessary conditions in the defining of a process model.

Given the model we may apply the noncooperative equilibrium solution

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41 The one attempt by DEBREU, "A Social Existence Theorem," Proceedings of the National Academy of the United States, 38, 1952, pp. 886-893 (see also ARROW and DEBREU, op.cit.) to completely formulate an economic model of trade and production as a process model resulted in a formulation with a central price agency carrying stocks to correct pricing errors. For a further critique of this model see M. SHUBIK, "Competitive and Controlled Price Economies: The Arrow-Debreu Model Revisited," CFDP 337, April 1972 (to appear in Schwodiauer, G. (ed.), Equilibrium and Disequilibrium in Economic Theory, forthcoming, Vienna, 1976).

and examine the conditions required for a noncooperative equilibrium to coincide with the competitive equilibrium. When this coincidence takes place, prices and the distribution of goods are the same, and the apparent need for all of the superstructure of markets, financial institutions and a trading technology disappears. The superstructure is most clearly visible at points of disequilibrium. Financial institutions and markets are a vital part of the dynamic guidance and control system of the economy.

3.4. Two Models and Many Solutions

It is suggested here that the great success of the general equilibrium theory in answering questions concerning the existence of efficient prices and the production and distribution of goods was aided by the parsimonious abstraction in modelling. A static nonstrategic solution concept was explored and the minimal modelling requirements called for a static or nonprocess model of the economy.

It is possible to build a process model without becoming fully involved in dynamics. This is exemplified in the use of the noncooperative equilibrium solution. Even though this solution concept is static the modelling requirements call for a process model of the economy in which the institutions emerge of necessity.

A full reconciliation of micro- and macro-economics calls for process models and dynamic solution concepts. We are in a position to build the process models, but our knowledge of satisfactory dynamic solutions is highly limited. Macroeconomic writings and the behavioral theory of the firm have many suggestions for the behavior of economic units, but they are for the most part, ad hoc and there is no generally accepted theory of economic dynamics.

Table 2 summarizes and compares the basic aspects of the general equilibrium and noncooperative equilibrium microeconomic theories and macroeconomic behavioral theories.

	General Equilibrium	Noncooperative equilibrium	Macroeconomic and other behavioral theories
Model	static	process or dynamic	process or dynamic
Solution	nonstrategic	static strategic	many *ad hoc* dynamic rules of behavior
Comments	noninstitutional	markets and financial institutions present	markets and financial institutions present

TABLE 2

4. MONEY GAMES AND NONCOOPERATIVE EQUILIBRIUM SOLUTIONS

In this section a specific example of exchange[42] modelled as a game and solved for a noncooperative equilibrium is given and contrasted with the general equilibrium model and solution.

4.1. Market Models and Trade in Money

Given n traders, trading in m commodities each trader i with an initial endowment of $(A_1^i, A_2^i, \ldots, A_m^i)$ where $A_j^i \geq 0$ for $i = 1, \ldots, n$ and $j = 1, \ldots, m$ and at least one $A_j^i > 0$ for each j. Suppose that

42 The modelling of an economy with trade and production poses some extra difficulties concerning information and the sequencing of moves. For example it may be unreasonable to assume that a stockholder knows what his dividends will be before a firm has produced and sold its produce.

each trader i has a preference ordering over all bundles of goods which can be represented by a concave utility function φ_i. We know from general equilibrium theory that under relatively general conditions there will exist at least one vector of prices $(p_1, p_2, \ldots, p_m)$ such that if each individual i maximizes $\varphi_i(x_1^i, x_2^i, \ldots, x_m^i)$ subject to $\sum_{j=1}^{m} p_j(x_j^i - A_j^i) = 0$ then for all j, $\sum_{j=1}^{n} A_j^i = \sum_{j=1}^{n} x_j^i$.

In order to model this economy as a game of strategy we must specify how price is formed. There are obviously many ways, but by making a few reasonable assumptions concerning trading technology and markets the methods can be limited.

1. Trade is in money.
2. Bids and offers are simultaneous.
3. The market mechanism selects a single price for each commodity.

Trade in money is introduced as an axiom. A money is defined as a utilitarian or fiat item which is used by all traders in the purchase of all commodities. If the money is a commodity with intrinsic worth beyond its role in exchange it will be included in the utility functions of the traders. Otherwise if it is merely a fiat money it will not appear in the utility functions. More details and discussion on the introduction of a money are given elsewhere.[43, 44, 45]

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43 M. SHUBIK and W. WHITT, "Fiat Money in an Economy with One Nondurable Good and No Credit," in Topics in Differential Games, A. Blaquiere (ed.), Amsterdam, North-Holland, 1973.

44 M. SHUBIK, "Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," Western Economic Journal, 10, 1972, pp. 24-38.

45 M. SHUBIK, "On the Number of Types of Markets with Trade in Money," CFDP 416, January 1976.

It is extremely important to be explicit about the information conditions as the number of strategies for any individual proliferate exponentially as a function of the amount of information he has available. The simplest situation is where all traders move without knowledge of what the others have done. This is a reasonable accurate model of a mass market such as a stock market; but is a less faithful representation of a mass retail market where the retailers usually announce prices on goods for sale before customers act.

If we limit ourselves to simultaneous bids then it can be shown that there are only a few types of bids which give rise to reasonable market mechanisms. If we require that the market mechanism generate a single price for each commodity then, as has been argued elsewhere[46] there are only around four basically different market models. An example of one of them is now given.

Suppose that the m^{th} commodity is used as a money. Then a strategy by an individual i is a vector of $2(m-1)$ components of the form $(b_1^i, q_1^i, b_2^i, q_2^i, \ldots, b_{m-1}^i, q_{m-1}^i)$ where b_j^i is the amount of money he is willing to spend on buying the j^{th} commodity and q_j^i is the amount of the j^{th} commodity that he offers for sale. We require the conditions:

$$b_j^i \geq 0 \ , \quad \sum_{j=1}^{m-1} b_j^i \leq A_m^i \quad \text{and} \quad 0 \leq q_j^i \leq A_j^i \quad \text{for} \quad j = 1, \ldots, m-1 \ .$$

We might expect that an individual who is willing to sell a quantity of commodity j will not also wish to buy it. This might be deduced as a property of a solution rather than be assumed.

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46 Ibid.

In any market j there will be $q_j = \sum_{i=1}^{n} q_j^i$ units for sale and $b_j = \sum_{i=1}^{n} b_j^i$ is the amount of money bid for them. We define price as $p_j = b_j/q_j$ and $p_j = 0$ if $b_j = q_j = 0$. The amount of good j obtained by individual i is $x_j^i = b_j^i/p_j$ for $j = 1, \ldots, m-1$. The amount of the m^{th} commodity, the money, that trader i will have is defined residually as:

$$x_m^i = A_m^i - \sum_{j=1}^{m-1} p_j x_j^i + \sum_{j=1}^{m-1} p_j q_j^i .$$

Shapley and Shubik[47] have analyzed a market model somewhat different to this one and have shown that if there are enough traders of each type and if each has "enough money" there will be a noncooperative equilibrium point arbitrarily close to any competitive equilibrium. The concept of "enough money" can be made precise and this is done elsewhere.[48] Intuitively however we can illustrate what it means. If an individual at a noncooperative equilibrium has his bids actively constrained by his cash limitations his ability to make optimal purchases is constrained. He would be better off if he had credit facilities available so that he could borrow to finance his purchases and pay back at the end of the period. If everyone had enough money optimality could be achieved without credit.

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47 L. S. SHAPLEY and M. SHUBIK, "Trade Using One Commodity as a Means of Payment," RAND, forthcoming 1976.

48 DUBEY and SHUBIK, op.cit.

4.2. Credit, Banking and Bankruptcy

If, in order to alleviate the cash constraint credit must be granted, a new host of modelling problems are faced. Is the credit granting agency a player or a mechanism outside of the model, i.e. is it an inside or an outside bank? If it is outside what are its credit granting rules, what is its information state and what happens at the end of the period if a borrower is unable to pay back a loan? If the bank is an inside bank we must specify the banker's payoff function and his strategies. Conditions forbidding a banker to lend to himself may be necessary if he is actually in a position to create credit.

If an attempt is made to construct a multistage model (two periods will do) a host of new considerations appear. In particular in a one period model there is no real reason for an individual to deposit money at the bank. In a multistage model the relationship between deposits and loans needs to be specified.

Once a bank has been introduced as a mechanism or as a player the way the price of money is set must be specified. If there are not a large number of banks, oligopolistic competition must be expected in the loan market. An example of price (Bertrand) competition as contrasted with credit rationing (Cournot) competition has been given elsewhere.[49]

If a bank is an inside bank and accepts deposits, then if a debtor to the bank fails to pay back his loans it is possible that the bank may be unable to cover its deposits, in which case a bank failure rule is needed.

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49 M. SHUBIK, "A Noncooperative Model of a Closed Economy with Many Traders and Two Bankers," forthcoming 1976, Zeitschrift für Nationalökonomie.

4.3. The Float

In the general equilibrium model the strategic role of money as a decoupling device which enables individuals to bid simultaneously without certain knowledge of prices or incomes, is not present. In the strategic game model it is crucial. Furthermore it is easy to observe that in a pure trade model the float, i.e. the amount of money and credit which is needed to finance trade and is essentially in transit in the system is, for the model in 4.1 $F = \sum_{i=1}^{n} \sum_{j=1}^{m-1} p_j(|A_j^i - x_j^i|)$.

In a single period model the basic use of money and credit is to facilitate trade, to "grease the system" or finance the float. Optimality conditions indicate that a government or outside bank could do that costlessly by issuing fiat at no interest rate. In a multiperiod model money and credit facilitate intertemporal trade and optimality conditions for this may require a rate of interest. This suggests that a distinction between a fiat issue that is interest free and a credit issue that is interest bearing may be relevant. This is discussed in more detail elsewhere.[50, 51]

4.4. Uncertainty, Insurance and Bankruptcy

It is my belief that a full exploration of the role of bankruptcy requires the presence of exogenous uncertainty, trade with money and credit and at least two time periods with a need for intertemporal trade. An

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50 M. SHUBIK, "Fiat Money, Bank Money, the Float and the Money Rate of Interest," CFDP 394, May 1974.

51 M. SHUBIK, "Fiat Money, Bank Money, the Force of the Rate of Interest and the Vanishing Float," CFDP 395, May 1974.

optimal bankruptcy rule in a model with trade in money and credit can be defined in reference to the general equilibrium model of the same economy. A bankruptcy rule will be optimal if there are limit noncooperative equilibria in the (sufficiently replicated) markets arbitrarily close to the competitive equilibria.[52] Further discussion of bankruptcy in the context of these models is given elsewhere.[53, 54]

Given trade in money and credit and the presence of exogenous uncertainty, then a role is created for insurance. In particular given a bankruptcy rule, then the introduction of insurance may be expected to improve credit opportunities.

The ways in which insurance can be introduced are as multiludinous as those for the introduction of banking. A decision must be made between an inside insurance company or one run by the government. These models have not yet been explored.

4.5. Dynamics, Stocks and Other Financial Instruments

The money game models discussed here and contrasted with the general equilibrium models have been for the most part one period models. In these one period models the reasons for the introduction of money, credit, specific markets, the float, bankruptcy rules and some rudimentary aspects of banking have been noted. An examination of trade through time is obviously needed to account for the many instruments and institutions of

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52 This definition is somewhat imprecise as a noncooperative equilibrium in a money game has different dimensions than a competitive equilibrium. We must constrain comparisons to price, distribution of goods and utility.

53 SHUBIK, "Commodity Money, Oligopoly, Credit...," op.cit.

54 SHAPLEY and SHUBIK, "Trade Using One Commodity...," op.cit.

an economy whose control system employs money and financial institutions.

It has been suggested elsewhere[55] that a critical distinction in economic and financial instruments appears as soon as there are two or more time periods to be considered. That is between instruments which are contracts between private "players in the game" and those which are not.

It is conjectured that a full economic dynamics with financial instruments will require eight basic units which are traded and that all instruments can be described as compound constructs of these units. They are:

(1) Services

(2) Goods

(3) Money

(4) Ownership Claims

(5) Service Contracts

(6) Futures Contracts

(7) Debt Contracts

and (8) Ownership Claim Contracts.

The dynamics of the system converts the last four items into the first four or otherwise transforms them into new contracts or wipes them out.

5. MATHEMATICAL INSTITUTIONAL ECONOMICS

The argument in this paper is that the task of bridging the gap between microeconomics and macroeconomics is twofold. It involves the construction of process models and the devising of solution concepts which

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55 M. SHUBIK, "On the Eight Basic Units of a Dynamic Economy Controlled by Financial Institutions," The Review of Income and Wealth, 21, 1975, pp. 183-201.

reflect the dynamics of economic behavior. The first task appears to be a necessary preliminary to the second if one's goals are to obtain consistency between micro and macroeconomics.

It is suggested here that by using the static solution concept of the noncooperative equilibrium, known since the time of Cournot, we are required to build process models even though the solution concept is static. The task of building these process models combined with the test of asking when do certain noncooperative equilibria approach the equilibria postulated by general equilibrium theory leads us to the mathematical invention of markets, financial instruments and institutions.

An appropriate name for this type of modelling is mathematical institutional economics. The institutions of the economy emerge as a necessity in the formulation of the process models. It is likely that many different market mechanisms for mass economies have equilibrium points which are "close" to those of general equilibrium theory. The mechanisms may nevertheless differ when there are few traders. Furthermore it appears that they may manifest different disequilibrium adjustment properties.

The noncooperative equilibrium solution appears to offer a way to construct a theory that is more general than general equilibrium theory. It is nevertheless still essentially static. Given, however that the models to which it is applied are process models, a further task is to consider behavioral solutions applied to these process models and to contrast these with the extension of solutions based upon explicit maximization principles to dynamics.