COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

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New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 385

ON DISEQUILIBRIUM ECONOMIC DYNAMICS

PART I

MICROFOUNDATIONS OF WICKSELLIAN DISEQUILIBRIUM DYNAMICS

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December 9, 1974

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PART I

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by

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1. Introduction to Part I and Part II

In this series of papers we want to present simple theoretical frameworks which can be used to analyze, not merely the characteristics of equilibrium, but also those of disequilibrium. Economics of disequilibrium has at least two great precursors--Knut Wicksell and John Maynard Keynes.[1] This series is an attempt to develop short-run macro dynamic theories, which are firmly based upon micro dynamic theories of the firm, and within which one can capture and examine some of their fundamental ideas.

Our point of departure is to banish 'Walras' Demon'--the market auctioneer--from our picture of the economy.[2] In the perfectly competitive

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*This paper is Part I of a series of papers on disequilibrium economic dynamics. This and Part II, "Wicksellian Disequilibrium Dynamics, Say's Law and the End of the Natural Rate Theory of Unemployment" (Cowles Foundation Discussion Paper No. 386), should be read together. Part III, "Microfoundations of Keynesian Disequilibrium Dynamics," and Part IV, "Keynesian Disequilibrium Dynamics and the Theory of Long-Run Phillips Curve," are forthcoming as Cowles Foundation Discussion Papers. Research for this paper was in part supported by grants from the National Science Foundation and the Ford Foundation.

[1]Wicksell [25, 26] and Keynes [10, 11, 12, 13]. We can also add Kalecki [9] to this list of great precursors in disequilibrium economics.

[2]See Arrow's pioneering work [1] on this problem.

world of Walrasian general equilibrium theory, all the price-setting power is delegated to a fictional market auctioneer whose role is to quote a price in each market, to signal it to all the market transactors and to raise it if there is an excess demand and lower it if there is an excess supply until all the transactions in the market are cleared. If this well-told fiction is abandoned, then somebody in the market who is actively participating in actual market transactions must take over the role of the price determination. The so-called _impersonal law of supply and demand_ must be somehow personalized.

The picture of the economy that we shall describe in the following pages as a workable and realistic alternative to the perfectly competitive, Walrasian general equilibrium theory is closely akin to that of the theory of imperfect competition, developed by Piero Sraffa, Joan Robinson, E. H. Chamberlain and Nicholas Kaldor among others in the late twenties and early thirties.[3] Each entrepreneur of the firm in this economy sets the price of his own product and fixes the money wage for his employees, but competes with the other entrepreneurs for the aggregate product demand and the aggregate labor supply. It is an individual firm that takes over the role of the Walrasian market auctioneer.

Our main concern is, however, not to re-work the partial equilibrium analysis of the established theory of imperfect competition, but to renovate its theoretical framework as a microeconomic foundation of disequilibrium economic dynamics. In Part I of this series, we shall

[3] Sraffa [22], Robinson [18], Chamberlain [2], and Kaldor [8]. See also Triffin [24] which attempts to extend the partial equilibrium framework of the imperfect competition theory in the direction of general equilibrium theory.

analyze the dynamic behavior of the firm whose entrepreneur must make sequential decisions upon money wage, labor employment, product price and sale of product under uncertainty in the world of imperfect competition. In this micro dynamic theory, the entrepreneur's current decisions upon product price, money wage, product sale and labor employment are primarily guided by his expectations about the current as well as future business conditions in the product and labor markets, with reference to his past expectations congealed into the goods in process and the existing capital equipment. It will be shown that if his expectations turn out to be unfulfilled in the light of the actual business conditions, then part of the 'price' of his mistakes is borne by his prospective customers in the form of unfilled orders and by his prospective employees in the form of 'involuntary unemployment.' In this sense our theory of the firm has some Keynesian feature at least in the short-run.

It is often claimed that the notion of equilibrium is always a central organizing idea in economics.[4] In the dynamic economy in which expectations about the future are influencing present actions, an appropriate notion of equilibrium would be that expectations are capable of being fulfilled at least in a statistical sense. For instance, J. R. Hicks defined "equilibrium over time" in his dynamic Walrasian general equilibrium theory as a state of the economy in which "the prices realized on the second Monday are the same as those which were previously expected to rule at that date." In this equilibrium, he observed:

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[4] See, for example, Hahn [5].

> the change in prices which occurs is that which was expected. If tastes and resources also remain what they were expected to remain, then in equilibrium nothing has occurred to disturb the plans laid down on the first Monday. So far as can be seen, no one has made any mistakes, and plans can continue to be executed without any revision.

Hicks, then, added that "the degree of disequilibrium marks the extent to which expectations are cheated, and plans go astray."[5]

In the area of macroeconomics and monetary theory, the so-called 'natural rate theory of unemployment,' advocated by Milton Friedman and Edmund Phelps among others,[6] has effectively employed the similar equilibrium notion to build the case against the existence of permanent trade-off between unemployment and inflation, which is often characterized by an empirical relationship called (long-run) 'Phillips curve.'[7] The natural rate theory maintains that there exists an average rate of unemployment, called the 'natural rate of unemployment,' which is consistent with equilibrium in the structure of real wage rates and whose level is invariant under any changes in the time-pattern of the rate of inflation. The real wage rates are considered to be in equilibrium if actual and anticipated changes in prices are on the average equal. If a sudden and unexpected change in prices occurs, then the real wage rates may deviate from the equilibrium level and as a result the actual rate of unemployment may deviate from the natural rate. There is a temporal trade-off between unemployment and 'unexpected' inflation. However, sooner or later, the

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[5] Hicks [6], p. 132.

[6] See, for example, Friedman [4] and Phelps [15, 16].

[7] Phillips curve relation was first found by Phillips [17].

public will come to learn about the inflation and become capable of forming correct expectations about it. Then, the real wage rates will again coincide with the equilibrium level, and the natural rate of unemployment, which is independent of the rate of inflation, will be restored in the labor market. Therefore, the natural rate theory claims that there is no permanent trade-off between inflation and unemployment.

In Part II of this series, however, we shall endeavor to show that the harmonious situation in which all the entrepreneurs' expectations are capable of being fulfilled is possible only when what we shall call <u>Say's equation</u>, that aggregate demand and aggregate supply are balanced with each other, holds.[7a] In a monetary economy in which money or other durable asset functions as a 'store of value'--the link between the present and the future--and in which the act of hoarding the store of value constitutes a completely different act from that of placing orders to some futures markets for the future delivery of specific products, <u>Say's law</u> that "supply creates its own demand" cannot be the universal truth. If Say's law can be disturbed and the aggregate demand actually deviates from the aggregate supply, then one can show that at least some of the entrepreneurs' expectations are bound to be falsified by their own aggregative pricing behavior. In this sense, we maintain that the natural rate theory of unemployment and, more generally, the Walrasian general equilibrium theory tacitly presuppose the validity of Say's law.

It is Keynes who remarked that:

> I doubt if many modern economists really accept Say's law that supply creates its own demand. But they have not been aware that they were tacitly assuming it.[8]

[7a] To be precise, our Say's equation is defined in Part II as the condition that the objective conditional expectation of the rate of gap between real aggregate demand and aggregate supply is equal to a constant value called the natural rate of excess product demand.

[8] Keynes [13], p. 223.

He regarded Say's law as "equivalent to the proposition that there is no obstacle to full employment," and maintained that

> If, however, this is not the true law relating the aggregate demand and supply functions, there is a vitally important chapter of economic theory which remains to be written and without which all discussions concerning the volume of aggregate employment are futile.[9]

We believe that Keynes' remarks quoted above apply equally well to the state of economic science today; though most of the "vitally important chapter of economic theory concerning the volume of aggregate employment" was already written by Keynes himself.

Once we are freed from the tyranny of Say's law, we are able to go deeper into the exploration of disequilibrium situations. We claim that the fundamental cause of disequilibrium lies in the disturbance of Say's equation. Of course, one can imagine a situation in which Say's equation happens to be satisfied, yet the entrepreneurs' expectations are not fulfilled and the economy is thrown out of equilibrium. In fact, disequilibrium situations treated by the dynamic Walrasian general equilibrium theory and its <u>enfant bâtard</u>, the natural rate theory of unemployment, have been confined to this very narrow case. In this kind of disequilibrium, the economy's adjustment process is reduced to the economic agents' error-learning process that is motivated by the discrepancies between anticipated and actual magnitudes of relevant economic variables.

However, if the disequilibrium is caused by the disturbance of Say's equation, then we can show that the process of adjustment is characterized by the <u>cumulative inflation or deflation process à la</u>

[9] Keynes [11], p. 26.

Wicksell.[10] If the aggregate demand exceeds the aggregate supply, both prices and money wages will keep rising at higher rates than the long-run equilibrium rates; and if the aggregate demand falls short of the aggregate supply, both prices and money wages will keep growing at slower rates than the long-run equilibrium rates and eventually start declining. The main motive force behind this cumulative inflation or deflation process is also the discrepancy between anticipated and actual prices and wages; but its salient feature is that this discrepancy will never disappear and the entrepreneurs' frustration will never be turned off, so long as the fundamental cause of the disequilibrium--the disturbance of Say's equation --continues to exist. Therefore, the question of 'stability' of long-run equilibrium is reduced to the following simple question: whether or not the cumulative inflation or deflation itself has the force to bring a balance between the aggregate demand and the aggregate supply. This is nothing but the well-known money wage problem of Keynes.

Hicks believes that "whenever prices are fairly steady, the system is likely to be quite adequately in equilibrium," and that "it is chiefly in times of rapid price-movement that acute disequilibrium is likely to occur."[11] Similarly, Milton Friedman maintains that "the temporary trade-off [between inflation and unemployment] comes not from inflation per se, but from unanticipated inflation, which generally means, from a rising rate of inflation."[12] Obviously, both of them regard rapid price changes

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[10] See Wicksell [25], pp. 81-121, or [26], pp. 190-208.

[11] Hicks [6], p. 132.

[12] Friedman [4], p. 11.

as a main 'cause' of disequilibrium. In contrast to their view, we assert that rapid changes in prices (and money wages) are rather the 'symptom' of the more fundamental cause of disequilibrium--the disturbance of Say's equation. This symptom itself becomes at the same time a cure for the disease, although it may sometimes turn poisonous and end up with killing the patient.

The crucial assumption made throughout Part I and Part II of this series is that entrepreneurs can adjust product prices and money wages costlessly at the beginning of every period, although they cannot be re-adjusted in the rest of the period. (We shall adopt the discrete-time analysis.) We shall call this kind of economy the <u>Wicksellian economy</u> in order to distinguish it from the more realistic and general economy called the <u>Keynesian economy</u>, in which it is costly for entrepreneurs to adjust money wages even at the beginning of period.

Our Wicksellian economy has still one (neo-)classical flavor; in a state of long-run equilibrium in which Say's equation happens to be satisfied and all the entrepreneurs' expectations are fulfilled, the average rate of unemployment coincides with the natural rate of unemployment, whose level is totally independent of changes in the monetary variables in the economy (<u>provided that</u> these changes do not disturb Say's equation). In other words, the Phillips curve is on average vertical in long-run equilibrium in the Wicksellian economy.

In Part III and Part IV of this series, however, we shall investigate the Keynesian economy and remove this last (neo-)classical element from our theory. There, we shall show that because of the existence of money wage adjustment costs the average rate of unemployment is correlated with the rate of inflation even in long-run equilibrium. If, furthermore, there exists some dynamic downward rigidity of money wages in the sense that it is more costly for entrepreneurs to lower money wages than to raise them, then we are able to derive a negatively sloped 'long-run equilibrium Phillips curve.'[13] In the Keynesian economy, therefore, there exists a trade-off between inflation and unemployment both in the short-run and in the long-run.

2. The World of Imperfectly Competitive Firms

Imagine a closed economy which is composed of many firms, many households and many other economic agents. Each household takes part in this economy as a buyer of products from and as a supplier of labor-services to firms. Each firm in turn takes part in it by employing labor-services supplied by households, and producing a single product which is more or less differentiated from other firms' products. The reasons for such differentiation of products may either be differences in physical and other characteristics of the products themselves; or differences in the geographical location of firms and households in cases where they are scattered over an area; or there may exist a certain 'good-will' or 'inertia' on behalf of the households because of their imperfect information about price differences among firms. The last reason which attributes product-differentiation to households' imperfect information is much emphasized in the recent development of economics of information, and is certainly an important one.[14] However, it is worthwhile to point out here that it can be rarely found as a sole cause of product-

[13] This idea can be traced back to Schultz [20] and Lipsey [14]. Recently, Tobin [23] pursued this idea further.

[14] The pioneering work in this area is Stigler [21]. A good survey is found in Rothchild [19].

differentiation; its major role seems to be to accentuate the already existing product-differentiation created by other causes.[15]

Whatever the causes, the product-differentiation provides the entrepreneur of each firm with a price-setting power. He has his own market for his own product, and he can decide his own product price. However, there are many pricing strategies open to him which can be adopted in his own product market. He may charge different prices to different customers by negotiating with each of them, or he may gather all the prospective customers in one place and determine one price by auction. In the present paper, however, we shall suppose that our entrepreneur adopts the <u>uniform</u>, <u>take-it-or-else-leave-it</u> pricing policy. By this we mean that he announces a single price for his product and asks his prospective customers to transact with him at this price or else leave his firm. In this way our entrepreneur can save the costs of troublesome price negotiations or costs of organizing an auction market, possibly at the expense of some extra revenue which could be earned if he could charge different prices to different customers by bilateral negotiations or if he could sell all his product supply by adjusting the price in his auction market. It should be emphasized that the uniform price thus announced by our entrepreneur is not only the mere figure of an exchange rate between the product and the means of payment (money) but also functioning as a <u>information signal</u> informing the prospective customers of trading opportunities in his market. Clearly, the larger the number of his dealings, the more effective can the uniform price function as an information

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[15] This point has been forcefully made by Kaldor [8] almost forty years ago before the appearance of Stigler [21].

signal; the price as an information signal has an attribute of the public good. It is important to note that for the price to work as an effective signal it must be announced before actual transactions taking place, and it must be fixed for a while to enhance its reliability.

There is only one kind of variable productive factor in our economy --labor-services supplied by households. In order to simplify our analysis, we shall assume that labor-services are a homogeneous and completely variable production factor by disposing of all their quasi-fixed nature from our model. All workers look the same to the eyes of the entrepreneur, and there are no distinctions between senior employees and newcomers. In this sense, our 'workers' might well be identified with the so-called secondary labor force in the theory of dual labor markets.[16]

That employees look the same to the eyes of employers does not imply that firms look the same to the eyes of workers who are contemplating on which firm they will apply for their jobs. Workers are often not indifferent to two firms even if they are offering the same money wages. They may be located in different places, or they may provide different non-pecuniary returns and different working conditions, or each of them may enjoy some workers' good-will because workers' searches for the highest paid jobs may be very costly. In other words, the money wages (or money), which can be interpreted as being bought by workers in exchange for their labor-services,are more or less differentiated to their eyes from firm to firm. This differentiation of money wages or money, so to speak, provides the entrepreneur of each firm with a power of fixing his own money wage at least in the short-run. We shall again assume that he adopts the

[16] See Doeringer and Piore [3].

uniform, take-it-or-else-leave-it pricing policy in his own labor market.

3. The Structure of Product Demand and Labor Supply Functions

In what follows we shall measure all the variables except for those which represent costs, revenues and profits in terms of 'logarithm,' and specify almost all functions as 'log-linear.' One reason for this is, of course, the mathematical tractability, but the primal reason lies in the fact that this will enable us to aggregate variables and functions very naturally so that we can later develop macroeconomic models which are based upon microeconomic relations without losing any rigorousness. In the imperfectly competitive economy which we described in the previous section, we assume that there exist m firms, indexed by $i = 1, 2, \ldots, m$, each of which employes a homogeneous and variable labor input and produces a single differentiated product. We shall suppose for simplicity that differentiation of products and differentiation of money wages, so to speak, among these m firms are symmetrical to the eyes of households and other economic agents. In other words, all the firms are supposed to enjoy the same degree of monopoly in their product markets and the same degree of monopsony in their labor markets.

Let us summarize the structure of product demands by households and other spending units in this economy. We shall assume that the demand for the i^{th} firm's product can be expressed by the following symmetric constant-elasticity demand function:

$$(1) \qquad y_t^d(i) = -\eta(p_t(i) - P_t) + Y_t^d + a_t(i) \ ; \quad \eta > 1 \ ; \quad i = 1, 2, \ldots, m \ .$$

The product demand function (1) consists of three components. The first one represents the i^{th} firm's <u>current relative price</u>--the difference between the log of its own announced price, given by $p_t(i)$, and the log of current, general price level, given by P_t. In this paper, we specify P_t as the economy-wide geometric average of individual firms' product prices:

$$(2) \qquad P_t \equiv \sum_{i=1}^{m} p_t(i)/m .$$

The price elasticity $\eta > 1$ is assumed to be invariant over time and common for all firms; that is, we assume that all the firms enjoy the same constant degree of monopoly $1/(\eta-1)$. The second component Y_t^d represents (<u>per firm</u>) <u>real aggregate demand</u> in the economy. It is the log of the maximum amount of real money balance (divided by the number of firms) which households and other economic agents as a whole wish to spend on current products in period t. In the standard textbooks of macroeconomics, how the real aggregate demand Y_t^d itself is determined is analyzed by the well-known Hicks-Hansen IS-LM apparatus or its numerous variations. In the present paper, however, we shall leave this part of the model unspecified. This is a deliberate choice, for our theory which will be developed from now on is so flexible that we can incorporate almost any models of real aggregate demand determination at later stages of the analysis. Finally, the third component $a_t(i)$ is a random disturbance summarizing all the factors, other than $p_t(i)$, P_t and Y_t^d, which specifically influence demand for the i^{th} firm's product, $y_t^d(i)$. To simplify our exposition, we shall assume that $a_t(i)$ is a time-independent, zero-mean random variable with cross-sectional average $\sum_i a_t(i)/m$

being always constrained to be zero. In words, neither good luck nor bad luck will persist in this economy, both intertemporarily and cross-sectionally.

If we aggregate individual product demand functions (1) over all firms, we obtain the following aggregate equation:

$$\text{(3)} \qquad \sum_i y_t^d(i)/m = \sum_i \{-\eta(p_t(i) - P_t) + Y_t^d + a_t(i)\}/m$$

$$= -\eta\{\sum_i p_t(i)/m - P_t\} + Y_t^d + \sum_i a_t(i)/m$$

$$= Y_t^d \ ;$$

so that the real aggregate demand (in log) is completely exhausted by real demands for individual products (in logs). It should be remarked, however, that this is not the so-called 'adding-up constraint' on demand functions derived from the optimization theory of consumer's behavior. In fact, our assumed demand functions (1) slightly violate the adding-up constraint in their original forms; though, if we replaced our definition of P_t , given by (2), by

$$P_t \equiv -\eta \log[\sum_i \exp\{a_i - (\eta-1)p_i/\eta\}/m]/(\eta-1) \ ,$$

then the adding-up constraint would be recovered.[17] Practically, we can

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[17] If a consumer's preferences are represented by a CES utility function: $\sum_{i=1}^{m} \exp(\frac{1}{\eta}a_i + \frac{\eta-1}{\eta}x_i)$, then the demand function for the i^{th} commodity can be written as:

$$x_i = -\frac{1}{\eta}(p_i - P) + (\tilde{Y}^d - P) + a_i \ ;$$

regard our P_t as an approximation of the 'true' P_t ; they would be indeed equal if all $p_t(i)$'s were equal.

The product demand functions (1) state that, once a level of current, real aggregate demand Y_t^d is given, only current relative prices, $p_t(i) - P_t$, matter to households and other spending units in the economy for their decisions on current product demands. They are free from any kind of money illusion. If the entrepreneur of a firm happens to charge the product price equal to the current, general price level, then he will on average receive a fair share of aggregate real demand, Y_t^d . This is due to the assumed symmetry of product differentiation. One important limitation of our demand functions (1) other than their log-linear specification is that they ignore any <u>direct</u> intertemporal influences of the firm's past history of prices, rates of unfilled-orders and other variables on the current demand for its product. However, this assumption is inessential to the main themes of this paper; although an introduction of these intertemporal elements into our model would certainly complicate our analysis very much.

The structure of labor supplies by households can be summarized as follows. We shall assume that the supply of labor to the i^{th} firm can be also expressed by the following symmetric, constant-elasticity labor supply function:

$$n_t^s(i) = \epsilon(w_t(i) - W_t) + N_t^s + b_t(i) \ ; \quad \epsilon > 0 \ ; \quad i = 1, 2, \ldots, m \ . \tag{4}$$

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where $\tilde{Y}^d$ is a given nominal expenditure level divided by m , the number of commodities, and

$$P \equiv -\frac{\eta}{\eta-1} \log[\sum_{i=1} \exp(a_i - \frac{\eta-1}{\eta} p_i)/m] \ .$$

The labor supply $n_t^s(i)$ is determined by three factors. First of all, it depends on the current relative money wage, the difference between the log of i^{th} firm's announced money wage, given by $w_t(i)$, and the log of current, general money wage level, given by W_t. In this paper, W_t is defined as the economy-wide geometric average of individual firms' money wages:

$$(5) \qquad W_t \equiv \sum_i w_t(i)/m .$$

The wage elasticity $\epsilon > 0$ or the degree of monopsony $1/(1+\epsilon)$ is assumed to be time-invariant and common for all t. The labor supply depends next on the (per firm) aggregate labor supply N_t^s, which is determined by work-leisure choices of households as a whole. We shall again leave the model of aggregate labor supply determination unspecified in this paper. Finally, the last component $b_t(i)$ in (4) is a random disturbance, amalgamating all the factors, other than $w_t(i)$, W_t and N_t^s, which are supposed to affect households' labor supply decisions. We shall again assume that $b_t(i)$ is a time-independent, zero-mean random variable with cross-sectional average $\sum_i b_t(i)/m$ being always equal to zero.

If we aggregate individual labor supply functions (4) over all firms, then just as the case of product demand functions we obtain the following aggregate equation:

$$(6) \qquad \sum_i n_t^s(i)/m = \sum_i \{\epsilon(w_t(i) - W_t) + N_t^s + b_t(i)\}/m$$

$$= N_t^s .$$

Labor supplies to individual firms (in logs) add up to the current aggregate labor supply (in log).

The labor supply functions (4) say that, once a level of aggregate labor supply N_t^s is given, only the current relative money wages, $w_t(i) - W_t$, concern the households in their current decisions on which firm to apply their jobs to. They are free from any money illusion. If the entrepreneur of a firm happens to fix the money wage $w_t(i)$ equal to the current general money wage level W_t, then he can hire on average N_t^s workers, because of the assumed symmetricity of firms' locations, non-pecuniary returns, working conditions, etc. Again, to simplify our analysis, we shall assume away any direct intertemporal interdependence between current and future labor supply schedules. This assumption is in conformity with our previous assumption that labor-services are a homogeneous and completely variable production factor, but seems less plausible than in the case of product markets.

Before proceeding our analysis, we must make explicit one important assumption. We shall assume throughout the present series of papers that the time-unit of one basic period is chosen sufficiently short so that each household can place an order of one specific commodity and apply his job only to one firm within one period. Therefore, a worker who is not hired by the firm he is willing to work with the announced money wage is actuarially unemployed during that period and a customer's order which is not honored by the firm whose announced price is agreeable to the sender of this order is actuarially unfilled during that period. In order to relax this assumption, we must presumably develop an elaborate model of households' searches in product and labor markets out of equilibrium. But this is clearly beyond the scope of this series of papers. In any case, our main results seem to be independent of this simplifying assumption.

4. The Basic Model of the Firm[18]

Let us consider the entrepreneur of a representative firm who must make decisions on product price, money wage, labor employment, production, sale of output, and so on, in the monopolistically-cum-monopsonistically competitive economy whose structure have been just outlined. In order to simplify our notation, we shall not bother to attach the label i to any variables, any parameters and any functions in the rest of this paper.

At the beginning of period, say t, the entrepreneur of a representative firm announces a money wage measured in log, w_t, and starts recruiting workers in his own labor market. The announced money wage is fixed at least throughout the remaining period; it is an information signal to the workers who are looking for their jobs in period t. During period t job-seeking workers visit the firm and apply jobs if w_t is aggreable to them in comparison with other firms' announced money wages, while some of the senior employees may find it disappointing and quit the firm. By the end of period t, the entrepreneur can observe the actual number of workers who are willing to work with w_t. Let us denote the log of the number of willing workers by n_t^s and call it the labor supply to the firm in period t. The entrepreneur must decide how many workers to employ among the workers willing to work in his firm. He does not have to employ all of them, of course. Let n_t denote the log of

[18] The model developed in this and subsequent sections is an extension of the first half of our previous paper; Iwai [7]. However, the present paper is completely self-contained.

the actual labor employment in period t . Then, we have the following obvious labor market constraint:

$$(7) \qquad n_t \leqq n_t^s \; .$$

These n_t workers start producing commodities with the help of fixed production factors endowed in the firm from the end of period t . We assume that the firm is engaged in a point-input-point-output production process and that it takes $\tau-1$ $(\geqq 0)$ gestation periods to complete one production cycle. We shall summarize this production process by the Cobb-Douglas production function:

$$(8) \qquad y_{t+\tau}^s = k_t + \gamma n_t \; ; \quad 0 < \gamma < \eta/(\eta-1) \; ;$$

where $y_{t+\tau}^s$ is the logarithmic level of output in period $t+\tau$, k_t is an index of productive capacity of fixed factors endowed in this firm at the end of period t . We shall call k_t 'capital stock' for short. In the present paper we shall not concern ourselves with the firm's long-term capital investment problems, so that from now on k_t will be treated as an exogenously determined variable which congeals all the long-term decisions made by the entrepreneur in the past. The labor-input elasticity γ is assumed to be constant and common for all firms. Since η will be assumed to be greater than unity, our production function (8) allows some degree of increasing returns to labor input

At the beginning of period $t+\tau$ the entrepreneur announces a product price (in log) $p_{t+\tau}$ and pegs it throughout the period. Then, orders are sent to him from customers who have found $p_{t+\tau}$ andvantageous to them. By the end of period $t+\tau$ he can observe the actual demand for his product in period $t+\tau$. The actual number of orders is denoted by $y^d_{t+\tau}$ and called the product demand in period $t+\tau$. He must decide how much to sell to the willing buyers. We shall assume that his product is 'non-durable' so that no inventory of final product can be carried over to the next period. (However, this does not imply that we have assumed away all the inventory problems from our model; the entrepreneur in our model can control inventory of 'goods in process' by adjusting labor employment and will speculate or hedge for anticipated changes in future demands for his product.) We shall also assume that he can backlog no unfilled orders; orders once declined to be honored will be lost forever. Under these assumptions, it is clear that the amount of the sold product in period $t+\tau$, whose logarithmic value is denoted by $y_{t+\tau}$, is either constrained by the current demand (or actual orders) $y^d_{t+\tau}$ or by the current output $y^s_{t+\tau}$; that is, we have

$$(9) \qquad y_{t+\tau} = \min[y^d_{t+\tau}, y^s_{t+\tau}] .$$

In other words, the actual sale of product is determined by the 'short' side of the product market.

Our entrepreneur's one-cycle activity which starts from his announcement of a money wage and ends with his sale of final product extends over τ periods. From this activity he receives sales revenue equal to $\exp(p_{t+\tau} + y_{t+\tau})$ at the end of period $t+\tau$ and pays wages $\exp(w_t + n_t)$ to workers at the end of period t; where $\exp(z_t)$ denotes the 'exponential' of the variable z_t. Note that both the revenue and the wage cost are <u>not</u> measured in logs. Then, if we denote a constant discount rate per unit period by δ, we can calculate the discounted value of profit earned by the entrepreneur from his one-cycle activity as follows:[19]

$$(10) \qquad r_t \equiv \exp\{(p_{t+\tau} + y_{t+\tau}) - \tau\delta\} - \exp(w_t + n_t) \ .$$

The most important assumption in Part I and Part II of this series is that the entrepreneur incurs no cost in adjusting money wage at the beginning of every period. The economy in which this assumption is satisfied is called the Wicksellian economy. In Part III and Part IV of this series, we shall introduce money wage adjustment costs into our model in order to capture the idea of 'money wage rigidity' in a dynamic context. The economy in which money wage adjustments are costly even at the beginning of period is called the Keynesian economy. It is clear that the Wicksellian economy is a special case of the more general Keynesian economy. We have also assumed away all the problems concerning inventory adjustment of final products and backlogging of unfilled-orders as well as any quasi-fixed nature of labor employment. We have also ignored any direct intertemporal interdependence of product demands and labor supplies in the present and in the future. Then our entrepreneur can decide the optimal price, wage and employment policies simply by maximizing the expected utility of the one-cycle profit defined by (10). In this paper we shall assume that he is risk-neutral and concerns only with its expected value.

However, even this very simplified model of the firm involves a

[19] If we assume instead that the entrepreneur concerns with the discounted value of real profit, then we should divide the first term of (10) by $\exp(P_{t+\tau})$ and the second term by $\exp(P_t)$. Our subsequent model can be easily modified so as to analyze this case.

sequential information structure. The information available to the entrepreneur when he announces $p_{t+\tau}$ differs from the information available to him when he determines n_t , which in turn differs from the available information when he announces w_t . He is continuously accumulating new information about his business environment from his actions and observations in the markets. The best way for us to handle this sequential nature of information is to reverse the chronological order of the sequence of our entrepreneur's decisions and solve his problem backwards; in other words, we shall adopt the technique of backward induction in the theory of dynamic programming. Consequently, we shall first examine the determination of the optimal price in the product market, then discuss the problem of the optimal employment policy and finally analyze the determination of the optimal money wage in the labor market. We hope that this will not confuse the reader.

5. The Optimal Product Price Policy

Let us now determine the optimal product price (in log) p_t^* to be announced in the market at the beginning of period t . In the following we shall denote either by $E_t(z_{t+\theta})$ or by ${}_t\hat{z}_{t+\theta}$ the entrepreneur's subjective expectation of a certain random variable $z_{t+\theta}$ conditional upon the information available to him at the beginning of period t . Then, the subjective expected value of the discounted profit which is to be maximized with respect to p_t^* can be expressed as:

$$(11) \qquad E_t(r_{t-\tau}|w_{t-\tau}, n_{t-\tau}, p_t) \equiv {}_t\hat{r}_{t-\tau}$$

$$= E_t\{\exp(p_t + y_t - \tau\delta) - \exp(w_{t-\tau} + n_{t-\tau})\} .$$

Since the wages paid out in period $t-\tau$ are already a sunk cost at the beginning of period t and the discount rate δ is assumed to be constant and non-stochastic, our entrepreneur has to maximize only the expected revenue with respect to p_t^*, which, in view of (9), can be written as:

$$E_t\{\exp(p_t + \min[y_t^d, y_t^s])\} \ . \tag{12}$$

In order to evaluate this expected revenue, we have to specify our entrepreneur's subjective picture of his market environment more explicitly. Accordingly, we shall assume that his subjective product demand schedule is of the same constant elasticity form as the true product demand function (1):

$$y_t^d = -\eta(p_t - P_t) + Y_t^d + a_t \ ; \quad \eta > 1 \ ; \tag{13}$$

where $\eta > 1$ here is the subjective elasticity of product demand, which is believed to be invariant over time. Note that the value of this subjective demand-elasticity of the i^{th} entrepreneur may be different from the value of the true demand elasticity given in (1), if his perception of his own product market is distorted.[20] That our entrepreneur's subjective product demand function (13) has the same functional form as the true schedule (1) (possibly with a different value of the demand-elasticity) does not imply that he knows the values of the aggregate variables P_t and Y_t^d. On the contrary, at least at the beginning of period t when he is about to announce p_t, he does not know their actual values and

[20] Recall we have omitted the superscript i.

must treat them as random variables, in addition to the random disturbance term a_t .

Let us put together three random variables, P_t , Y_t^d and a_t and amalgamate them into a single random variable α_t in the following manner:

$$(14) \qquad \alpha_t \equiv \eta P_t + Y_t^d + a_t \ .$$

Then, the subjective product demand function (13) can be rewritten simply as

$$(15) \qquad y_t^d = -\eta p_t + \alpha_t \ .$$

Our entrepreneur must attach a certain subjective probability distribution to the single random variable α_t in order to be able to evaluate the expected profit (12). Let ${}_t\hat{\alpha}_t$ denote his <u>subjective expectation</u> of α_t conditional upon his information available at the beginning of period t :

$$(16) \qquad E_t(\alpha_t) \equiv {}_t\hat{\alpha}_t = \eta_t\hat{P}_t + {}_t\hat{Y}_t^d + {}_t\hat{a}_t \ .$$

Then, we shall assume that the deviation of the actual value of α_t from its expected value ${}_t\hat{\alpha}_t$ is believed to be a time-independent random variable with the following <u>subjective probability distribution</u>:

$$(17) \qquad \Pr\{\alpha_t - {}_t\hat{\alpha}_t \leq z \,|\, {}_t\hat{\alpha}_t\} \equiv \Psi(z) \ .$$

By construction the mean of this deviation equals zero. In other words, our entrepreneur believes that his subjective expectation ${}_t\hat{\alpha}_t$ is an unbiased predictor of α_t with a time-independent random prediction error. As is explained in the footnote below, this is an innocuous assumption.[21] In order to simplify our exposition somewhat we shall assume, not implausibly, that the entrepreneur correctly believes that the mean of the random disturbance a_t is equal to zero; that is, we shall assume that:

$$(18) \qquad E_t(a_t) \equiv {}_t\hat{a}_t = 0 \ .$$

Before evaluating the expected revenue (12) explicitly, let us define by g_t the logarithmic difference between demand for and supply of the firm's product in period t :

$$(19) \qquad g_t \equiv y_t^d - y_t^s = -\eta p_t + \alpha_t - y_t^s \ .$$

We shall call g_t <u>the rate of excess product demand</u> in period t for short.[22] Its subjective conditional expectation can be easily calculated

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[21]We can show that the sequence of prediction errors $\{\alpha_t - {}_t\hat{\alpha}_t\}$ is a serially uncorrelated (or 'white') random series (from the subjective viewpoint of the entrepreneur) simply by using the fact that the expectation is the expectation of a conditional expectation: $E(Z) = E\{E(Z|I)\}$, or, equivalently, for $\theta \geqq 1$

$$E\{(\alpha_{t-\theta} - {}_{t-\theta}\hat{\alpha}_{t-\theta})\cdot(\alpha_t - {}_t\hat{\alpha}_t)\}$$

$$= E[E_t\{(\alpha_{t-\theta} - {}_{t-\theta}\hat{\alpha}_{t-\theta})\cdot(\alpha_t - {}_t\hat{\alpha}_t)\}$$

$$= E\{(\alpha_{t-\theta} - {}_{t-\theta}\hat{\alpha}_{t-\theta})\cdot E_t(\alpha_t - {}_t\hat{\alpha}_t)\}$$

$$= E[(\alpha_{t-\theta} - {}_{t-\theta}\hat{\alpha}_{t-\theta})\{E_t(\alpha_t) - {}_t\hat{\alpha}_t\}] = 0 \ .$$

Therefore, if the entrepreneur believes that the prediction error $\alpha_t - {}_t\hat{\alpha}_t$ is a stationary and Gaussian random variable, then it must be time-independent.

as

$$(20)\qquad {}_t\hat{g}_t \equiv E_t(g_t) = -\eta p_t + {}_t\hat{\alpha}_t - y_t^s = g_t - (\alpha_t - {}_t\hat{\alpha}_t) .$$

Note that the logarithmic level of output y_t^s in period t should be regarded as a given datum to our entrepreneur at the beginning of the period, because his firm's production activity involves no technological uncertainty and the levels of capital stock and labor employment were determined and known to him already at the end of period $t-\tau$.

If we substitute (14)-(20), the expected revenue (12) can be evaluated as follows:

$$\begin{aligned}(21)\qquad & E_t\{\exp(p_t + \min[y_t^d,\ y_t^s])\} \\ & = E_t\{\exp(p_t + y_t^s + \min[g_t,\ 0])\} \\ & = \exp(p_t + y_t^s)\cdot E_t\{\exp(\min[{}_t\hat{g}_t + (\alpha_t - {}_t\hat{\alpha}_t),\ 0])\} \\ & = \exp\left\{\frac{\eta-1}{\eta}y_t^s + \frac{1}{\eta}({}_t\hat{\alpha}_t - {}_t\hat{g}_t)\right\}\left\{\int_{-\infty}^{-{}_t\hat{g}_t} \exp(z + {}_t\hat{g}_t)d\Psi(z) + 1 - \Psi(-{}_t\hat{g}_t)\right\} .\end{aligned}$$

For given y_t^s and ${}_t\hat{\alpha}_t$, this can be regarded as a function of ${}_t\hat{g}_t$, which is in turn a linear function of p_t as is expressed by (20). We assume that this function has a unique global maximum.[23] Then, differentiating

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[22] Let x and y denote two variables, measured in logs, whose values are very close, then we can easily show that $x-y \cong \{\exp(x) - \exp(y)\}/\exp(y)$. Therefore, if the values of y_t^d and y_t^s are not widely different we can regard g_t as an approximation of the true rate of excess product demand, $\{\exp(y_t^d) - \exp(y_t^s)\}/\exp(y_t^s)$.

[23] In Iwai [7], we have shown that if $\Psi(z)$ is an exponential distribution function, there exists a unique global maximum of (21).

(21) with respect to p_t or equivalently with respect to ${}_t\hat{g}_t$ and equating the derivative with zero, we obtain the first-order condition for our entrepreneur's optimal product price policy:

$$(22) \qquad (\eta-1)\int_{-\infty}^{-{}_t\hat{g}_t} \exp(z + {}_t\hat{g}_t)d\Psi(z) - \{1 - \Psi(-{}_t\hat{g}_t)\} = 0 \ .$$

Notice that this is a function of ${}_t\hat{g}_t$ only, independently of the values of y_t^s and ${}_t\hat{\alpha}_t$. If the expected revenue (21) viewed as a function of ${}_t\hat{g}_t$ attains the global maximum at

$$(23) \qquad {}_t\hat{g}_t = g^* \ ,$$

then obviously this maximum solution must satisfy the first-order condition (22). Consequently, this g^* can be regarded as a constant whose value is solely determined by a given subjective demand elasticity η and by a given subjective probability distribution $\Psi(z)$, because they characterize equation (22) completely. We shall call g^* the normal rate of excess product demand.

Solving (20) with respect to p_t and noting that by (23) ${}_t\hat{g}_t$ can be now regarded as a given constant g^* , we can derive the following log-linear formula for the optimal product price:

$$(24) \qquad p_t^* = \frac{1}{\eta}({}_t\hat{\alpha}_t - g^* - y_t^s) \ ,$$

which in view of (16) and (18) can be rewritten as

$$(25) \qquad p_t^* = {}_t\hat{P}_t + \frac{1}{\eta}({}_t\hat{Y}_t^d - y_t^s - g^*) \ .$$

This optimal product price equation says that, other things being equal, an anticipated increase in current general price level or in current aggregate product demand boosts up the current product price, whereas an increase in current product supply depresses the current product price. The response coefficients of ${}_t\hat{Y}^d_t$ and y^s_t are determined by the subjective demand elasticity η. However, the most important property of the optimal product price equation (25) is that the coefficient of ${}_t\hat{P}_t$ is unity, which implies that if the entrepreneur expects that the current general price is growing at ten per cents, other things being equal, he lets his product price grow at the same ten per cents. It has the <u>money-neutrality property</u>.

It is worthwhile to remark here that whether or not the entrepreneur charges p^*_t above or below the price level which is expected to clear his own product market is determined according to whether ${}_t\hat{g}_t = g^* \lesseqgtr 0$, which in turn depends upon the value of the subjective demand elasticity or the subjective degree of monopoly. It can be shown that if $\eta\Psi'(-g^*) > 1 - \Psi(-g^*)$, g^* is negatively correlated with $1/(\eta-1)$; so that we can say that the harder firms are competing with each other for product demands the lower is the product price charged relative to its expected market clearing level. This seems the normal case.

The constant g^* was called the normal rate of excess product demand simply because the optimal product price policy for our entrepreneur amounts to finding p^*_t which is expected to bring the rate of excess product demand g_t into equality with this constant rate. Define by j_t <u>the rate of unfilled-orders</u> in period t :

$$(26) \qquad j_t \equiv y_t^d - y_t = y_t^d - \min[y_t^d,\ y_t^s] = \max[g_t,\ 0] \ ,$$

and by ${}_t\hat{j}_t$ its subjective expectation:

$$(27) \qquad {}_t\hat{j}_t \equiv E_t(j_t) = E_t(\max[g_t,\ 0]) = E_t(\max[{}_t\hat{g}_t + (\alpha_t - {}_t\hat{\alpha}_t),\ 0])$$

$$= \int_{-{}_t\hat{g}_t}^{\infty} z d\Psi(z) + {}_t\hat{g}_t\{1 - \Psi(-{}_t\hat{g}_t)\} \ .$$

Then it is clear from the nature of the optimal product price policy that ${}_t\hat{j}_t$ should be also brought into equality with a constant value j^*, which is obviously defined by:

$$(28) \qquad j^* \equiv \int_{-g^*}^{\infty} z d\Psi(z) + g^*\{1 - \Psi(-g^*)\} \ .$$

We shall call j^* <u>the normal rate of unfilled-orders</u>.

6. The Optimal Employment Policy

Let us now turn to the examination of the optimal labor employment n_t^* (in log) which is decided by our entrepreneur at the end of period, say, t. The labor employment n_t determined in period t will produce $y_{t+\tau}^s = k_t + \gamma n_t$ amount of output in period $t+\tau$, which will, in view of (22), (23) and (24), yield in that period the expected revenue equal to:

$$(29) \qquad E_{t+\tau}\{\exp(p_{t+\tau}^* + \min[-\eta p_{t+\tau}^* + \alpha_{t+\tau},\ y_{t+\tau}^s])\}$$

$$= \frac{\eta\{1 - \Psi(-g^*)\}}{\eta - 1} \exp\left\{\frac{\eta - 1}{\eta} y_{t+\tau}^s + \frac{1}{\eta}({}_{t+\tau}\hat{\alpha}_{t+\tau} - g^*)\right\} \ .$$

Therefore, the entrepreneur's subjective expectation of discounted profit evaluated at the end of period t can be expressed as:

$$(30)\quad E_t\{r_t|w_t,\ n_t\} = E_t\left[\frac{\eta\{1-\Psi(-g^*)\}}{\eta-1}\exp\left\{\frac{\eta-1}{\eta}y^s_{t+\tau} + \frac{1}{\eta}({}_{t+\tau}\hat{\alpha}_{t+\tau} - g^*) - \tau\delta\right\} - \exp(w_t + n_t)\right]$$

Our entrepreneur knows that he will accumulate new information concerning the random variable $\alpha_{t+\tau}$ through his market activities from period t to period $t+\tau$, so that at the end of period t he regards his own subjective expectation ${}_{t+\tau}\hat{\alpha}_{t+\tau}$, which is conditional upon his future information available at the beginning of period $t+\tau$, as a random variable. (To simplify our analysis somewhat, we shall assume that his available information at the time of employment decision, i.e., at the end of period t, remains the same as the one available to him at the beginning of period t.) Let us denote by ${}_t\hat{\alpha}_{t+\tau}$ the conditional expectation of $\alpha_{t+\tau}$ evaluated at the beginning of period t. Then, we have

$$(31)\qquad {}_t\hat{\alpha}_{t+\tau} \equiv E_t(\alpha_{t+\tau}) = E_t\{E_{t+\tau}(\alpha_{t+\tau})\} = E_t({}_{t+\tau}\hat{\alpha}_{t+\tau})\ ;$$

because the (conditional) expectation of a conditional expectation is the (conditional) expectation. In words, (31) states that ${}_t\hat{\alpha}_{t+\tau}$ can be regarded not only as the conditional expectation of the random variable $\alpha_{t+\tau}$ but also that of the subjective expectation ${}_{t+\tau}\hat{\alpha}_{t+\tau}$ evaluated at the beginning of period t. As before, we shall assume that the deviation of ${}_{t+\tau}\hat{\alpha}_{t+\tau}$ from its expected value ${}_t\hat{\alpha}_{t+\tau}$ is believed to be distributed with the following time-independent <u>subjective probability distribution</u>:[24]

$$\Pr\{{}_{t+\tau}\hat{\alpha}_{t+\tau} - {}_t\hat{\alpha}_{t+\tau} \leq z \mid {}_t\hat{\alpha}_{t+\tau}\} = \Psi^{(\tau)}(z) \ . \tag{32}$$

Substituting (31) and (32), we can calculate the expected discounted profit (30) as follows:

$$\left[\frac{\eta\{1 - \Psi(-g^*)\}}{\eta-1}\int_{-\infty}^{\infty} \exp\left(\frac{z}{\eta}\right) d\Psi^{(\tau)}(z)\right] \exp\left\{\frac{\eta-1}{\eta}k_t + \frac{(\eta-1)\gamma}{\eta}n_t + \frac{1}{\eta}({}_t\hat{\alpha}_{t+\tau} - g^*) - \tau\delta\right\} \tag{33}$$

$$- \exp(w_t + n_t) \ .$$

Our entrepreneur wishes to maximize this function with respect to the labor employment n_t subject to the labor market constraint (7). This is the simplest constrained maximization problem. Since (33) is strictly concave in $\exp(n_t)$ under the assumption of $\gamma < (\eta-1)/\eta$ we can apply the well-known Kuhn-Tucker theorem to obtain the following formula of the optimal employment policy:[25]

$$n_t^* = \min[n_t^s, n_t^d] \ ; \tag{34}$$

where n_t^d is defined by

$$n_t^d \equiv \frac{\eta}{\sigma}\{\varphi - \tau\delta + \frac{\eta-1}{\eta}k_t + \frac{1}{\eta}({}_t\hat{\alpha}_{t+\tau} - g^*) - w_t\} \tag{35}$$

$$= \frac{\eta}{\sigma}\{\varphi - \tau\delta + \frac{\eta-1}{\eta}k_t + {}_t\hat{P}_{t+\tau} - w_t + \frac{1}{\eta}({}_t\hat{Y}^d_{t+\tau} - g^*)\} \ ;$$

[24] If the entrepreneur believes that the sequence of random variables, $\{{}_{t+\theta}\hat{\alpha}_{t+\tau}\}$, $\theta = 0, 1, 2, \ldots,$ constitutes the random walk up to $\theta = \tau+1$, where by convention ${}_{t+\tau+1}\hat{\alpha}_{t+\tau} \equiv \alpha_{t+\tau}$, then $\Psi^{(\tau)}(z)$ can be interpreted as the "τ-fold convolution" of $\Psi(z)$ with itself.

[25] Since the constraint function is linear, the constraint qualification is satisfied in this model.

and φ and σ in (25) are constant respectively defined by

$$\varphi \equiv \log[\gamma\{1 - \Psi(-g^*)\}\int_{-\infty}^{\infty} \exp\left(\frac{z}{\eta}\right) d\Psi^{(\tau)}(z)] , \tag{36}$$

$$\sigma \equiv \eta(1-\gamma) + \gamma = \eta - \gamma(\eta-1) . \tag{37}$$

Note that $0 < \sigma < \eta$ and that $\sigma \gtreqless \gamma$ according to $\gamma \lesseqgtr 1$.

We shall call n_t^d simply <u>the demand for labor</u> in period t . It is the level of labor employment that would maximize the expected discounted profit (32) were it not for the labor market constraint (7). The labor demand n_t^d is determined by a discount rate δ , by a given capital stock k_t , by a pre-determined money wage w_t and by the entrepreneur's conditional expectations of $P_{t+\tau}$ and $Y_{t+\tau}^d$ at the time of labor employment, denoted by ${}_t\hat{P}_{t+\tau}$ and ${}_t\hat{Y}_{t+\tau}^d$. The salient feature of this labor demand function is that the <u>current</u> demand for labor is influenced by the <u>entrepreneur's anticipation of demand for his 'finished' output, formed at the time when he commits himself to starting the process which will produce it</u>. For the amount of labor demand today depends on the level of sales revenue which the entrepreneur expects to receive from the corresponding output z periods later. So, our labor demand function is totally consistent with Keynes' theory of employment on which he built <u>the principle of effective demand</u> that forms the core of "General theory of Employment, Interest and Money."[26]

Our optimal employment policy given by (34) is simply saying that 'short' side of the current labor market dominates the current actual

[26] See Keynes [11], Chapters 3 and 5.

employment of labor. It is capable of explaining the Keynesian involuntary unemployment at least as a short-run phenomenon. When the actual labor supply n_t^s falls short of the current demand for labor n_t^d, our entrepreneur must hire all the workers who are willing to work in his firm with the announced money wage w_t. However, when n_t^s turns out to be larger than n_t^d, his optimal employment behavior is to hire only n_t^d workers, because he is in fact maximizing his expected profit at this level of employment. A part of workers who are willing to work with the announced money wage are thus involuntarily unemployed in this firm's labor market. This is due to the fact that in order that a money wage might function as an effective information signal it must be announced prior to actual employment contracts and fixed at least temporarily by our entrepreneur. No wonder that this short-run rigidity of money wage brings about the short-run Keynesian phenomenon. However, whether the Keynesian involuntary unemployment will persist even in the long-run or not is an entirely different matter, and we shall investigate this problem and attempt to give some answer to it in Part III and Part IV of this series.

The short-side-of-the-market-dominates-the-actual-transaction form of the optimal employment policy (34) allows us to define unambiguously the rate of excess labor supply, to be denoted by h_t, and the rate of involuntary unemployment, to be denoted by u_t, in our representative firm's labor market. We define:

$$(38) \qquad h_t \equiv n_t^s - n_t^d ,$$

$$(39) \qquad u_t \equiv n_t^s - n_t = n_t^s - \min[n_t^s,\ n_t^d] = \max[h_t,\ 0] .$$

The rate of unfilled vacancy v_t may be also defined as the difference between u_t and h_t :

(40) $$v_t \equiv n_t^d - n_t = u_t - h_t \ .$$

7. The Optimal Money Wage Policy

Finally, we must analyze our entrepreneur's optimal money wage determination at the beginning of period, say, t . He wants to choose the optimal money wage rate w_t^* so as to maximize the conditional expectation of the discounted profit defined by (10) evaluated at the beginning of period t . Substituting (34)-(37) into (33), we can express it as:

(41) $$E_t(r_t|w_t)$$
$$= E_t\left[\frac{\eta}{\gamma(\eta-1)}\exp\{\varphi - \tau\delta + \frac{\eta-1}{\eta}k_t + \frac{(\eta-1)\gamma}{\eta}\min[n_t^s,\ n_t^d] + \frac{1}{\eta}({}_t\hat{\alpha}_{t+\tau} - g^*)\}\right.$$
$$\left. - \exp(w_t + \min[n_t^s,\ n_t^d])\right] \ .$$

In order to evaluate (41) explicitly, we again have to specify our entrepreneur's perception of the economic environment surrounding his labor market. We shall assume that his <u>subjective labor supply schedule</u> is of the same constant elasticity form as the true labor supply function (4):

(42) $$n_t^s = \epsilon(w_t - W_t) + N_t^s + b_t \ ; \quad \epsilon > 0 \ ;$$

where $\epsilon > 0$ here is the subjective elasticity of labor supply which is believed to be invariant over time. The value of ϵ may vary from firm to firm, even though the true elasticity is uniform over all firms. Since at the beginning of period t our entrepreneur cannot know the values of W_t, N_t^s and b_t, he must attach certain subjective probability distributions to these random variables.

Let us define a random variable β_t as follows:

$$(43) \qquad \beta_t \equiv -\varepsilon W_t + N_t^s + b_t \ .$$

Then, the subjective labor supply function (42) can be rewritten simply as

$$(44) \qquad n_t^s = \varepsilon w_t + \beta_t \ .$$

This is a constant-elasticity supply function with a single multiplicative random variable. Denote the entrepreneur's <u>subjective expectation</u> of β_t conditional upon his information up to the beginning of period t by ${}_t\hat{\beta}_t$:

$$(45) \qquad {}_t\hat{\beta}_t \equiv E_t(\beta_t) = -\varepsilon\,{}_t\hat{W}_t + {}_t\hat{N}_t^s + {}_t\hat{b}_t \ .$$

Then, we shall assume once again that he believes that the deviation of the actual value of β_t from its expected value ${}_t\hat{\beta}_t$ is distributed with the following time-independent <u>subjective probability distribution</u>:

$$(46) \qquad \Pr\{\beta_t - {}_t\hat{\beta}_t \leq z \,|\, {}_t\hat{\beta}_t\} \equiv \Phi(z) \ ;$$

where by definition the mean of this deviation is equal to zero. He believes his expectation ${}_t\hat{\beta}_t$ be an unbiased predictor of β_t with 'white' prediction errors. In order to simplify our exposition, we shall again assume that the entrepreneur correctly believes that the mean of the random disturbance term b_t is zero:

(47) $$E_t(b_t) \equiv {}_t\hat{b}_t = 0 \ .$$

We can now evaluate the expected discounted profit (41) explicitly by substituting (43)-(47) into it. After tedious calculations we obtain:

(48) $$E_t(r_t|w_t)$$

$$= \exp\left[\frac{1+\epsilon}{\eta+\epsilon\sigma}\{\eta\varphi - \tau\delta + (\eta-1)k_t + {}_t\hat{\alpha}_{t+\tau} - g^* + \frac{\eta-\sigma}{1+\epsilon}\,{}_t\beta_t\}\right]$$
$$\cdot\left[\exp\left(-\frac{\eta-\sigma}{\eta+\epsilon\sigma}\,{}_t\hat{h}_t\right)\left\{\frac{\eta}{\eta-\sigma}\int_{-\infty}^{-{}_t\hat{h}_t}\exp\left\{\frac{\eta-\sigma}{\eta}(z + {}_t\hat{h}_t)\right\}d\Phi(z)\right.\right.$$
$$\left.\left. - \int_{-\infty}^{-{}_t\hat{h}_t}\exp(z + {}_t\hat{h}_t)d\Phi(z) + \frac{\sigma}{\eta-\sigma}(1 - \Phi(-{}_t\hat{h}_t))\right\}\right] \ ;$$

where we denote by ${}_t\hat{h}_t$ the entrepreneur's conditional subjective expectation of the rate of excess labor supply h_t at the beginning of period t ; i.e.,

(49) $${}_t\hat{h}_t \equiv E_t(h_t) = E_t(n_t^s - n_t^d)$$

$$= (\epsilon w_t + {}_t\hat{\beta}_t) - \frac{\eta}{\sigma}\{\varphi - \tau\delta + \frac{1}{\eta}({}_t\hat{\alpha}_{t+\tau} - g^*) + \frac{\eta-1}{\eta}k_t - w_t\} \ .$$

The expected profit evaluated by (48) can be regarded as a function of ${}_t\hat{h}_t$, which is in turn by (49) a linear function of w_t . We assume that it has a unique global maximum.[27] Then, by differentiating (48) with respect to ${}_t\hat{h}_t$ and equating the derivative with zero, we can deduce the

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[27] In Iwai [7], we have proved that if $\Phi(z)$ is an exponential distribution, there exists a unique global maximum of (48).

the following first-order condition for our entrepreneur's optimal money wage policy:

$$(50)\qquad \{1 - \Phi(-{}_t\hat{h}_t)\} + (1+\epsilon)\int_{-\infty}^{-{}_t\hat{h}_t} \exp(z + {}_t\hat{h}_t)\,d\Phi(z)$$

$$- \epsilon\int_{-\infty}^{-{}_t\hat{h}_t} \exp\left\{\frac{\eta-\sigma}{\eta}(z + {}_t\hat{h}_t)\right\} d\Phi(z) = 0 .$$

Note that this is a function of ${}_t\hat{h}_t$ only. If the expected discounted profit (48) as a function of ${}_t\hat{h}_t$ attains the global maximum at

$$(51)\qquad {}_t\hat{h}_t = h^* ,$$

then obviously this maximum solution must satisfy the first-order condition (50). As a result, h^* can be regarded as a constant whose value is determined by given subjective elasticities ϵ and η, by a given technological elasticity γ, and by a given subjective probability distribution $\Phi(z)$. We shall call h^* the normal rate of excess labor supply.

Solving (49) with respect to w_t and noting that by (50) ${}_t\hat{h}_t$ can be treated just as a constant h^*, we can derive the following log-linear formula for the optimal money wage w_t^*:

$$(52)\qquad w_t^* = \frac{\eta}{\eta+\epsilon\sigma}\{\varphi - \tau\delta + \frac{\eta-1}{\eta}k_t + \frac{1}{\eta}({}_t\hat{\alpha}_{t+\tau} - g^*) - \frac{\sigma}{\eta}({}_t\hat{\beta}_t - h^*)\} ,$$

which can be rewritten as:

$$(53)\qquad w_t^* = \frac{\eta}{\eta+\epsilon\sigma}\{\varphi - \tau\delta + \frac{\eta-1}{\eta}k_t + {}_t\hat{P}_{t+\tau} + \frac{\sigma\epsilon}{\eta} + {}_t\hat{W}_t + \frac{1}{\eta}({}_t\hat{Y}^d_{t+\tau} - g^*) - \frac{\sigma}{\eta}({}_t\hat{N}^s_t - h^*)\} ,$$

in view of (45) and (47). This optimal money wage equation states that, other things being equal, an anticipated increase in $P_{t+\tau}$, W_t or $Y^d_{t+\tau}$ or an increase in k_t tends to raise the current money wage w^*_t whereas an anticipated increase in N^s_t or an increase in δ tends to lower it. The constant response coefficients of these explanatory variables are determined by given subjective elasticities ϵ and η and by a technological elasticity γ. Notice that the above optimal money wage equation has also the <u>money-neutrality property</u> in the sense that the coefficients of ${}_t\hat{W}_t$ and ${}_t\hat{P}_{t+\tau}$ add up to one. When the entrepreneur expects that both W_t and $P_{t+\tau}$ grow at a certain percentage rate at the beginning of period t, then other things being equal, he tends to raise w^*_t at the same percentage rate.

It should be clear from the nature of the optimal money wage policy discussed above that whether the entrepreneur fixes w^*_t above or below the money wage level which is expected to clear his own labor market depends on whether ${}_t\hat{h}_t = h^* \gtreqless 0$. But we can show in many cases that $h^* < 0$, implying that he usually sets w^* below the expected market clearing level.[28] It must be emphasized, however, that this result is <u>not</u> due to the existence of uncertainty in the labor market <u>but</u> due to the well-known property of monopsony pricing that a monopsony tends to keep some excess capacity of productive factors even in the situation of no uncertainty.[29] On the other hand, we can usually show that h^* is decreasing in $1/(1+\epsilon)$ and increasing in $1/(\eta-1)$ and γ.[30] Therefore,

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[28]This is certainly the case in our previous model in Iwai [7].

[29]See, for example, Robinson [18].

[30]This is again the case in our previous model in Iwai [7].

the more competitive the labor market and the less competitive the product market, the higher does w_t^* tend to be fixed by our entrepreneur.

Let us denote by ${}_t\hat{u}_t$ the entrepreneur's conditional subjective expectation of the rate of involuntary unemployment u_t in his own labor market evaluated at the beginning of period t : Then, ${}_t\hat{u}_t$ can be regarded as a function of ${}_t\hat{h}_t$ only, for we have:

$$(54)\qquad {}_t\hat{u}_t \equiv E_t(u_t) = E_t(\max[h_t, 0]) = E_t(\max[{}_t\hat{h}_t + (\beta_t - {}_t\hat{\beta}_t), 0])$$

$$= \int_{-{}_t\hat{h}_t}^{\infty} z d\Phi(z) + {}_t\hat{h}_t\{1 - \Phi(-{}_t\hat{h}_t)\} .$$

It is easy to see that ${}_t\hat{u}_t$ is an increasing and convex function of ${}_t\hat{h}_t$, which is bounded below by zero; that is, if the density of $\Phi(z)$ given by $\Phi'(z)$, exists, we have

$$(55)\qquad \frac{\partial({}_t\hat{u}_t)}{\partial({}_t\hat{h}_t)} = 1 - \Phi(-{}_t\hat{h}_t) \geqq 0 \quad (> 0 \text{ for some } {}_t\hat{h}_t) ,$$

$$(56)\qquad \frac{\partial^2({}_t\hat{u}_t)}{\partial({}_t\hat{h}_t)^2} = \Phi'(-{}_t\hat{h}_t) \geqq 0 \quad (> 0 \text{ for some } {}_t\hat{h}_t) .$$

Since the optimal money wage policy amounts to choosing w_t^* that brings ${}_t\hat{h}_t$ into equality with a constant h^* , it also implies that ${}_t\hat{u}_t$ be equated with a constant u^* defined by

$$(57)\qquad u^* \equiv \int_{-h^*}^{\infty} z d\Phi(z) + h^*\{1 - \Phi(-h^*)\} .$$

We shall call u^* <u>the normal rate of involuntary unemployment</u>. Its value is determined by given subjective elasticities ϵ and η , by a given technological elasticity γ and finally by given subjective probability distributions $\Phi(z)$ and $\Psi(z)$.

For a later reference, it is convenient to calculate the expected value of the rate of profit (not the level of profit) conditional upon the information available at the beginning of period t . In view of (19), (26), (24), (8), (34), (39) and (35) we can show that

$$(58) \quad E_t(p_{t+\tau} + y_{t+\tau} - w_t - n_t)$$

$$= E_t\{E_{t+\tau}(g_{t+\tau} - j_{t+\tau} + p^*_{t+\tau} + y^s_{t+\tau}) - w_t - n_t\}$$

$$= g^* - j^* + E_t\left\{\frac{{}_t\hat{\alpha}_{t+\tau} - g^*}{\eta} + \frac{\eta-1}{\eta}k_t + \frac{\gamma(\eta-1)}{\eta}n^*_t - w_t - n^*_t\right\}$$

$$= g^* - j^* + E_t\left\{-\frac{\sigma}{\eta}(h_t - u_t) - \varphi + \delta\tau\right.$$

$$\left. - \frac{\sigma}{\eta}n^d_t + \left(\varphi - \delta\tau + \frac{{}_t\hat{\alpha}_{t+\tau} - g^*}{\eta} + \frac{\eta-1}{\eta}k_t - w\right)\right\}$$

$$= g^* - j^* - \frac{\sigma}{\eta}(h^* - u^*) - \varphi + \delta\tau ,$$

which is constant. We shall call this constant the firm's normal rate of profit. Its value is determined by η , ϵ , γ , $\Psi(z)$, $\Psi^{(\tau)}(z)$ and $\Phi(z)$.

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