COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 365

THE GENERAL EQUILIBRIUM MODEL IS THE WRONG MODEL AND A NONCOOPERATIVE STRATEGIC PROCESS MODEL IS A SATISFACTORY MODEL FOR THE RECONCILIATION OF MICRO AND MACROECONOMIC THEORY

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November 12, 1973

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THE GENERAL EQUILIBRIUM MODEL IS THE WRONG MODEL AND A NONCOOPERATIVE STRATEGIC PROCESS MODEL IS A SATISFACTORY MODEL FOR THE RECONCILIATION OF MICRO AND MACROECONOMIC THEORY*

by

Martin Shubik**

1. Introduction

This paper is devoted to a few basic comments on the modeling of economic processes. The detailed explanations of the assertions made here are given elsewhere.[12,13,16-29]

There are a few technical concepts required for the understanding of these remarks. Several are well known but several require somewhat more knowledge of the description of information conditions and the meaning of a noncooperative equilibrium in a multiperiod strategic process.

2. What is Wrong with General Equilibrium?

The general equilibrium analysis in its mathematical formulation has at least three major flaws, all of which are critical. They are as follows:

(1) It is independent of the number of competitors.

(2) It is a totally static tightly coupled system. It is error-free. As the individuals are nonstrategic they are not permitted by the solution concept to commit errors.

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*The research described in this paper was undertaken by a grant from the Office of Naval Research.

**The author is indebted to his colleagues Brainard, Koopmans, Scarf, Shapley, Slighton, Whitman and Whitt for many helpful discussions and in some cases collaborations.

(3) It implicitly requires symmetric information conditions, and, as has been indicated by Radner[9] does not survive other information conditions.

There is a literature on the relationship between the core of an economy and the competitive equilibria.[2] Some have believed that because when one uses the mathematical device of a continuum of economic agents a proof of the coincidence of the core and the competitive equilibria is obtained that somehow the logical justification of the role of numbers in competition has been provided. This is false. The core is a cooperative solution concept and is essentially as static and almost as nonstrategic and nonprocess oriented as are the mathematical models of the competitive equilibria.

The major solution concept which is both strategic and process oriented is the noncooperative solution first put forth by Cournot[4] and generalized by Nash.[8]

3. Some Basic Definitions

3.1. The Extensive Form and Information: The End of Tattonement Handwaving

One of the key contributions of von Neumann and Morgenstern[30] was to provide a method to describe a strategic process in detail. In particular this means that a complete description of a market requires the total specification of the method of exchange. The type of casual comment that has frequently been given to describe recontracting or bargaining is not sufficient to build a logically complete and consistent process model. This does not matter for general equilibrium analysis or for cooperative game theory because they are static and the bargaining, trading, communication and recontracting are implicitly assumed to take place costlessly outside of the model. This is a key way in which consideration of dynamics and process are swept under the carpet in those theories.

Consider a simple model with two traders A and B who borrow from the bank and them buy a product. Two examples of the extensive form of the game are displayed in Figure 1 and Figure 2. In Figure 1 the notation

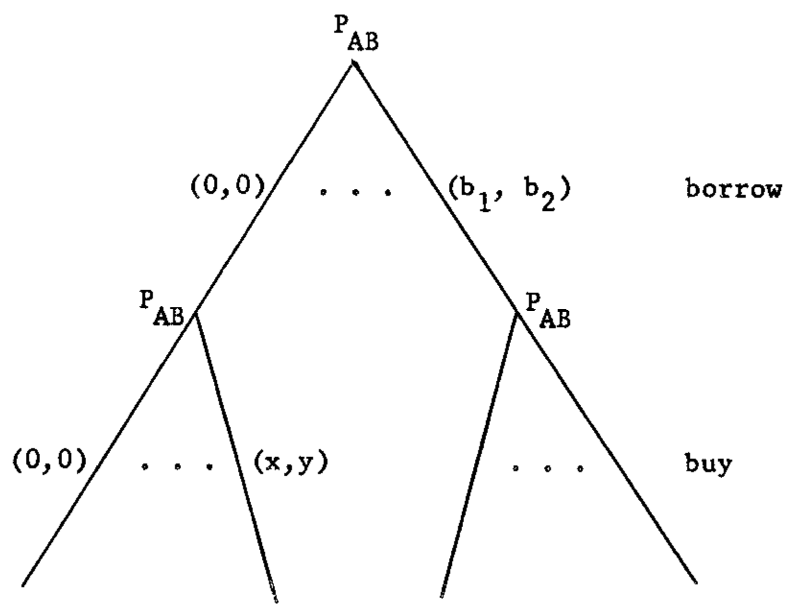

FIGURE 1

P_{AB} indicates that traders A and B move simultaneously when borrowing. They are then informed of the state of borrowing and move simultaneously again when buying. Each branch represents a pair of choices one by A and one by B, thus (b_1, b_2) indicates that A and B simultaneously borrow b_1 and b_2. In Figure 2 there is less information. Traders

A and B select both their borrowing and spending without knowledge of each other's action. Each branch is now indexed by 4 numbers the first two represent the borrowing and buying actions of trader A and the second two of trader B .

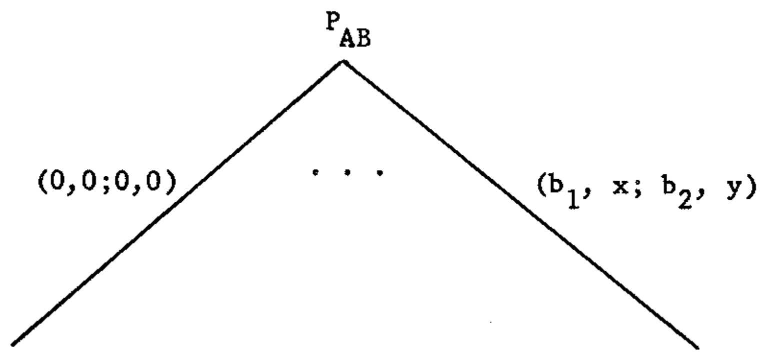

FIGURE 2

One might argue that such microdetail is not only too picayune but does not catch the flavor of partial knowledge in a market where sometimes A and B move without knowledge and sometimes A knows what B has done or viceversa. This can be modeled formally as is shown in Figure 3 for borrowing. In the first stage there are 4 states of Nature. They can be described as follows:

P_1 A borrows first and B finds out but A does not know

P_2 A borrows first and B does not find out but A does not know

P_3 B borrows first and A does not find out but A does not know

P_4 B borrows first and A finds out but B does not know

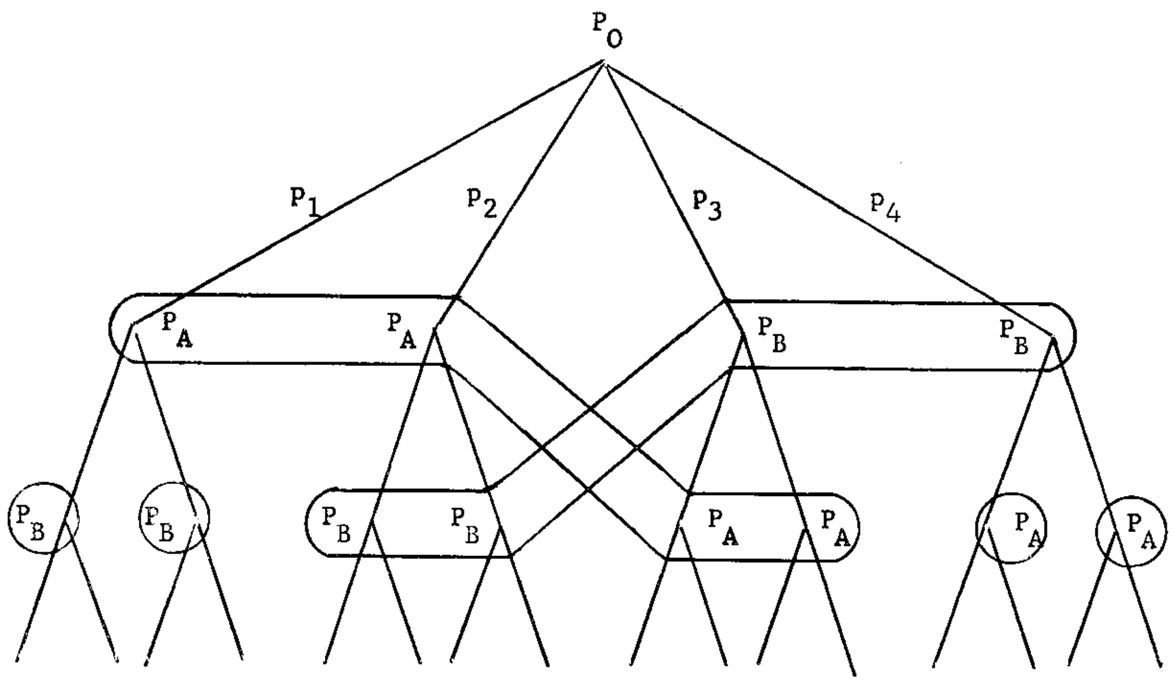

FIGURE 3

The points encircled by the closed curves are information sets, thus for example A is unable to distinguish among three states determined by p_1, p_2, and p_3 this is why when he is called upon to move the information set indicates 4 nodes among which A cannot distinguish. If the last event with probability p_4 were to happen then A would be informed of B's acts and the remaining two nodes calling for A's decision can be clearly distinguished by A.

Information conditions may easily have in important role to play in determining the behavior of a system. The methods of the extensive form provide the means to model any order, or probablistic order of market moves thus any tattonement or adjustment process can be made explicit and there is a relatively straightforward way to perform a sensitivity analysis over changes in trading information.

3.2. Noncooperative Solutions: State and Historical Strategies

Consider a simple game where two players A and B must simultaneously select a number 1, 2, or 3. They obtain a payoff and they play again. This can be shown in extensive form in Figure 4. Suppose that the payoffs are

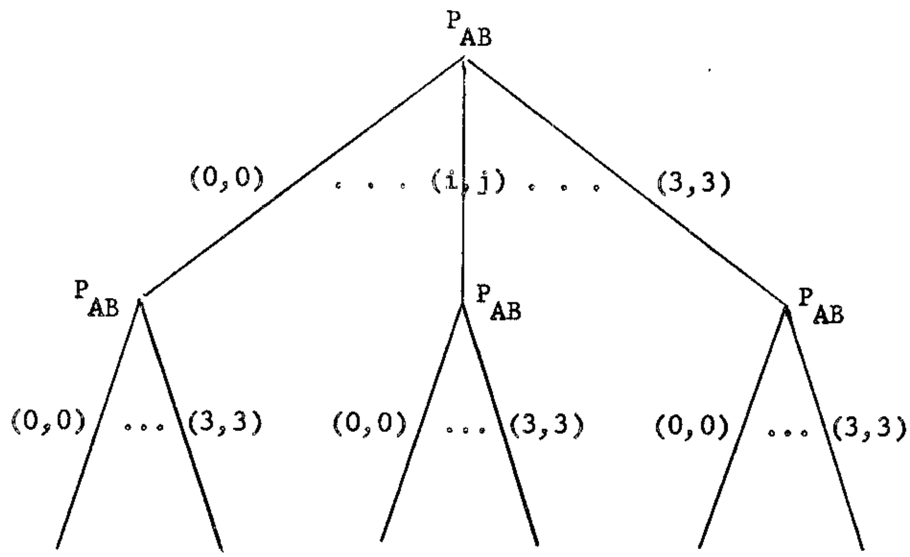

FIGURE 4

the same in each period and that the overall payoff to a player is the amount he obtains in Periods 1 and 2 added together. Table 1 shows what the payoffs are. The first number in each cell is the payoff to the first player and the second is the payoff to the second.

TABLE 1

	1	2	3
1	5,5	1,7	-10,-1
2	7,1	0,0	-10,-1
3	-1,-10	-1,-10	-10,-10

We may now illustrate a noncooperative equilibrium of two types: (1) A state strategy or perfect equilibrium and (2) a historical equilibrium.

Suppose the payoff to Player 1 is $P_1(s_1, s_2)$ where s_1 is a strategy for Player 1 and s_2 a strategy for Player 2. A pair of strategies (s_1^*, s_2^*) form an equilibrium if no player is motivated to change his strategy. This can be expressed as:

$$\max_{s_1} P_1(s_1, s_2^*) \Longrightarrow s_1 = s_1^*$$

$$\max_{s_2} P_2(s_1^*, s_2) \Longrightarrow s_2 = s_2^* .$$

In the example above where the 3x3 matrix game is to be played twice we can consider two types of strategy. One depends only on the state an individual finds himself in and the other depends on the history that carried him into that state.

Regardless of how the matrix game has been played the first time, the players at the second time period face a 3x3 matrix game. If they use a state strategy then each individual plays this new game with no reference to how the previous game was played, i.e. without contingency plans based on behavior in previous states. In this instance, as can be checked from Table 1, the strategy pair (2,2) form an equilibrium point in that game. If Player 1 selects his second strategy Player 2 can do no better than also choose his strategy 2 and viceversa.

By using a backward induction given that we know how the players will act in the last period we can now examine the first period. The players face the same 3x3 matrix game at that point hence the state strategy pair (2,2) will form an equilibrium in that game. Thus the overall strategy for each can be described as:

s_i = "I will choose 2 in the first game and 2 in the second game."

This pair can be called a <u>perfect equilibrium</u> in the sense that it is not only an equilibrium in the two stage supergame considered as an entity, but its components also form equilibria in every subgame considered independently.

A <u>historical strategy</u> makes explicit use of the history of the process. An example illustrates this. The historical strategy is as follows:

h_i = "I will choose 1 to start, if my competitor chooses 1 in the first period I choose 2 in the second period, if he does anything else I choose 3 in the second period."

A pair of strategies in this type form an equilibrium in the overall two stage game, but their components considered separately are not necessarily in equilibrium in the subgames. Thus the pair (1,1) is not in equilibrium in the first period.

The state strategies yield as payoffs $P_1(s_1, s_2) = P_2(s_1, s_2) = 0 + 0 = 0$ and the historical strategies* yield $P_1(h_1, h_2) = P_2(h_1, h_2) = 5 + 0 = 0$.

The historical strategies pick up the basic elements of threat and trust in the evolution of the process. These features are not necessarily high information content phenomena. A mass process can still have individuals use historical strategies. Thus for example a policy of the following sort may be interpreted as such a strategy:

> "If the rate of inflation has risen in the last two periods I will double the size of my debt; if it has stayed constant or fallen I will pay back ten percent of my debt."

The behavior in this case is contingent on an aggregate historical statistic.

The noncooperative behavior model of an economy is naturally designed to describe information states in detail, perform sensitivity analyses on changes in information and to distinguish state and historical strategies. The general equilibrium approach is not suited to take care of any of these features.

4. The Need for Money and Its "Disappearance" in Equilibrium

It is asserted here that the general equilibrium model provides a structure for trade where all futures markets exist and *a fortiori* trust is complete among all traders. Whereas in the actual world few

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*By formal mathematical redefinition it is possible to complicate the state space description to include previous periods as part of the current state thus preserving a state description which includes some history. It is not fruitful to do this here.

futures markets exist and money and ownership claims are used heavily to substitute for the need for trust. A noncooperative model with trade in money[13,18] provides the appropriate way of modeling the second system instead of the first.

The Paradox: When the noncooperative model is in a state equilibrium it gives the same results as those given by the general equilibrium model. In this equilibrium the complete monetary mechanism apparently disappears. No cash flow constraints are binding and only the budget constraint plays a role.

When the noncooperative process is not in a state equilibrium then the monetary mechanism along with institutions and laws (such as bankruptcy) needed to define the rules for calculating payoffs in the disequilibrium state all appear. The disequilibrium is guided by cash flow constraints and other aspects of the financial mechanism.

The general equilibrium model describes the functioning of the system in only a "straightjacketed" state. The type of handwaving done with discussing recontracting or tattonement, the ignoring of bankruptcy conditions and the slurring over the description of information conditions are all the earmarks that show why the general equilibrium model is unable to even fully describe the state of the economic system when in disequilibrium.

4.1. The Missing Degree of Freedom and the Edgeworth Box Paradox

One needs no more complicated a situation than the Edgeworth bilateral monopoly model to see why there is a strategic need for an extra degree of freedom in describing a noncooperative game model of the economy.

Unlike the general equilibrium model the noncooperative description must specify outcomes for all positions of disequilibrium as it must provide a complete process description of the economy.

Suppose we were to try to model an n-person closed trading economy as a noncooperative process. If we wish to give each trader strategic freedom then we must have a mechanism for accommodating a mismatch in plans. Otherwise the specification of the strategies for $n-1$ individuals would be tantamount to specifying the outcome for the n^{th} individual independent of his strategies. A natural way to introduce the extra degree of freedom is to have trade in fiat money.[18,29]

When the system is in equilibrium, all of the n strategies or plans completely mesh and all commodity trades balance hence the system apparently gets along without having to resolve the problem of meshing n degrees of strategic freedom with only $n-1$ degrees of freedom in having all the books balance. The open models of noncooperative behavior in oligopoly theory such as those of Cournot and others[3,14] work because the money mechanism involving exchange of physical goods for money in the open system provides the extra strategic freedom. When the strategies of $n-1$ oligopolists have been specified the n^{th} individual can still do as he wishes because the oligopoly open model conserves neither goods nor money.

4.2. Disequilibrium Institutions and Law

The noncooperative game model is messy. To a certain extent it appears to call for a great amount of ad hoc modeling. Institutional details such as "If no one bids any money for the goods in the k^{th} market what happens to the goods?" have to be resolved. Precise orders of moves must be specified. Even though one can see plainly that in a given system there are a reasonable set of optimal outcomes which should contain the solution it is still necessary to give the rules to cover the possibility that all traders take (what appears to be ex post facto) a set of ridiculous acts. If the acts are feasible no matter how far the outcomes may be from any reasonable equilibrium the mechanism to define the payoffs must be defined.

In order to define the payoffs in all disequilibrium states, rules covering items such as supply but no demand, bankruptcy, seniority of creditors must be specified even if they cover states that would be deemed by all to be suicidal. The criterion is feasibility not desirability.

The law, trading customs and financial institutions provide the ad hoc rules needed to completely define the noncooperative process. One suspects that there are many variants in defining all of the rules needed. But they have to be fully defined somehow before the market can be analyzed adequately. Thus a mathematical institutional economics[14] is called for to fully specify the process.

How do items such as banks, fiat money, ownership claims and so forth fit into an overall model of the economy? Modeled as a game with trade in money they fit relatively easily because the system is being expressly modeled to stress the loose coupling. People are permitted to make strategic blunders and to have plans fail to match. The cost price of this type of modeling is that more rules and institutional detail than are aesthetically pleasing are called for.

In equilibrium when plans do match the rules and the mechanism provided by the financial infrastructure apparently vanish but they are present to provide the control system when equilibrium is not present.

5. Three Limiting Processes

It is argued here that there are three key factors which need to be considered before an adequate model of mass economic behavior can be formulated. They are the role of:

(a) numbers

(b) time

and (c) information.

5.1. The Role of Numbers in an Economy

The general equilibrium model is static and independent of numbers of participants. The core of an n person economy is also static even though it depends on the number of participants.[7,15]

Recently the work of Aumann utilizing a continuum of players has established an equivalence between the equilibrium points and the core.[2] This result is by no means as general as it may seem, primarily because it is based upon a static model of the closed economic system.

The Arrow-Debreu model[1] is nonstrategic. The players are not modeled as minute agents with strategic freedom to commit errors. They are modeled as automata. There is no need in either the general equilibrium or in the core model to ever examine the system in disequilibrium in explicit detail.

The paradox of the possible coincidence of the price system in a socialist economy with the price system described by Walras,[31] Arrow and Debreu[1] is no paradox at all. Both are nonstrategic and static. The mathematical structure that has been solved does not contain the key aspect of a competitive system--to wit competition and all its attendant disequilibrium. The recent work on the core and other cooperative game theoretic solutions also do not contain the key element for the formation of a competitive price system. On the contrary the core is a high information and communication solution concept. As an early proponent of the core and of the replication process for studying mass economic behavior[15] I am completely willing to admit that to a great extent the results on the core helped to direct attention from the understanding of the competitive process in low information and communication decentralized mass markets.

The strategically oriented solution concept that at least tries to capture the elements of decentralized competition is the noncooperative solution that was first described by Cournot[4] and considered under replication by Shubik[14] and others. As has already been noted these models are open or partial equilibrium models. The books do not have to balance. The basic difficulty is to get these models into the context of a closed economic system, then to see what happens to the nature of competition as numbers increase.[18]

5.2. The Role of the Infinite Horizon

In a model with a finite horizon, a banking system, a nonzero interest rate and a nonconstant supply of money the financial books cannot balance in general. With an infinite horizon the books never have to balance. The system can function without default as long as debtors' incomes equal or exceed the burden of the debt.

5.3. Information Needs Related to Time and Numbers

An underlying assumption in most verbal descriptions of competition is the idea that as the number of competitors increases they use relatively less communication and have less detailed information. The process becomes more impersonal and decentralized and is based more on information in the form of aggregated statistics than detailed information on individual competitors.

The key aspect of many economic activities that differentiates them from the viewpoint of information processing and coding from say political or societal activities or from abstract games is that a natural metric exists on many of the strategies. In mass markets, for example, for wheat, the information that 2,000,000 tons were produced last season is probably more useful to most buyers or sellers than is a detailed list of the quantities produced by each individual farmer.[28]

In verbal descriptions on the evolution of competitive markets it is common to start to describe a market with few traders in close communication and with presumably considerable information, then to invoke the "force of competition," "recontracting" or some other *deus ex machina* to support the claim that with many competitors an efficient price system emerges. In neither the verbal nor the mathematical formulations are the changes in information conditions between the cases with few and many traders made explicit. It is reasonable to expect that as numbers of individuals and goods and time periods become large the information used per individual must become more and more highly coded or aggregated. The macroeconomic models present essentially this type of information representation. However it is also possible to model this type of information limit formally in a microeconomic model.[28]

When the number of competitors is few* there is little doubt that the nature of the disaggregation of information plays a critical role in shaping oligopolistic competition.

6. Planning, Expectations and Dynamics

6.1. Solution Concepts and Macroeconomics

It is argued here that the noncooperative solutions contain the general equilibrium solutions and are far more general, far more institutional, process oriented and well defined than the latter. This is because the defining of payoffs for all disequilibrium states and the act of being explicit about information conditions provides the means to cut the Gordian knot that has tied up microeconomic analysis for many years. That is the description of tatonnement or recontracting.

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*Although the advanced methods involving the use of measure theory to describe class of small economic agents may be fruitful to obtain certain results, this type of mathematics obscures and makes more difficult the analysis of information conditions and the extensive form.

The model presented in the papers related to this[12,13,16-29] is simplicity in itself. It portrays markets as simple clearing houses where an amount of simultaneously bid money chases an amount of goods. It might well be argued that this model is too simplistic. That is not the point. It is well defined and logically consistent and gives interesting results. It does not avoid the detailed description of trading. It gives a completely specified, albeit simplistic description of how trading takes place in equilibrium or disequilibrium. Furthermore it specifies both money flows and ownership paper and goods flows.

It is my belief that many other, more complex, more refined and more "realistic" models of the trading process could be defined to specify slightly different noncooperative processes than the one modeled here. Without the presence of exogenous uncertainty* it is likely that there is a large class of somewhat different noncooperative models which differ from each other in institutional details to handle disequilibrium yet have the same set of state equilibria which turn out to be the competitive equilibria of the system modeled without attention paid to its institutional and strategic details.

It has been noted in 3.2 that a distinction can be made between historical and state strategies. Furthermore in 5.3 the role of aggregate information was stressed. The historical strategy models based on aggregated information can be interpreted easily in terms of macroeconomic models of behavior. Thus it can be shown that strategic models of ex ante

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*The presence of uncertainty introduces problems in the evaluation of risk and it is possible that if linear utility functions are assumed it may be such that there are stronger limitations on optimal laws and institutions, such as bankruptcy, than otherwise.

rational behavior constructed from microeconomic principles yield macroeconomic behavior models. Furthermore *ex ante* rational behavior by no means implies an *ex post* desirable outcome as much business cycle theory and game theory concerning the Prisoners' dilemma illustrates.

The above observation is *not inconsistent* with the static general equilibrium solutions also being solutions to a microeconomic noncooperative strategic market model.

6.2. Financial Control not Real Goods Dynamics

It is suggested here that trade in mass markets with independent decisionmakers calls for a process that can at best be an extremely low information phenomenon given the exponential increase of information states with numbers. Furthermore it is argued that *ceteris paribus* trade with individual trust calls for more information and coding of information than does trade which does not require as much individual trust.

It is further suggested that a mass economy with individual decisionmakers making independent decisions under low information calls for a *process* which handles the attendant disequilibrium and mismatch arising from the imperfect meshing of separate plans. A monetary system with the exchange of consumer goods and ownership claims backed by assets provides such a process.[26]

In the running of the economic system in disequilibrium the guiding mechanism is provided primarily by the monetary mechanisms and trade in ownership claims. *Money is not a veil*; the monetary and financial system is the neural network, or controlling cocoon surrounding and guiding the real system.

In order to study the evolution of the time path of the real economy and its stability properties the type of dynamic general equilibrium models which are constructed to deal with excess demand or supply perturbations relating solely to the real goods are not adequate to deduce the time paths, the disequilibrium or equilibrium states of economies which use money and trade ownership claims.

Even though a real goods economic system may have multiple equilibria and a simple nonmonetary dynamic model may indicate instability or cycles among all of them; the dynamics associated with the same physically describable system with a monetary control mechanism added will be different. The cash flow constraints, banking rules, bankruptcy and other features needed to completely define the system states in disequilibrium provide extra guidance and constraints on the system dynamics.[24]

6.3. The Role of Uncertainty

There are three fundamentally different sources of uncertainty which must be noted. They are:

(1) Strategic uncertainty,

(2) exogenous uncertainty,

and (3) endogenous uncertainty due to information aggregation.

The meaning of strategic uncertainty is discussed in 6.4 in the context of individual and mass behavior and prediction of behavior.

The other two types of uncertainty influence the structure of any financial system needed to guide an economy in somewhat different but related ways.

Exogenous uncertainty arises from natural phenomena such as weather changes or unknown natural resource distribution; perceptual problems in evaluating the success of new processes or products and so forth.

If we are willing to make the assumption that in this world information is not a free good and that the processes of encoding and decoding information are also not free, then it follows that even if it were technologically feasible for each individual to obtain full or perfect information concerning the state of his environment, almost always he would not do so. The economically viable information conditions call for a high level of aggregation. But a system with aggregated information is a system with uncertainty. Even if there were originally no exogenous uncertainty in the system if each individual obtains only aggregated information he receives only a statistic on the system state, and not a full description. Hence at least for some decisions he is faced with uncertainty. This, of course, is one of the fundamental problems faced in the design of accounting systems.[32]

When an economic system has exogenous or endogenous uncertainty present it becomes necessary to define rules which cover the array of phenomena covered in the actual world by insurance, bankruptcy laws and contract renegotiation. In a world without uncertainty the presence of enough assets may serve as a sufficient supply of hostages.[26] In which case, trust and forward contracts are not needed because all trade can be secured. In a world with uncertainty this is not so. If A lends B his cow and B puts up his house as security and both value the house above the cow, in a world with certainty A will get his cow back. In a world with uncertainty the cow can die and the house burn down. In other words the system can reach states where contracts cannot be fulfilled.

An adequate treatment of mass markets with uncertainty and with trade in money will probably show behavior akin to that of mass particle motion in statistical mechanics, however these models have not yet been investigated.

6.4. Planning, Prediction and Behavior: The End of Perfect Foresight

The formal definition of a state noncooperative equilibrium in a dynamic game[19,29] (see also 3.3) may appear to be somewhat stilted and give rise to the same sort of mental gymnastics concerning perfect foresight that has appeared in the literature over the years.

Fortunately given the underlying structure of the process in terms of spot markets, a simple straightforward planning or behavioral interpretation of process can be given. The noncooperative state equilibria are maintained essentially by all individuals having self-fulfilling predictions of the future. The existence of the noncooperative equilibrium is equivalent to stating that there exists a set of predictions concerning future prices and supplies such that if each economic agent has these predictions and each maximizes his welfare on the assumption that these predictions are true they will in fact be true.

If the predictions are wrong the system will not necessarily be in equilibrium yet the models for trading, consumption and income flows in the papers referred to[12,13,17-29] are still defined. The specific dynamics of adjustment pose deep unsolved problems. Shubik and Whitt[29] have considered some simple examples solved as parallel dynamic programs however one might wish to adopt a more direct behavioral planning approach based on specifying methods for generating finite horizon predictions and operating on these predictions to determine the next move of each agent. The literature on business cycles is rich with models of this type.

It should be stressed that as money and spot markets play a critical role in the models referred to here the dynamics in even a modified simple model for a phenomenon such as the hog cycle will be different. As any farmer knows cash flow constraints and foreclosures modify the cycle.

The role of banking is not discussed in this paper. Some observations on the problems entailed in introducing banking are presented elsewhere.[27] One comment is called for here. When there are no laws of conservation concerning the amounts of money in a dynamic system it may well be that the same physical distribution can be associated with a fixed, inflating or deflating set of prices. Setting aside the governmental control or political aspects of deflation or inflation a reason for running say a stationary system with stationary rather than inflating or deflating prices is that for planning purposes it provides a most efficient coding of information. Put less fancily if there is no particular economic purpose served or denied by inflating or deflating a price system then a good criterion to select among different economically satisfactory price systems is to minimize the information requirements of the economic planning units.

6.5. The Government Sector and Public Control

The work referred to here has been directed towards building a microeconomic model of an economy with a monetary system. A monetary system does not imply a banking system or a government financial system. However, once a monetary system is in place it is fairly straightforward to see how a banking and governmental financial system can be connected in easily and naturally for control purposes.

As far as the private sector is concerned the government and banking system will supply an extra set of inputs which must be regarded as (1) exogenous inputs, (2) inputs which can be predicted from behavioral considerations or (3) a new set of strategic players. Regardless of which one is selected this indicates that ploys such as "open mouth" operations and factors such as erratic national policy decisions must have a real and possibly important effect on the dynamics of the system in the sense that the dynamics depends directly on the individual forecasting. This observation, of course, is not new. What may possibly be new is that in the type of model suggested here not only is it not new but it is also consistent with a microeconomic model which in turn contains the general equilibrium microeconomic model as a special and incompletely specified case.

6.6. Accounting, Income and Barter

The construction and analysis of microeconomic models in which money plays a central role is simplified considerably by the device of imagining that at every period all of the goods in the economy are put up for sale. Thus for an individual to continue to own his house or factory he must buy it back every period. This, at first, sounds bizarre. However it has some important good features. In particular as all of economic life is thereby monetized it means that the actual income of every individual in terms of money is measured every period. This is a considerable improvement on the system that exists in most parts of the world where wealth maximization depends heavily on avoiding having to account for many income flows.

Looking at modern economies from the viewpoint of their degree of monetization and comparing them with barter systems it is probably fairly accurate to say that no more than 30% of any economy is out of the barter stage in the sense that most economic transactions are self-barter transactions where year in year out individuals exchange their own assets with themselves without having to report the resultant gains or losses until they engage in a monetary trade with someone else.

The 30% suggested above was arrived at by a very crude rule of thumb calculation that in the U.S. most consumer goods and services (except housewife services and other home labor) go through the markets and monetary system. The asset to final product ratio is of the order of somewhere between 5:1 and 15:1 say 10:1. Around 20% of the assets or the papers representing their ownership may be exchanged each year. The remaining 80% does not get monetized they are in terms of the modeling suggested here bartered by their owners to themselves in unreported transactions.

6.7. On Futures Markets

The noncooperative process that has been formalized in the papers referred to utilizes only spot markets. In contrast the general equilibrium analysis calls for the simultaneous existence of all futures markets. In the world we live in it is important to note that there is a limited amount of futures trading (compared with spot) but even every trade in a future has a spot component. You may buy a wheat future for cash now. You pay for the wheat on delivery in the future but you pay for the wheat future in the spot market for futures.

If spot futures markets were to be introduced into the noncooperative models constructed they would change the cash flow requirements and would also provide the appropriate intermediate model between a pure noncooperative spot market in present goods on the one hand and a totally cooperative model which has all futures markets operating simultaneously.

7. Concluding Remarks

7.1. On Avoiding Misleading Problems

It is my belief that the essence of a viable mathematical model of a competitive process was within the writings of Cournot[4] but it is easy to be sidetracked by premature calls for "realism" or relevance in modeling. Furthermore simple mathematical descriptions are at a considerable disadvantage when compared with verbal descriptions because it is far easier to spot the lack of realism and the weaknesses of the former than it is to spot the logical inconsistencies or lack of completeness of the latter.

Among the more important red herrings serving to misdirect research effort away from the key aspects of understanding an economy with money and financial institutions were the following:

(1) The barter model of Edgeworth as the paradigm for competition.

(2) The core which preserves the role of numbers in a mass market model but is implicitly static and calls for a highly coordinated high information content friction free system.*

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*mea culpa

(3) The general equilibrium theory which is not a model of competition. It is independent of the number of competitors. It does not fully specify the strategic alternatives for each economic agent in the sense that the payoffs for all actions including all disequilibrium states are not defined and it is essentially vague on both information conditions and information processing requirements.

(4) The concept of perfect foresight which was nothing more than a monument to the type of semantic confusion that can arise when one fails to well define a model in the detail necessary.

(5) The verbal discussion of tatonnement and recontracting which amounted to an avoidance of being explicit about disequilibrium payoffs, processes and information states.* This may have come about by a misplaced worry concerning the lack of "realism" of the market processes modeled in most explicit microeconomic models and macroeconomic models of bargaining processes or business cycles.

(6) The concern for multiple equilibria and real goods dynamics which can provide a limitless source for nontractable mathematical problems. There are clearly a host of open questions concerning the dynamics of systems with money and financial institutions. But given that one has initial conditions which involve not merely the distribution of physical goods but of money and ownership claims as well; the dynamics may be

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*The work of J. Cross provides a worthwhile exception.[5]

considerably different. Even were this not the case it is quite likely that the resolution of the multiple equilibrium problem lies not so much with the mathematics as with the introduction of more economic process detail. An economic system that functions may well be an inelegant and somewhat institutional mechanism. Mathematical elegance is by no means the earmark of good applied mathematics. The use of mathematics in economics is as applied mathematics.

(7) "Real" transactions costs which divert one's attention from the more critical transactions costs which are due to information gathering encoding and decoding. Transactions costs such as transportation costs provide us with a veritable mine of elegant and only tangentially relevant problems for our understanding of a monetary economy. For example we might wish to calculate how many bilateral trades are needed to achieve equilibrium in an economy with trade with a money but only pairwise trading. Problems such as this may well be elegant exercises in combinatorics and may even suggest certain optimal design features for markets. But if one's goal is to illuminate the role of money and financial institutions in the economy the returns from this type of work may not be commensurate with the effort required.

7.2. Mathematical Difficulties and Behavioral Systems

The multiperiod models which have been formulated appear as simultaneous interlinked dynamic programs. It is conjectured that there will be two different levels of mathematical difficulty encountered in dealing with them. For the models for which it is expected that there is a coincidence of the competitive equilibrium points and the noncooperative state strategy equilibria there may be a possibility to use much of the formal theory of the former to extend the results to the latter.

The investigation of actual dynamics does not appear as promising. As has been noted in 6.4 an alternative mathematical approach is to use direct and simpler models of behavioral processes.[10]

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