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KEYNESIAN MODELS OF RECESSION AND DEPRESSION

by

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1. Do Market Adjustments Assure Full Employment?

Keynes's General Theory attempted to prove the existence of equilibrium with involuntary unemployment, and this pretension touched off a long theoretical controversy. Pigou, in particular, argued effectively that there could not be a long run equilibrium with excess supply of labor. The predominant verdict of history is that, as a matter of pure theory, Keynes failed to prove his case.

Very likely Keynes chose the wrong battleground. Equilibrium analysis and comparative statics were the tools to which he naturally turned to express his ideas, but they were probably not the best tools for his purpose. For one thing, he explicitly confined the General Theory to a time period in which are given "the existing skill and quantity of available labor, quality and quantity of available equipment, the existing technique" and other factors. As he said, "... in this place and context, we are not considering or taking into account the effects and consequences of changes in them." But his model produces a solution in which, in general, the stock of capital,

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and other stocks, are not constant. Changes in these stocks will in turn alter investment, saving, and other behavior. For this reason alone, the solution of Keynes's model cannot be stationary, even in its own endogenous variables; and on this ground alone it fails to qualify as an equilibrium. The evolution of Keynesian equilibrium as stocks change is receiving a great deal of attention these days, and I shall not dwell on this point here.

The second important point, the one on which Pigou insisted, is that excess supply of labor must cause money wages to decline. Even if this did not succeed in eliminating unemployment, one might not call a situation in which money wages and prices are persistently falling an equilibrium. But of course Pigou went further in contesting Keynes's claim that a "trap" might exist from which the economy could not be rescued however low the wage and price level.

Keynes tried to make a double argument about wage reduction and employment. One was that wage rates were very slow to decline in the face of excess supply. The other was that even if they declined faster employment would not -- in depression circumstances -- increase. As to the


second point, he was well aware of the dynamic argument that declining money wage rates are unfavorable to aggregate demand. But perhaps he did not insist upon it strongly enough, for the subsequent theoretical argument focused on the statics of alternative stable wage levels.

The real issue is not the existence of a long-run static equilibrium with unemployment, but the possibility of protracted unemployment which the natural adjustments of a market economy remedy very slowly if at all. So what if, within the recherché rules of the contest, Keynes failed to establish an "underemployment equilibrium?" The phenomena he described are better regarded as disequilibrium dynamics. Keynes's comparative statics were an awkward analytical language unequal to the shrewd observations and intuitions he was trying to embody. If the purity of neo-classical equilibrium is preserved, this verdict is no real blow to Keynes or solace for Pigou. The Great Depression is the Great Depression, the notorious "Treasury View" is still ridiculous, whether mass unemployment is a feature of an equilibrium or of a prolonged disequilibrium.

4/...it would be much better that wages should be rigidly fixed and deemed incapable of material changes, than that depression should be accompanied by a gradual downward tendency of money-wages, a further moderate wage reduction being expected to signalise each increase of, say, 1 percent in the amount of unemployment. For example, the effect of an expectation that wages are going to sag by, say, 2 percent in the coming year will be roughly equivalent to the effect of a rise of 2 percent in the amount of interest payable for the same period. The same observations apply mutatis mutandis to the case of a boom." General Theory, p. 265.
The issue is by no means dead. Today "full employment" has become the "natural rate," and "equilibrium" often allows for any steady rate of deflation or inflation, not just zero. But the proposition which Keynes was questioning is once again strongly argued in the profession and in public debate. Once again it is alleged that the private market economy can and will, without aid from government policy, steer itself to full employment equilibrium. This is the basis for advocacy of fixed rules of monetary growth and fiscal policy, as against active discretionary policy responding to information fed back from the private economy. At this very moment it is the basis for a policy of letting the recession run its course, in confidence that in a relatively short run -- two or three years -- equilibrium will be restored at full employment with reduced or even zero inflation.

2. **Keynesian and Marshallian Price Dynamics.**

Milton Friedman has pointed out that Keynes was a "Marshallian in method" and translated the supply-demand framework of Marshall from individual markets to the whole economy. "Where he deviated from Marshall, and it was a momentous deviation, was in reversing the roles assigned to price and quantity. He assumed that, at least for changes in aggregate demand, quantity was the variable that adjusted rapidly, while price was the variable that adjusted slowly,
at least in a downward direction." Friedman is correct that this was a momentous deviation, and one way to appreciate the point is to look explicitly at the dynamic stability implications of Walrasian v. Marshallian assumptions about quantity adjustment.

Marshallian adjustment in a particular market is that quantity adjusts to the difference between demand price and supply price for existing quantity. Walrasian adjustment is that quantity adjusts to the difference between demand and supply at existing price.

Let us now apply these two adjustment assumptions to a simple macro-economic model. Let \( Y \) be aggregate real output, and \( Y^* \) its value at full employment, i.e., at the "natural rate" level of unemployment. Let \( E \) be aggregate real effective demand, which can differ in short run disequilibrium both from \( Y \) and from \( Y^* \). Given the nominal stock of outside money \( M \) and other exogenous or policy-set variables effective demand \( E \) is a function \( E(p, x, Y) \) of three variables: \( p \) the price level, \( x \) its expected rate of change, and \( Y \) the level of output and real income.

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In finer detail, \( E \) is the sum of consumption \( C \), private investment \( I \), and government purchases \( G \):

\[
E = C(Y, Y^*, -T, -R, x \frac{M}{p}, W) + I(Y, Y^*, -K, -R) + G
\]

Here the \( C \) and \( I \) functions have positive derivatives in all their arguments. \( T \) represents taxes, a function of \( Y \) and \( Y^* \). \( W \) is private wealth, equal to \( \frac{M}{p} + qK \), where the coefficient \( q \) is the ratio of market valuation of capital equity to replacement cost. An increase in the real interest rate \( R \) relative to the marginal efficiency of capital makes \( q \) fall, and makes investment fall. The marginal efficiency of capital depends positively on \( Y \) and \( Y^* \), negatively on \( K \). The real interest rate \( R \) depends inversely on both \( M/p \) and \( x \), and rises with \( Y \) and \( W \).

The price level effect \( E_p \) on demand is negative, for the following familiar combination of reasons. First is the Keynes effect. A given nominal quantity of money will be a larger real quantity at a lower price level. Consequently the interest rate may be lower, and investment demand higher. The Keynes effect is expected to be weaker the larger the real supply of money relative to output \( Y \), and to vanish altogether in the "liquidity trap." This will tend to make \( E_p \) smaller in absolute value at low levels of \( \frac{Y_p}{M} \).
Second is the Pigou effect, the wealth effect on consumption. The lower the price level, the higher the real value of those components of net private wealth fixed in nominal value. The relevant components are outside money (and some part of any non-monetary public debt in existence). Consumption demand is expected, ceteris paribus, to respond positively to increases of wealth.

The short-run Pigou effect is very likely weaker than the long-run effect, and may not even have the same negative sign. And it is the short-run effect which is relevant for Keynesian theory and for the dynamics of this paper. The difference arises as follows: Among the stocks fixed in the short run are private debts in the unit of account. These are a heavier burden to debtors the lower the price level, and there are good reasons why transfer of real income and wealth to creditors spells a net deficit of aggregate demand. Debtors are debtors because they have high propensities to spend. Many of them are liquidity-constrained, and as their debt/equity ratios increase their credit lines dwindle or, in case of bankruptcies, disappear. Although these are "only" distributitional effects, they may be more important than the real value of outside money and debt.

The long run comparative-static Pigou effect, in contrast, assumes that each alternative price level has prevailed for a sufficiently long time so that inside debts are scaled to that price level -- although strangely enough exogenous outside money is not. In this counter-historical
"as if" mental experiment, debtors are no more burdened at one price level than another.

As for $E_x$, there are several effects. A decrease in the expected inflation rate raises the real rate of interest. This increase discourages investment, and it also deters consumption both directly and by lowering the market value of equity capital, one component of wealth. On the other hand, expected capital gains on money holdings $x \frac{M}{p}$ are favorable to consumption. This is a "flow Pigou effect," to be distinguished from the stock effect. The question here involves the size of the marginal propensity to spend from expected real capital gains. Econometric evidence has been that this marginal propensity is small, although capital gains eventually affect consumption via the wealth effect. I have assumed that the other effects of expected inflation dominate the flow Pigou effect.

The marginal propensity to spend $E_y$ is taken to lie between 0 and 1 on usual Keynesian grounds. As is well known, a high response of investment demand to contemporaneous income could easily make $E_y$ exceed one. But Keynes typically regarded investment to be determined more by long-run sales and profit expectations than by current business activity. The likelihood that, in prolonged departures from full employment, investment will come to be governed more by contemporaneous than by full employment sales and profits is a source of possible instability and of prolonged disequilibrium to which I shall return later in the paper.
In equilibrium, the following three conditions hold:

(2.1) \( E(p, x, Y) - Y = 0 \)

(2.2) \( Y - Y^* = 0 \)

(2.3) \( x = p/p = 0 \). (I shall also denote \( p/p \) as \( \pi \))

I shall call the first dynamic version of this model the WKP model (Walras-Keynes-Phillips). All the adjustment functions which follow will conform to the notation \( A_y z \), where \( y \) is the variable adjusting, \( z \) the variable on which the adjustment depends, and \( A_y \) is a positive constant.

The WKP model is as follows:

The WKP Model

\[
\begin{align*}
(2.1.1) & \quad \dot{Y} = A_y (E - Y) \\
(2.2.1) & \quad \pi = A_p (Y - Y^*) + x \\
(2.3.1) & \quad x = A_x (\pi - x)
\end{align*}
\]

Equation (2.1.1) says that production \( Y \) moves in response to discrepancies of \( E \) and \( Y \). This implements the Keynesian view that in the very short run money wages and prices are set and output responds to variations of demand.

How can \( E \) and \( Y \) diverge even for an instant? Many words have been spilled, both by Keynes himself and by others, on this question, usually posed in terms of the possibility of inequality of Saving
and Investment. In our present context, let \( D \) be the demand which must always equal \( Y \) to preserve the national income identities. Let \( D \) be a function of \( Y \) as well as of \( x, p, \) and \( Y \). Then

\[
D(Y, x, p, Y) = Y, \quad E(x, p, Y) = D(O, x, p, Y).
\]

Equation (2.1.1) follows from a negative value of \( \partial D/\partial Y \), which means that demand is lower, at given \( Y \), when \( Y \) is increasing. Lags in consumption spending lead to this sign, and so does unintended inventory decumulation. The investment accelerator works in the other direction, but for the reason already given it is not a Keynesian idea.

Equation (2.1.2) is a natural rate version of the Phillips curve. The short-run Phillips curve is the obvious Keynesian version of price dynamics. Throughout this paper I am condensing product and labor markets into one sector, and assuming with Keynes that prices are determined by marginal variable costs, i.e., by labor costs. Excess labor supply and \( Y-Y^* \), the "Okun gap," are linked, -- when one is zero so is the other. So it is the gap which causes wage rates to fall. But to "fall" does not mean to decline absolutely; it means to decline relative to \( x \), the accustomed and expected rate of inflation of both labor costs and prices. This is the more modern wrinkle. By here assuming (2.1.2) I do not mean necessarily to associate myself, -- much less Keynes! -- with the natural rate hypothesis in all its power and glory.
The third equation (2.3.1) is the well known model of adaptive expectations. There is nothing particularly Keynesian about this equation, and the same formulation will carry over to the non-Keynesian dynamic model. Keynes himself would scorn it and stress instead the stochastic and historical sources of expectations. But like so many of his observations, these do not lend themselves to simple formal analysis.

As two extremes of interest I shall wish to consider:

(2.3.2) \( x = \pi \) (extrapolation of current rates of price change)

(2.3.3) \( x = 0 \) (extrapolation of current price level)

The alternative dynamic version may be called the M model (Marshall). The equations are:

\[
\text{The M Model}
\]

(2.1.2) \( \pi = B_p (E-Y) + x \)

(2.2.2) \( Y = B_Y (Y^*-Y) \)

(2.3.1) \( x = A_x (\pi-x) \) (or 2.3.2 or 2.3.3)

As compared with the WKP model, the adjustment roles of the first two equations are interchanged. The first equation now says that the immediate impact of excess demand for goods and services is to raise prices, or more strictly to raise them faster than they had been expected to rise. (It is not entirely accurate to regard (2.1.2) as non-Keynesian. When there is an inflationary gap \( E > Y^*, Y = Y^* \) this looks
very much like the Keynesian model of inflation. But in Keynes's inflation theory, $Y^*$ is considered an absolute short-run constraint on production, as in war-time. In normal conditions, Keynes would, I think, regard $Y^*$ as a medium-run labor market equilibrium with normal margins of excess capacity and of frictional unemployment, a level of output which could be at least temporarily exceeded. ) In any event, equation (2.1.2) is one way to inject into the model the view that prices respond quite flexibly to changes in excess demand for goods, whether or not the economy is close to full employment.

The non-Keynesian partner of this price adjustment equation is (2.2.2), where the "gap" between potential and actual output inspires adjustments of production and employment. This is because they are associated with gaps of the same sign between the demand price for labor (the value of its marginal product) and its supply price. The idea is that when $Y^*$ exceeds $Y$ the real wage is less than marginal productivity. Competitive employers therefore add to their work forces and their production. In Keynesian theory, on the other hand, production increases only when demand at existing prices expands.

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6/. In the General Theory, Keynes discusses frictional and involuntary unemployment on p. 6, and in defining involuntary unemployment on p. 15 says, "Clearly we do not mean by 'involuntary' unemployment the mere existence of an unexhausted capacity to work."

7/. This is true even if the labor supply curve is downward sloping, provided it is closer to vertical than the schedule of marginal productivity of labor.
3. **Local Stability of the Two Models.**

Let us now consider the local stability of the WKP and M models, around their equilibrium values \( Y=Y^*, \ p=p^*, \ x=0 \). For this purpose it is convenient to substitute in the third equation the value of \( \pi-x \) drawn from the second or first equation. Thus the third equations in the WKP and M models become respectively:

\[
\begin{align*}
(3.1) \quad x &= A_x A_p (Y-Y^*) \\
(3.2) \quad x &= A_x B_p (E-Y)
\end{align*}
\]

For the WKP model, the linearized equations are:

\[
\begin{bmatrix}
\dot{Y} \\
\dot{p} \\
\dot{x}
\end{bmatrix}
= \begin{bmatrix}
A_Y (E_Y - 1) & A_Y E_p & A_Y E_x \\
A_p p^* & 0 & p^* \\
A_x A_p & 0 & 0
\end{bmatrix}
\begin{bmatrix}
Y - Y^* \\
p - p^* \\
x
\end{bmatrix}
\]

(3.3)

The critical necessary condition for stability is:

\[
(3.4) \quad \frac{p^* E_p}{p} + A_x E_x < 0
\]

The first term of (3.4) is negative, and the second term positive. As would be expected, a strong negative price level effect on aggregate demand, a weak price expectation effect, and a slow response of price expectations to experience are conducive to stability. In one extreme case (2.3.3), where \( x=x=0 \), the system is of course stable. In the other extreme case (2.3.2), where \( x=\pi \), the first term of (3.4) drops out and the system is necessarily unstable.
The M model is quite different. It is separable into output and price equations. Equation (2.22) is a stable differential equation in the single variable Y. The stability of the price system depends on (3.4), in the same way as the stability of the WKP model. The formal system is:

\[
\begin{bmatrix}
\dot{Y} \\
\dot{p} \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
-B_Y & 0 & 0 \\
B_p (E_Y - 1)p & B_p E & B_p E \\
A_p B_p (E_Y - 1) & A_p E & A_p E
\end{bmatrix}
\begin{bmatrix}
Y - Y^* \\
p - p^* \\
x
\end{bmatrix}
\]

(3.5)

As Friedman surmised, Keynes's choice of adjustment mechanisms is a crucial element of his theory. In particular, the Walras-Keynes-Phillips adjustment model allows the distinct possibility that slumps from full employment will not be automatically remedied by market forces. Keynes could also be interpreted to hold the view that price-level effects \( \frac{E}{p} \) are weak relative to speculative effects \( \frac{E}{x} \). I shall discuss this interpretation further in the next section.

4. Irreversible Recessions and Deep Depressions.

Let us take a more global look at the equilibrium condition \( E=0 \) (2.1). In Figure 1 are shown in \((p,x)\) space several loci along which the condition is met. The slope of such a locus, \( \frac{-E}{p} \), is positive. Each locus is for a given value of \( Y \); a reduction in \( Y \) shifts the locus to the left. In the Figure, the right-most locus is for full employment output \( Y^* \).
Figure 1

$E = Y$ loci

$Y_2 = E(p, x, Y_2)$

$Y_1 = E(p, x, Y_1)$

$Y^* = E(p, x, Y^*)$

$\pi = x + A(p)(Y - Y^*)$

$\frac{\dot{Y}}{A(p)}$

$T_1$

$S$

$O$

$x$

$\pi$

$Y^*$

$Y_1$

$Y_2$

$T_2$

$\frac{\dot{Y}}{A(p)}$
The weakening or vanishing of the "Keynes effect" at low values of
Y and p tends to reduce E, in absolute value. This is re-
flected in the curvature of the loci.

Consider an initial position T₁ at levels of E and Y
short of Y*. Prices begin to decline because Y₁ is less than Y*. To a degree that depends on the speed of adaptation, expectations of
price change become negative. The arrow indicates the direction of
movement. As drawn, the movement is stabilizing, taking the economy
to higher E and Y, toward Y*.

The lower panel of Figure 1 concerns the direction of the
arrow, the relationship of π and x. The horizontal axis matches
in origin and scale that of the upper panel. The lines are parallel
45° lines, for Y*, Y₁, and Y₂, the same output levels as in the
upper diagram. The points S, T₁, T₂, correspond to the similarly
labeled points above. At S, π = x = 0. At T₁, π is negative. So
is x, by an amount proportional to the difference between π and
x, shown horizontally as x/Δx.

Consider instead an initial position T₂ in the two panels. At
T₂ both the slope of the E=Y locus and that of the arrow are steeper.
The reason that the arrow is steeper can be seen in the lower panel:
x
has doubled, but π has more than doubled. The net outcome could
Δx
go either way. The possibility illustrated is that at $T_2$ the locus $E=Y$ is so steep that the movement is destabilizing. The system might be stable for small deviations from its equilibrium but unstable for large shocks. The failure of automatic market processes to restore full employment would be reinforced if large and prolonged recession caused investors to gear their estimates of the marginal efficiency of capital more to current than to equilibrium demand and profitability.

Under these adverse circumstances, and in the absence of countercyclical policy, the economy could slip into a deep depression.

In non-linear non-monetary business cycle models like those of Kalecki, Goodwin, and Hicks, a long depression phase occurs with the economy at a floor. At this floor the capital stock is excessive and gross investment is zero; production is solely to meet minimal private and social consumption requirements, which are independent of income and wealth. The depression phase lasts a long time, while depreciation slowly whittles the capital stock down to the amount needed for floor level production.

It is not part of this paper to provide a model of such a floor. The relevant question is whether deflation will by itself lift the economy from the floor. Will deflation so augment private wealth that consumption rises above its floor level? Clearly this will not happen unless condition (3.4) is met at the depression income level.
But at the floor, $E_x$ is higher than in the normal regime. An increase in the deflation rate $-x$ lowers the value of the capital stock. The physical capital stock declines slowly. But its value -- its real value -- can decline rapidly; when no gross investment is taking place the existing stock will be valued well below replacement cost. At the liquidity trap, the real interest rate is the irreducible nominal rate $\overline{r}$ plus the expected rate of deflation $-x$.

The value of a unit of capital is $(\rho-\delta)/(r-x)$ where $\rho-\delta$ is the marginal productivity of capital net of depreciation. Although the attrition of the stock slowly raises $\rho$, deflation rapidly raises $\overline{r-x}$.

5. **Concluding Remarks.**

God may have made the world so that full employment equilibrium exists and is stable. Perhaps the divine design guarantees that capitalist market economies will never be trapped in depressions with involuntary unemployment, and will never need to depart from fixed no-feedback rules of fiscal and monetary policy. But Keynes had good empirical and theoretical reason to suspect otherwise. He did not establish an underemployment equilibrium. But he did not really need to. Even with stable monetary and fiscal policy, combined with price and wage flexibility, the adjustment mechanisms of the economy may be too weak to eliminate persistent unemployment.