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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART XII

A DYNAMIC ECONOMY WITH FIAT MONEY WITHOUT BANKING AND WITH AND WITHOUT PRODUCTION GOODS

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A DYNAMIC ECONOMY WITH FIAT MONEY WITHOUT BANKING AND WITH AND WITHOUT PRODUCTION GOODS*

by

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1. Introduction

This paper sketches a set of conjectures concerning the functioning of an economy using fiat money with no futures markets and no barter. Examples are provided to indicate the plausibility of the conjectures. Hopefully in a further joint paper with W. Whitt a more careful and general mathematical presentation of models together with proofs of several of the conjectures will be given.

2. The Basic Model

We limit our consideration to concave utility functions of the form:

(1) $$U_i = \sum_{t=0}^{\infty} \beta_i^t \varphi_i(x_{1,t+1}^i, \ldots, x_{k,t+1}^i) ,$$

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where there are k commodities and n individuals. Although it is conjectured that the further remarks hold for utility functions of type (1) and for economies with production obeying certain bounding conditions, the selection of examples is confined mainly to the starkly simplified case with

$$U_i = \sum_{t=0}^{\infty} \beta_i^t x_{t+1}^i \quad (2)$$

Furthermore unless otherwise specified production and inventorying of goods is not considered.

The Commodities

There is one commodity each period. It cannot be stored. It appears each period in the central warehouse where it is immediately auctioned in a manner described below.

The commodity input stream is:

$$A_1, A_2, A_3, \ldots, A_t, \ldots$$

which has an upper bound.

The Money Supply

For the model without banking we may assume that there exists a fixed finite amount of fiat money M (which could for example be regarded as even having a physical existence). This money has an initial distribution of

$$\gamma^1, \gamma^2, \ldots, \gamma^n .$$

Without further loss of generality we can assume $\sum_{i=1}^{n} \gamma^i = M = 1$.

Without any banking or credit facilities in the economy the amount of money in each period is constant; all trade is at spot prices and all money must be used either in trade or it must be hoarded.

Conservation of money is preserved for all periods in the finite or infinite horizon models.

Ownership and Trade

As in previous papers[2,3,6] the convention for ownership and trade is as follows. The i^{th} individual has claims to α_t^i of the amount of goods A_t at period t . Thus $\sum_{i=1}^{n} \alpha_t^i = 1$.

The meaning of ownership is that after the pile of goods A_t have been sold in period t trader i obtains at the start of period $t+1$ the amount α_t^i of the money expended to buy A_t . This is shown in Figure 1.

The price mechanism is extremely simple. All traders are assumed to bid an amount no larger than their cash on hand. Cash on hand will be the sum of the income from last period together with hoarding, or $S_{t+1}^1 = S_t^1 - x_t + \alpha_t(x_t + y_t)$. Where S_t^i is the total monetary assets of trader i at the start of period t . In Figure 1 the market is illustrated for two traders where $S_1^1 = \gamma$ and $S_1^2 = 1-\gamma$.

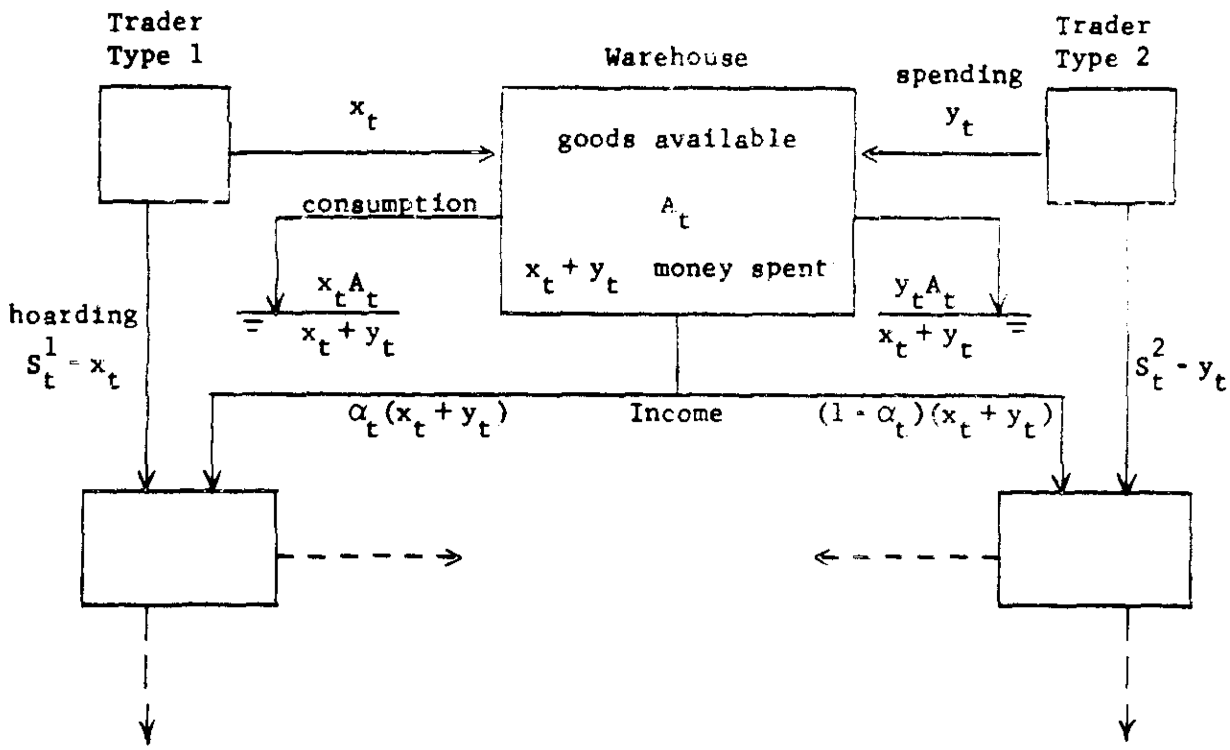

FIGURE 1

2.1. The Simple Market with Constant Inputs

In a previous paper Shubik and Whitt[6] have provided a detailed exploration of the market with fixed ownership claims and the same amount of commodity entering each period. This can be described briefly for the two person market as follows:

Commodity supply	1, 1, 1, ...	
Ownership claims	α , $1-\alpha$	
Initial money	$\alpha+\gamma$, $1-\alpha-\gamma$	$0 \leq \gamma \leq 1-\alpha$
Utility functions	$U_i(\tilde{x}) = \sum_{i=0}^{\infty} \beta_i^t x_{t+1}$	

It was possible to characterize the noncooperative policies for the traders and to show that the trader who initially begins with a larger proportion of the money than his ownership claim may under certain conditions hoard money, whereas the trader with money in proportion less than his ownership will always spend everything if $\beta_1 = \beta_2$. Thus we have an economy with a fixed amount of resources each period and a fixed amount of money with hoarding and a gradual rise in price level until a stationary state with a fixed price level is reached.

It is easy to observe that in this extremely simple model if $\gamma = 0$ there is an immediate stationary state solution in which the traders all spend everything. The same amount of money recirculates back to each, each period and they each obtain an amount in proportion to their ownership claims.

A natural set of questions to ask of this simple model is what would happen in the amount of the commodity were varied each period, what would happen with variations in preferences or with variations in ownership claims. In particular a question of interest is does this model have a noncooperative perfect equilibrium point[3] solution which produces a Pareto optimal distribution of goods that can be associated with a competitive equilibrium solution to the same market.

The answer to this question appears to be yes under certain qualifications involving hoarding and ownership claims.

In a previous paper[5] an example was given of a market with a period shift in the preferences of two classes of traders. In this market there is always net hoarding in the economy and the distribution of resources is Pareto optimal. A numerical example is summarized below:

Example 1. Suppose there are two types of traders, $2n$ *in toto*, where n is large. There are n units of money in the economy and n units of real good each period. A trader of Type 1 starts with γ units of money and Type 2 starts with $1-\gamma$. Suppose that the utility functions for the traders are respectively:

$$U_1 = x_1 + \epsilon_1 \beta x_2 + \beta^2 x_3 + \epsilon_1 \beta^3 x_4 + \ldots$$

$$U_2 = y_1 + \epsilon_2 \beta y_2 + \beta^2 y_3 + \epsilon_2 \beta^3 y_4 + \ldots$$

The competitive equilibrium prices, given $\epsilon_1 > \epsilon_2$ and $\beta < 1$ are: $1, \epsilon_1\beta, \beta^2, \epsilon_1\beta^3, \beta^4, \ldots$ with the distribution of real resources being for:

Trader 1: $(1 - \epsilon_1\beta)/2, 1, (1 - \epsilon_1\beta)/2, 1, \ldots$

and Trader 2: $(1 + \epsilon_1\beta)/2, 0, (1 + \epsilon_1\beta)/2, 0, \ldots$

Their total payoffs are for:

Trader 1: $\dfrac{1 - \epsilon_1\beta}{2(1-\beta^2)} + \epsilon_1\beta \dfrac{1}{(1-\beta^2)} = \dfrac{1 + \epsilon_1\beta}{2(1-\beta^2)}$

Trader 2: $\dfrac{1 + \epsilon_1\beta}{2(1-\beta^2)} + 0 \qquad = \dfrac{1 + \epsilon_1\beta}{2(1-\beta^2)}$

For ease in following a specific example suppose $\beta = 2/3$, $\epsilon_1 = .9$ and $\epsilon_2 = .1$. We have for competitive equilibrium:

"futures" prices	1,	.6,	.44,	.264,	.193, ...
Trader 1 consumption	.2,	1,	.2,	1,	.2, ...
Trader 2 consumption	.8,	0,	.8,	0,	.8, ...
Trader 1 payoffs			1.44		
Trader 2 payoffs			1.44		

The noncooperative equilibrium which gives the same distribution of resources has the following monetary features:

Initial money distribution	$\gamma = 3/11$, $1-\gamma = 8/11$
Total money	n, n, n, n, ...
Spot prices	10/11, 6/11, 10/11, 6/11, ...
Trader 1 hoarding	1/11, -- 1/11 --
Trader 2 hoarding	-- 5/11 -- 5/11

This solution tells us that if banking were available then there will exist a money rate of interest for which traders of Type 2 would borrow during odd periods and Type 1 would borrow during even periods. This point is examined in detail in the next paper.

2.2. Failure of Pareto Optimality

Example 2a. Consider the same conditions as in Example 1, except that the utility functions and the initial money holdings are different. In particular:

$$U_1 = \sum_{t=0}^{\infty} \beta_1^t x_t \quad \text{where} \quad 1 > \beta_1 > \beta_2$$

and

$$U_2 = \sum_{t=0}^{\infty} \beta_2^t y_t \ .$$

The competitive equilibrium associated with this market is as follows:

"futures" prices $1, \beta_1, \beta_1^2, \beta_1^3, \ldots$

Trader 1 consumption 0, 0, 0, ..., 0, 1, 1, 1, ...

Trader 2 consumption 1, 1, 1, ..., 1, 0, 0, 0, ...

Traders of Type 2 consume up to a switch-over point determined by calculating t^* such that:

$$\frac{1 - \beta_2^{t^*+1}}{1 - \beta_2} = \beta_1^{t^*+1}\left\{\frac{1}{1 - \beta_1}\right\}.$$

For example given $\beta_1 = (\sqrt{33} - 3)/4 = .68616$ and $\beta_2 = .5$ then $t^* = 2$ and $U_1 = U_2 = 1.50$.

It is immediately obvious that there is no initial distribution of money that will give the competitive equilibrium resource distribution from the noncooperative spot trading. After the first t^* time periods traders of Type 2 will have some monetary assets, but for the system to give the competitive equilibrium distribution they would be required to hoard all of their money indefinitely.

Although it is possible to maintain equilibrium with net hoarding positive it is not possible to have one group hoard every period. An expenditure of the hoard at any point where goods of worth are available yields an improvement in payoff.

2.3. One Period Hoarding as a Means of Limited Credit

Example 2b. There exists a noncooperative equilibrium which is not Pareto optimal with $\gamma = .21076$ and $m = .15694$ where prices are given as:

Spot prices	.84306,	.578474,	.84306,	.578474
Trader 1 hoarding	.15694	--	.15694	--
Trader 2 hoarding	--	.421526	--	.421526
Trader 1 consumption	.06384	1	.06384	1
Trader 2 consumption	.93616	0	.93616	0

Payoff to Type 1 = .75(1/.52918) = 1.4173

Payoff to Type 2 = .93616(4/3) = 1.2482

The difference between the solution to Example 2a, and the solution here provides us with a <u>measure of the worth of trust</u>. A noncooperative state equilibrium is generally different from a general equilibrium solution. They only coincide when the level of trust implicit by the later turns out to be unnecessary for the former. The solution to Example 2a was not self-policing there is no internal mechanism that gives Traders of Type 2 an incentive to hoard their money for ever after period t^*. There is a clear incentive (which would be even clearer if U_1 and U_2 were strictly concave) in Example 2b for all parties to conform.

2.4. The Restoration of Pareto Optimal Trade by Assets

In Example 2a there is no enforcement mechanism in the stage by stage behavior to force traders of Type 2 to stop spending at period t^*+1. Without unlimited trust the traders can settle for a noncooperative equilibrium that is not Pareto optimal.

By introducing durable goods or assets of worth we can restore the possibility for obtaining a noncooperative equilibrium that is optimal and provides the same distribution of real goods as does the competitive equilibrium.

Heuristically, the assets can serve as <u>hostages</u> for the noncooperative enforcement of the equilibrium.[4] Mathematically, the enforcement required can be described as follows: one cannot have an equilibrium in which any one type of trader hoards money in an uninterrupted method for ever. This amounts to saying that he has more money always than any of his future cash constraints call for.

Unfortunately in order to achieve Pareto optimality, as has been shown in <u>Example 2a</u> such a situation may be required. If we could always guarantee that the individual will have a cash flow requirement in the indefinite future at which he will need all of his cash we may be able to produce a situation which is operationally like hoarding forever. This can be done by monetizing all assets every period and having individuals buy back their own assets.

Example 3. This is essentially the same as Example 2 with one major modification. There exists a quantity of a completely durable producer good which produces the consumer good. For example suppose that we have as input 1 unit of the producer good at time t and as outputs 1 unit of the producer good plus 1 unit of the consumer good at time $t+1$.

We assume initial endowments as follows: Real goods at $t = 1$, n units of producer good, n of consumer good. Ownership claims:

	P	C
Trader Type 1	1/2	1/2
Trader Type 2	1/2	1/2

P = producer good

C = consumer good

We now consider two parallel markets. In the first the consumer good is sold. In the second ownership claims are sold. The proceeds from the sale of the consumer goods are paid to the last owners as are the proceeds from the sale of the ownership claims.* Figure 2 illustrates the markets. It, of course, may make a difference if they are considered in sequence or in parallel as the information conditions are different. For ease we select the simplest, which is a parallel market.

*There exists a definitional problem to cover what happens when nothing is bid in a market. In a maximizing process such as this the probability that this could happen is extremely small, but the case has to be handled.

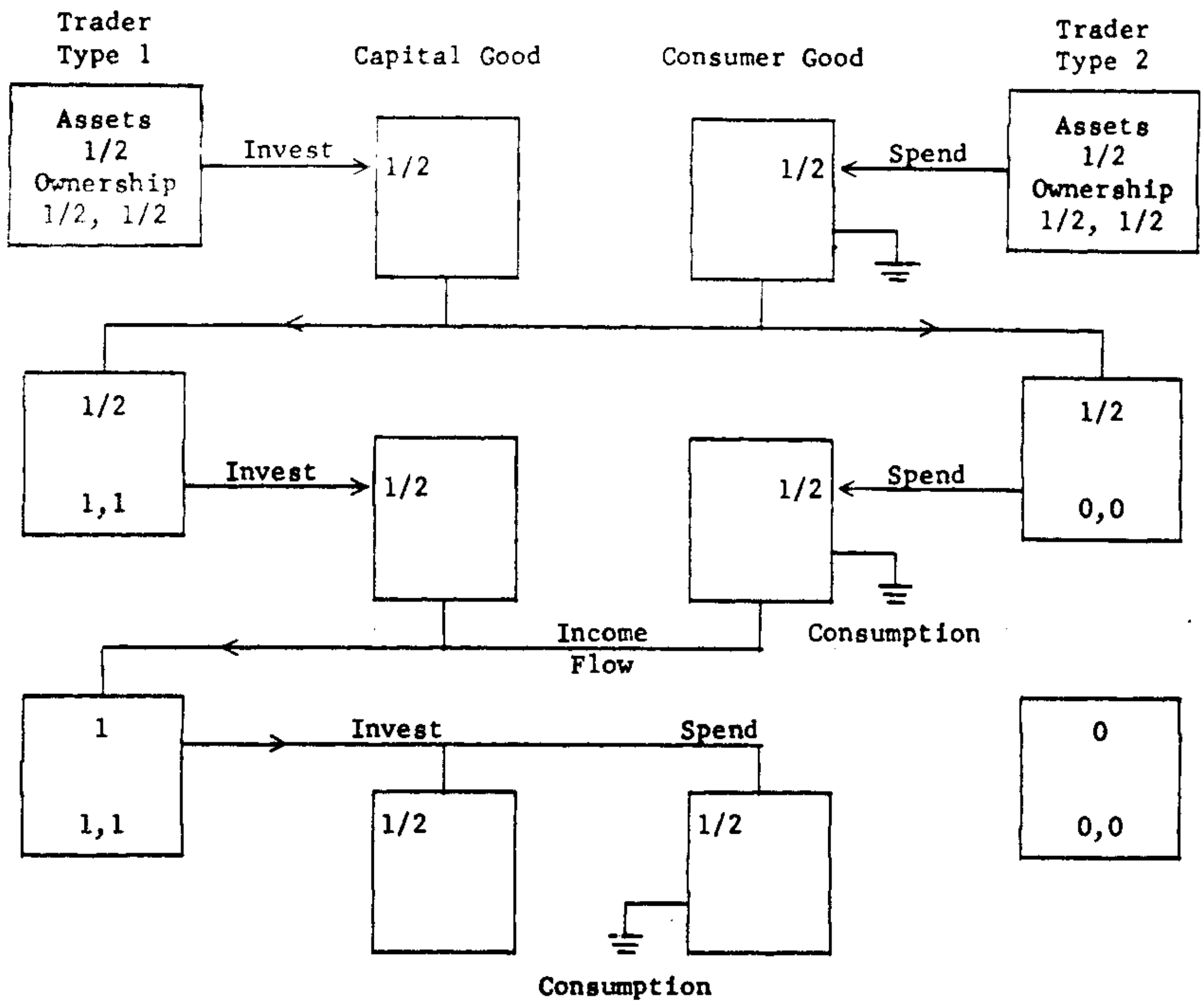

FIGURE 2

We now have a clear model of ownership transfer in the sense that the purchase of the producer good entitles the owner to the income deriving from selling the producer good next period and the income from selling the consumer good (C_2) that has been produced.

We now show the existence of a noncooperative equilibrium point*

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*The lack of hoarding in this simply example may be due to its simplicity. The introduction of assets provide security not necessarily a total substitute for hoarding.

which produces the same distribution suggested in Example 2a for the competitive equilibrium. It is already illustrated in Figure 2, however Table 1 shows the transactions and the type of individual maximization problem faced by each individual of Type 2 is illustrated below.

TABLE 1

	Spot Prices		Trader Type 1				Trader Type 2			
Period	p_1	p_2	Spend	Invest	Consume	Income	Spend	Invest	Consume	Income
1	1/2	1/2	0	1/2	0	1/2	1/2	0	1	1/2
2	1/2	1/2	0	1/2	0	1/2	1/2	0	1	1/2
3	1/2	1/2	1/2	1/2	1	1	0	0	0	0
4	1/2	1/2	1/2	1/2	1	1	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

On the assumption that there are many traders of each type we may describe the maximization faced by an individual of Type 2 as follows:

$$\text{Maximize} \quad \sum_{t=0}^{\infty} (1/2)^t x_{t+1}$$

$$\begin{aligned} \text{Subject to} \quad & x_1 \leq 1 \\ & x_2 \leq 1 \\ & x_3 \leq 2(1 - x_1) \\ & x_4 \leq 2(1 - x_2) \\ & \vdots \end{aligned}$$

By inspection we can observe that the maximum is at a policy where $x_1 = x_2 = 1$, $x_3 = x_4 = \dots = 0$.

The lags are interpreted as investment now provides ownership for next period and ownership now provides income claims for next period.

Conjecture. When durable assets and trade in ownership claims are introduced into a closed regular production and exchange economy then it is almost always possible to obtain a noncooperative perfect equilibrium which is Pareto optimal and produces the same distribution as a competitive equilibrium.

3. An Interpretation of the Examples

3.1. Their Mathematical Meaning

It is suggested that by formally describing a multistage model where markets and trading are given in detail it is possible to view an economy with n individuals mathematically as n parallel concave programming problems connected by a set of boundary conditions in common.

If we consider a finite horizon problem as treated by general equilibrium we have n parallel maximization problems where there is one boundary condition on each--the budget constraint. This is tantamount assuming total trust and the simultaneous existence of all futures markets.

By abandoning the general equilibrium approach, switching to the noncooperative equilibrium state strategy model and introducing trade in fiat money then each maximization problem in a T time period model is required to satisfy T constraints which are the cash flow constraints. This model has trading only in spot markets and no trust or credit.

For the noncooperative equilibrium to provide a Pareto optimal solution at most the maximum cash flow constraint (or constraints if there is more than one maximum) must be encountered by the program. Every trader must meet at least one constraint once, for otherwise this would imply indefinite individual hoarding which cannot be optimal.

In a model with a fixed money supply of fiat money it is possible to adjust relative prices by hoarding. However as Example 2b shows Pareto optimality may require that one set of traders hoard indefinitely. By introducing assets which produce the consumer goods and by requiring their monetization--as long as they have a future productive worth they serve as a "cover" for the fiat money.

3.2. Information Conditions

The formulation of the noncooperative game models calls for care in being explicit about the information conditions. Regardless of the lack of symmetry in information conditions postulated it appears that a noncooperative equilibrium of some sort will exist even though in general Pareto optimality may be lost. This boils down to saying that ignorance may have a cost in terms of failure to achieve the efficiency that would be present with costless knowledge.

The general equilibrium system does not appear to be able to handle general information conditions.

3.3. State Strategies or Historical Strategies

The solution concept Suggested here makes use of a simple state strategy equilibrium in a game with symmetric information conditions. This is at most what the general equilibrium analysis can be interpreted as doing. This analysis however by no means exhausts the more general set of equilibria that may be present under more complicated sets of strategies which use more previous history.

The more complex historical strategies can be regarded as more congenial to macroeconomic models. They are at least within the logical formulation of the noncooperative model they are not within the general equilibrium formulation.

3.4. Planning Horizons and Dynamics. Who Needs Equilibrium?

At one level the noncooperative model with its appeal to equilibrium considerations appears to have all of the static difficulties in interpretation encountered by the general equilibrium model. This is not so.

Although it is conjectured that the mathematical exploration of disequilibrium states may be considerably more difficult than the study of equilibrium, at least the models are well defined for the disequilibrium state and there appears to be a natural set of algorithms for calculating the next stage of an evolving process.

The natural interpretation of the noncooperative state equilibrium is in terms of a dynamic planning process where at any point an individual makes his decision based on his current state and his prediction of future prices. The noncooperative equilibrium will have the "self-fulfilling prophecy" property--if all individuals have the appropriate prediction on future prices these prices will indeed evolve.

If we assume that the individuals have misperceived the future prices then the system will still evolve if each maximizes using his prediction of future prices. One still needs to specify a behavioral mechanism for the generation of further price predictions. Such an interpretation, might up to a point be applied to a general equilibrium model; however it appears to me that this would call for a clearer definition of information conditions, a definition of payoffs in positions of disequilibrium and also a price prediction mechanism.

3.5. On Accounting

It must be stressed that the type of monetary model constructed here can be far more easily formulated by monetizing all assets every period thereby simplifying the concept of income far more than do current accounting practises.

The mechanism of total monetization, apart from simplifying the problems of modeling attempted here, appears to provide a better device for control and prediction in the economy than do current accounting practises.

3.6. Credit, Contracts, Assets and Bankruptcy

This paper has been confined to noncooperative situations where there is essentially no trust save in the ownership of consumer goods, claims to producer assets and fiat money. Loans, banking, escrow arrangements, the use of assets for security against loans all require somewhat more elaborate modeling of strategies and consideration of conditions on contract.

These added features appear to be needed when transactions costs are high, making the outright sale of an asset or ownership claims to the asset inconvenient or expensive. They also clearly come into play when uncertainty is introduced.

Especially when we consider a dynamic interpretation of the model, uncertainty may come from three different important sources. They are (1) exogenous uncertainty; (2) game theoretic or strategic uncertainty and (3) perceptual or cognitive uncertainty. Examples of each are (1) uncertainty arising from exogenous statistically describable phenomena such as probability of the outbreak of fire, or other mishaps for which the insurance mechanism exists; (2) strategic uncertainty is highly related to fewness in numbers of competitors where because of their importance in size relative to the market an inability to guess individual strategies introduces a serious uncertainty into the efficiency of planning procedures; (3) possibly the most underestimated by economic theorizing and the most important form of uncertainty in economic life comes in the inability of individuals to evaluate opportunities or to correctly perceive the problems they face. This is clearly illustrated by the problems in investment banking where many individuals will have highly different subjective estimates of the probability for the success for the same investment.

As long as each individual owns more than he borrows (in the sense that his security is always evaluated more highly by the borrower than is the loan) then the system need never evolve to a state where contract must be violated, i.e. where conditions for the return of a loan or surrender of an asset cannot be met. As soon as there is sufficient uncertainty in a system this no longer need be true. Thus merely to well define the problem laws or rules must be given specifying what is to be done when conditions contracted for cannot be met.

Neither the general equilibrium analysis nor static cooperative game theory need to deal or are equiped to deal with these problems. They are essentially static and non-institutional. In contrast the noncooperative solution is both strategically or process oriented and peculiarly institutional. It is institutional in the sense that because payoffs and strategies must be defined for all states of the system (not merely those near some suspected equilibrium), a full definition of rules such as bankruptcy or contract renegotiation is called for. The mathematician might regard this as an unfortunate feature of the noncooperative model as it delays generalization. It may however be the right feature for the construction of a viable mathematical institutional economics.[1]

3.7. Banking

In the next paper it is shown that in a system without futures markets banking and a variable monetary supply may be introduced as a substitute for hoarding and a fixed money supply. The security, contractual and bankruptcy problems may be increased upon introducing banks.

FOOTNOTES

[1] Shubik, M., Strategy and Market Structure, Wiley, New York, 1959, Part II.

[2] Shubik, M., "A Theory of Money and Financial Institutions: Fiat Money and Noncooperative Equilibrium in a Closed Economy," International Journal of Game Theory, 1, 4, 1972, pp. 243-268.

[3] Shubik, M., "Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," Western Economic Journal, 10, 4, 1972.

[4] Shubik, M., "A Theory of Money and Financial Institutions, Part VII," CFDP No. 336, 1972.

[5] Shubik, M., "A Theory of Money and Financial Institutions, Part XI," CFDP No. 363, 1973.

[6] Shubik, M. and W. Whitt, "Fiat Money in an Economy with One Nondurable Good and No Credit," Topics in Differential Games, A. Blaquiere (ed.), 1973, pp. 401-448.