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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART XI

TRADE WITH FIAT MONEY BUT NO INDIVIDUAL TRUST

A PRELIMINARY STAGE TOWARDS BANKING

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by

Martin Shubik

1. Introduction

This paper and the immediately subsequent one on banking have no general theorems proved. They contain several simple models which in some cases are worked out in detail. They also contain several conjectures and theorems to be proved.

This is written in the belief that general equilibrium theory has more or less reached a dead end. The proof of the existence of a price system in a static, complete information, non-institutional economy was a valuable result in the study of production and distribution. The proof was based on important but by no means sufficiently general conditions to be a decent representation of more than a small part of economic life as we know it.

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At the other end of the spectrum we have had a great growth in macroeconomic models which incorporate many of the aspects of the financial infrastructure and individual institutions of our society. But these models are not easily reconciled with the elegant non-institutional maximizing models of static general equilibrium economics.

Primitive societies as well as modern ones develop institutions and methods to handle trade, distribution, trust, saving, borrowing and even rudimentary banking and insurance. Yet in spite of adequate evidence that the institutions and economic methods and customs of virtually all of our societies indicate that money, banking, brokers and insurance fill _essential_ roles in a modern economy the microeconomic models leave them out and the macroeconomic models treat them in an _ad hoc_ manner.

The economy as a whole behaves as a loosely-coupled feedback system. People make errors. Sometimes sales are lost through lack of supplies. Inventories rot as non disposible surplus. Firms and individuals renegotiate or repudiate contracts; some go bankrupt. If the level of error and failure is kept relatively small the system functions smoothly and adjusts to blunders and minor disasters.

The dynamics of the system are the product of individual behavior _and_ the institutional environment. The institutions are the _carriers of process_. It could also be argued that institutions and laws contain the embodiment of the socio-economic and politico-economic goals and values of our society as a whole. If this is the case then many of the attempts to study the stability of the general equilibrium system by studying virtual displacements or writing differential equations based on excess supply and demand may give rise to interesting problems in mathematics but are tangential to our understanding of economic dynamics.

Some years ago I stressed that one of the reasons for the study of the core of a market game was because it could be regarded as offering a completely different concept of stability than is offered by the competitive equilibrium. This is still a static view. I now believe that the time has come to consider dynamic systems including those which never necessarily reach a steady state equilibrium. However it is easier to say this than do it. The remainder of this article deals with carrying out the basic modeling needed to start to consider dynamic economic systems. Gross oversimplifications are dealt with, and as we encounter inadequacies in the simple models the need for the elemental aspects of money and banking emerge. As some of the inadequacies are overcome by adding features new difficulties are encountered which call for more modeling.

It is my belief that in the last forty years of the growth of mathematical microeconomics the stress has been on theorem proving more than on modeling and the testing of the robustness of the models. The stress here is on modeling. Even if every theorem suggested by the models were easy to prove the task of understanding the relevance and the limitations of the models and theorems would still be of considerable importance.

2. On Money

In most parts of the world most trade is carried on by most people using an "institutional stuff" called money. The main forms are coins, bills, checks and in some cases accounting transfers in a double entry bookkeeping system. In increasingly less parts of the world, but nevertheless, still to some extent commodity monies may be used. These include ingots of gold and silver or bars of salt, bricks of tea and so forth.

A simple way of describing a monetary commodity or a fiat money is that at least one central identifying feature of a money is that it enters into trade in a manner that distinguishes it strategically from other commodities. In particular if there are $m+1$ commodities in existence then if one of them is selected as a money as a good first approximation we mean that the individual usually exchanges any of the m commodities for money, i.e. he "buys" or "sells" for money.

It must be stressed that the nonsymmetry is in the strategic treatment of the money commodity not in how it figures in the individual's utility function. Depending upon the actual reasons why individuals or societies use a commodity money we may be able to deduce various properties that a commodity money must have and thus distinguish a commodity money from other commodities in the utility functions of the traders.[3]

Assumption 1. A "money" is distinguished strategically from other commodities.

Assumption 2. The use of money decreases the need for individual trust. With a commodity money the lessening of the need for individual trust between traders can be attributed to two major factors:

(a) the individual utilitarian worth of the commodity,

(b) its "trading trust" worth due to custom and social acceptance.

Assumption 3. A fiat money decreases the need for individual trust by providing a symbol of law and its enforcement as a substitute for individual trust. This includes its use in taxation and the discharge of other debts.

Assumption 4. There are many diverse historical reasons that may have resulted in various societies adopting commodity money and fiat money. The need for a common measure of value, and the need for a surrogate for trust are among them. However the initial causes for the introduction of a monetary system may have little if anything to do with the stability of the system once it is in existence.

An important property of a noncooperative equilibrium in a mass market is that it is stable against individual deviation even if it might not be stable against deviations by large groups. Thus, for example, if enough individuals use a money and if each feels that it is not adequately backed but that others will use it then all will use it until a large enough group organizes to act against its use.

The barter model of trade so frequently described in its two person version in microeconomic theory is a poor and basically misleading prototype model for a mass economy. The very nature of the two person exchange is more related to the social-psychology of bargaining than the functioning of a mass economy through organized markets.

As numbers increase in an economy not only do transactions costs due to transportation and communication become important but so do many aspects of information including identification and evaluation of trust.

<u>Assumption 5</u>. Fiat money and coin are the most anonymous of the class of monies and credit devices used by individuals in modern society. The ownership of fiat money and coin entails no responsibilities whatsoever. With some commodity monies at least the responsibility for quality and weight rested with the holder. Apart from being forbidden to knowingly pass on a forged bill even quality control is not a responsibility of a fiat money holder.

Fiat money transactions can be and are frequently completely anonymous. When a check is used this is not the case, at least temporarily until the check has cleared. The other array of credit instruments and financial paper have varying degrees of individuality and lack of anonymity. They come with various arrays of ownership responsibilities and contractual arrangements.

<u>Assumption 6</u>. Money and credit are <u>decoupling devices</u> needed for the running of a dynamic and possibly always non-stationary economy. They provide the looseness of coupling which enables error correction to take place. Up to a point the elasticity of the value of money is precisely one of its most valuable properties in permitting the decentralization of decisionmaking.

<u>Assumption 7</u>. Modern mass economies strive to minimize the need for trust. The less there is a need for trust in either the judgment or honesty of individuals, the less is there need for information concerning them and their histories.

Much of commercial banking involves masses of individuals in routine transactions hence in comparison with investment banking it requires far less information processing of the individual. The former activity can be reasonably well described in terms of *state strategies*, i.e. little history is needed to determine the actions of a commercial bank with say a depositor. The latter activity frequently depends in detail on much previous history. In Wall Street slang: the client's "track record" is extremely important when investment banking funds are being sought.

Assumption 8. Assets play a critical role in the enforcement of contract conditions. In escrow and other arrangements they act as hostages. The richer an economy is in terms of real assets the more easy it becomes *ceteris paribus* to enforce contract without the need for trust and hence *a fortiori* with little need for detailed information. This is of course why mortgage markets can develop as relatively low information mass markets in rich economies.

3. Many Commodities, Commodity Money and One Period Trade

3.1. The Core and Tatonnement

Consider n individuals trading in $m+1$ commodities. If we considered them as engaged in a cooperative game we could consider the $2^n - 1$ nonempty coalitions they could form. When n is a number of the order of 10 to 100 million we suspect that an exhaustive consideration of coalition possibilities is not a good model of human economic activity in the market place.

Similarly tatonnement processes and the crying out of prices by autioneers, recontracting and so forth do not catch the essence of mass economic behavior.

3.2. The Noncooperative Models

By trying to solve an abstract utterly impractical but completely well defined problem in oligopoly theory some insight may be gleaned concerning strategies, trading and markets. The Cournot model for oligopolistic competition has each firm select a single number, its rate of production, as its strategy.[1] One can argue about the lack of realism of the model and the unsatisfactory feature of it being utterly static, but at least it is consistent and complete and the equilibria are reasonably interesting and show desirable properties as numbers of competitors increase.

The Cournot model is an open or one sided market. Only the sellers are treated as strategic entities. The new problem is how to model a general closed economy as a single period noncooperative game in strategic form with strategies akin to the Cournot model. Limiting ourselves to n traders and m commodities we must search for the simplest of trading arrangements in the sense that each individual must be in a position to announce a noncontingent strategy (a single move), i.e. his actions are decentralized or loosely coupled to the actions of the others in the sense that he need not completely know the bids and offers of all others before being able to specify his own.

Three models suggest themselves. All of them have important differences in interpretation. They are based upon the observation that although overall trade is multilateral, markets and exchange are essentially bilateral and that this is reflected in mass markets in terms of supply and demand.

Model 1

There are n traders, m commodities and $m(m-1)/2$ markets, i.e. there is a market for every pair of commodities: apples for oranges, apples for steel...etc. A strategy for an individual trader i is to offer quantities of his commodities in exchange in as many of the $m(m-1)/2$ markets as he chooses.

Suppose his initial endowments are: $(A_1^i, A_2^i, \ldots, A_m^i)$ then his strategy is to name an amount ${}^i q^s_{r,s}$ or ${}^i q^r_{r,s}$ in each market where the first symbol stands for an offer by i to sell the commodity s in exchange for commodity r in the exchange market between r and s. The second symbol refers to the other side of the market.

The restrictions on the strategies are as follows:

$$ {}^i q^s_{s,s} = 0 . $$

There is no market where a completely fungible commodity is exchanged for itself.

$$ {}^i q^s_{r,s} = {}^i q^s_{s,r} . $$

The market in which apples are exchanged for oranges is the same as the market in which oranges are exchanged for apples.

$$ \sum_{r=1}^{m} {}^i q^s_{r,s} \leq A^i_s \quad \text{for} \quad s = 1, \ldots, m . $$

This states that an individual cannot offer for sale more of a commodity than he owns.

Trader i will obtain the following amounts of the s^{th} commodity:

$$x_s^i = A_s^i - \sum_{r=1}^{m} {}^i q_{r,s}^{s} + \sum_{s=1}^{m} \left\{ \frac{{}^i q_{r,s}^{r}}{\sum_{j=1}^{n} {}^j q_{r,s}^{r}} \left(\sum_{k=1}^{n} {}^k q_{r,s}^{s} \right) \right\} .$$

This merely states that his final holding equals his initial holding minus his sales (as we assume that markets clear, all goods up for sale are assumed to be sold* (as in the Cournot model), plus his purchases.

In any market we might suspect that the same individual will not be on both sides of the market at the same time. However it is more desirable to be able to prove this than assume it. Wash sales and reserve prices for buy backs exist. One might wish to complicate the strategies by allowing for a reserve level of exchange to be specified in each market.

In general as is well known the noncooperative equilibria in the noncooperative trading game with $m(m-1)/2$ markets are not Pareto optimal.

Proof is by counterexample. Suppose that they were. Then consider a three person economy without double coincidence of wants. The utility functions of the traders are $\varphi_1(x,y,z) = z$, $\varphi_2(x,y,z) = x$ and $\varphi_3(x,y,z) = y$. The initial endowments are $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ respectively. There are three markets for the three goods but none of them will go active in a one move trade. If trade is multiperiod, individuals may reach optimality, but this of course is under the assumption that the inventorying and transportation of goods is costless.

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*There is a technical difficulty encountered when one side of a market is inactive. This is discussed elsewhere.[4]

Furthermore there are several interesting combinatorial problems in calculating how few trades will get a specific trading system to a Pareto optimal equilibrium. However these models are not necessarily the most relevant or fruitful ones in furthering our microeconomic understanding of money.

Consumers are not traders in general in a mass economy. They may be so on a Melanesian island; but it is gilding the mathematical lily to believe that somehow <u>the</u> proof of the need for a money depends on a relatively poor abstract model of the consumer as a trader. Consumers in modern societies do not trade very much for a host of reasons. Transportation costs and communication costs are only a few of them. Far more important is the fact that <u>trading</u> is a profession in a complex society. Training, knowledge and discernment are frequently needed as well as good communications and inventory space. One does not need a very fancy model or theorem to prove that most individuals do not carry 200,000 prices in their heads nor do they go through seventy-five combinations of exchanging wood for fish for bricks, etc. before they finally end up with the half dozen eggs they want.

The noncooperative model with $m(m-1)/2$ markets cuts down on the type of combinatories required for the core by many orders of magnitude. But as has been noted above even it is not satisfactory. Our next step is to introduce a commodity money and cut down the number of markets to m.

Model 2

The properties of the commodity money model have been discussed elsewhere[4] however a sketch is given here. Suppose that there are n individuals and $m+1$ commodities where by law or custom and convenience the $m+1^{st}$ commodity is distinguished strategically from all others.

Assume that individual i has as his initial endowment $(A_1^i, A_2^i, \ldots, A_{m+1}^i)$ and a utility function $\varphi(x_1^i, x_2^i, \ldots, x_{m+1}^i)$. We begin by making no special assumptions concerning the existence of distinguishing features for a commodity money in the utility function of the individual. If there are special properties that must exist for a commodity money they must be deduced from the key operational difference which distinguishes the role of a commodity money in the strategy space of the individual and not in his utility function.

We assume that there exists m markets in which the first m goods are exchanged for the $m+1^{st}$ good. As always it is important to keep in mind that any economic model is usually meant to be an approximation of reality. As such it will tend to simplify and possibly oversimplify the description of some phenomena. In a society which has no credit but uses commodity money the odds are that some of the trade will be done by barter. But it is much easier to model the pure cases such as all barter, all commodity money trading or all credit.

A strategy in this model can best be thought of as a $2m$ dimensional vector of buy or sell decisions in each of the first m commodities. Buying involves the exchange of the commodity money for one of the m commodities and selling is vice versa. We see immediately that under this arrangement a cash flow constraint in introduced.

Let s_j^i = the supply of the j^{th} commodity by the i^{th} trader

d_j^i = the demand for the j^{th} commodity by the i^{th} trader.

We may assume $(s_j^i)(d_j^i) = 0$, i.e. a trader either offers to buy or sell or is inactive.

The cash flow condition appears as:

$$\sum_{j=1}^{m} d_j^i \leq A_{m+1}^i \ .$$

This constraint is far stronger than the usual budget constraint in the standard general equilibrium models.[2] The truth appears to lie in between the cash flow and budget constraints. We may now interpret the noncooperative model as one extreme and the Walrasian model as the other extreme. This noncooperative model has no credit hence is governed by the cash flow constraint which is independent in the short run of the other ownership claims of the individuals. The Walrasian system has unlimited trust and hence credit is granted to all traders, not based on merely their commodity money but their total net worth is implicitly assumed as totally liquid and ex post and ex ante prices are considered as given and the same, hence the budget constraint can be calculated.

It is easy to see that without credit a commodity money model solved noncooperatively may not be able to achieve Pareto optimality. If each individual does not have "enough" commodity money trade is hampered by the cash flow constraint. A simple example illustrates this.

Consider two types of individuals with endowments $(A,0,M)(0,A,M)$ and utility functions of the form $U(x,y,z) = \sqrt{xy} + z$. The competitive equilibrium is at $(A/2, A/2, M)$ for each. The noncooperative equilibrium is given by:

$$\max_{x_i z_i} \sqrt{(A - x_i) \frac{z_i}{z_1} y + M - z_i} \quad \text{where} \quad z_i \geq 0 \ , \quad \leq M \ , \quad x_i \leq A$$

$$\max_{y_j z_j} \sqrt{\frac{z_j}{z_2} x (A - y_j) \quad + M - z_j} \qquad\qquad z_j \geq 0 \ , \quad \leq M \ , \quad y_j \leq A \ .$$

The last term is a <u>cash</u> <u>flow</u> constraint, and it follows immediately if $M = 0$ then no trade takes place. If M is large enough Pareto optimality is achieved.

In the above x and y are the total offerings of the commodities z_i and z_j the cash bids of individual traders of type 1 and 2. z_1 and z_1 are the total sums bid.

<u>Model 3</u>

It has already been noted that in a modern economy most people are <u>not</u> traders. They are consumers and predominantely workers for a contractual wage. They buy many and sell very few commodities and services. A trader model may be a poorer approximation of a modern economy than a consumer model.

At the level of simplicity being striven for here the simplest structure which captures this appears to be one with m simultaneous markets where all of the m commodities are auctioned off to the n individuals who each hold only supplies of the $m+1^{st}$ commodity; the commodity money. We may view this model as a natural preliminary to a dynamic model where we give the individuals ownership claims to the real commodities but as all m commodities must be sold via the markets even given ownership claims there is a cash flow problem introduced by the one period separation between ownership claims and control of the assets being sold and the income derived therefrom. A Walrasian model assumes this away by taking the essentially totally static approach already noted.

In economies such as those of the United States, England, France, Japan and others some of the major difficulties in the control and understanding of the economies come about through the imperfections in accounting systems. This is especially true when we consider several periods and the close relationship between accounting conventions and tax laws. Accounting "fudge factors" help give rise to many forms of tax shelter by enabling individuals to exercise great latitude in defining "income."

In a dynamic world the model being suggested here is unreal in the sense that although it would be a far better scheme for economic control and accounting than the one we use, it is probably far too hard to administer and has other difficulties when we consider transactions costs. However basically if forces the <u>complete</u> monetization and liquidation of all privately owned goods each period thus producing a much closer measure of income than is done by current trading and accounting practises.

Suppose each of the n individuals has an amount of commodity money, A^i_{m+1} for the i^{th} individual. We can assume either that there are piles of goods $A_1, A_2, \ldots, A_m$ in the m markets without specifying their ownership, or we can specify the ownership rights to each. Thus for commodity j, $A_j = \sum_{i=1}^{n} A^i_j$ where A^i_j is the ownership right of individual

i for good j. In this model we note the innate restriction that differentiates the right from full strategic control. The individual has the right to the income received from the sale of goods to which he has an ownership claim. He however is required to put them up for sale each period. Once more it must be noted that the model is innately dynamic in the sense that the process can be best interpreted in terms of money flowing in and back to be used again for purchases next period.

In this model, price is given by

$$p_j = A_j / \sum_{i=1}^{n} d_j^i .$$

This model is the easiest of the three to analyze[3] and is probably the best model of the consumer. A more "realistic" model can easily be well defined. This would have consumers as in Model 3 and producers and traders modeled as in Model 2. However such detail would be misplaced until the simpler models have been sufficiently explored.

4. On Credit, Banking and Bankruptcy

4.2. Credit and Bankruptcy

The models above are all well defined with a commodity money and with the understanding that no credit is granted to anyone. However as we know this assumption may well be as unreasonable as the assumption that credit is fully granted to all.

Suppose that we wish to introduce the granting of credit. The logic of defining a consistent and complete model of the market as a noncooperative game calls for several basic problems to be faced.

The most important problem which cannot be avoided is that a bankruptcy law must be defined. In contrast with the general equilibrium model, whereby the very definition of the solution and the lack of strategic freedom of the traders no errors can take place; in the noncooperative game the players have strategic freedom and the payoffs must be defined for all combinations of strategies. These combinations may well include ones in which some players are unable to repay what they have borrowed.

The bankruptcy law is needed to well-define the payoffs. The existence of a bankruptcy law does not necessarily imply that a solution will involve individual firms actually going bankrupt. It well may be that without sufficient exogenous uncertainty no one ever goes bankrupt. The concept of an optimal bankruptcy law has been discussed elsewhere.[4]

There are several alternatives which enable us to model credit and bankruptcy. These are discussed when banking is considered.

4.2. Banking

Before we discuss banks in detail the following models can be distinguished:

(1) Barter with no credit or futures

(2) Barter with futures

(3) Trade with commodity money and no credit

(4) Trade with fiat money and no credit

(5) Trade with fiat money and bilateral credit among traders

(6) Trade with fiat money and an internal market for money

(7) Trade with fiat money and an outside banking mechanism (described by a fixed strategy

(8) Trade with fiat money and an $n+1^{st}$ player who controls banking and has a strategic goal.

The first four models are obvious. The second is the Debreu model. The distinction between (5) and (6) is important in terms of the evolution of banking. If we have few traders say 5 or 6 it is reasonable to expect that loans or extension of credit is between identifiable pairs. When there are millions we expect a "market" or a bank to come into existence. Such an institution accepts deposits from lenders and makes the loans to the borrowers on a many to one and one to many basis. Thus it becomes necessary to specify not only bankruptcy rules, but with the existence of a bank seniority of creditor rules, credit rationing and bank failure rules.

Further discussion of banking is given in a subsequent paper. In the final section of this paper an infinite market with fiat money but without banking is considered. Without credit facilities there is no need for rules guarding against failure to honor debts.

5. Fiat Money and Hoarding

Conjecture. A trade and production economy that trades in fiat money but has no credit or futures trading, where the traders have concave utility functions which are bounded has at least one noncooperative perfect equilibrium point.

(1) If the traders are not "many" the equilibrium may not be Pareto optimal.

(2) If the process is finite the equilibrium may not be Pareto optimal.

(3) If the process is infinite then there will be a noncooperative equilibrium point which provides the same Pareto optimal distribution as a competitive equilibrium point,* provided the returns from the infinite horizon to each trader at every point

*There are some modeling qualifications concerning ownership. See the discussion of the role of β in the following example.

are sufficient to maintain the noncooperative equilibrium.*

(4) Relative prices are adjusted by hoarding.

(5) The hoarding solution is a boundary solution. By introducing banking the market for loans provides a "futures" market and yields an analytical solution provided, in some cases, the money supply must be made variable.

An extremely simple example is illustrative of this conjecture. Consider an economy with two types of traders each entitled to 1/2 ownership of the single good which appears each period. Suppose the stream of "manna" is A, A, ... each period.

Traders of type 1 have a utility function of

$$U_1 = \epsilon \sum_{t=0}^{\infty} \beta^{2t} q^1_{2t+1} + \sum_{t=0}^{\infty} \beta^{2t+1} q^1_{2t+2}$$

and type 2,

$$U_2 = \sum_{t=0}^{\infty} \beta^{2t} q^2_{2t+1} + \epsilon \sum_{t=0}^{\infty} \beta^{2t+1} q^2_{2t+2} \;;$$

where $\beta < 1$, $\beta > \epsilon$.

Suppose the total amount of money issued to traders of type 1 is 1/3 and traders of type 2 is 2/3. Then there exists a noncooperative equilibrium where during periods $t = 1, 3, 5, \ldots, 2k+1$ traders of type 1 hoard all of their money and type 2 spend all of theirs. Thus of the money supply of n in the economy $n/3$ is permanently hoarded. The price level is $2n/3A$ each period.

*A full discussion of this together with a counterexample is given in Part III.

Heuristically the key aspect to the conjecture given above is as follows. Suppose we have solved for the prices and trades in an infinite general equilibrium system. Thus for k commodities each period we have a sequence of prices $p_{11}, p_{21}, \ldots, p_{k1}; p_{21}, p_{22}, \ldots, p_{k2}; \ldots p_{1t}, p_{2t}, \ldots, p_{k,t}; \ldots$. These are of course essentially futures not "spot" prices thus for example if there were a natural discount in all utility functions of say β then this would be reflected in the future prices. In particular there would be a factor of β^t differentiating the spot from future prices. Applying this factor to obtain spot prices we can calculate the cash flow requirements of every period. If our process is bounded there will be an upper bound to the cash flow required to finance trade in any period in the future. In this model with all goods auctioned before they can be consumed all the real wealth of the economy is monetized. Figure 1 shows the cash flow requirements. Suppose the 4^{th} period would call for

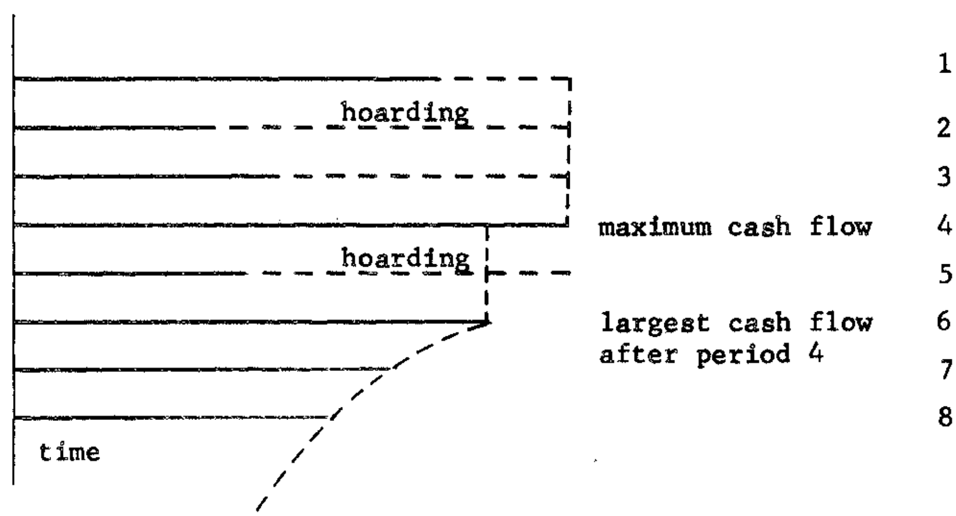

FIGURE 1

the maximum cash flow then prices prior to this period can be brought in the appropriate ratios across time periods by hoarding. Suppose the second largest cash flow occurs at period 6. The noncooperative game with a fixed money supply would have period 6 prices inflate until they use all the money supply.* There will be hoarding in period 5. Suppose the needs for cash flow then dropped exponentially from then on. All the money may be used in every period with a progressive inflation of prices resulting.

It is important to stress that there is no futures trading in this model. Spot prices are the only prices at which trasnactions take place.

It is easy to get a counterexample to Pareto optimality for a finite horizon model. Two periods suffice. For example consider $U_1 = x_1 + .9x_2$, $U_2 = y_1 + .1y_2$ the competitive prices are $p_1 = 1$, $p_2 = .9$. Assume supplies such that $x_1 + y_1 = 1$ and $x_2 + y_2 = 1$. Then the competitive equilibrium yields (.95, .95) and the noncooperative equilibrium yields prices of $p_1 = p_2 = 1$ and payoffs of (.95, .55) which are not Pareto optimal.[6]

Consider the infinite version of this example with a natural discount β .

$$U_1 = \sum_{t=0}^{\infty} \beta^{2t} q^1_{2t+1} + .9 \sum_{t=0}^{\infty} \beta^{2t+1} q^1_{2t+2}$$

$$U_2 = \sum_{t=0}^{\infty} \beta^{2t} q^2_{2t+1} + .1 \sum_{t=0}^{\infty} \beta^{2t+1} q^2_{2t+1} ,$$

competitive equilibrium prices are .1 and $.9\beta$.

*As the next example shows this is not quite accurate, it is possible that money is always hoarded hence all of the money supply is never used.

Does there exist a noncooperative equilibrium with hoarding which gives the same as the competitive equilibrium? Under the latter income to each is $.5(1 + .9\beta)$. The traders obtain $(.5 - .45\beta, .5 + .45\beta)$, of the first period good and $(1,0)$ of the second, with a utility worth of $(.5 + .45\beta)$ each.

The noncooperative equilibrium is as follows. Let γ and $1-\gamma$ be the initial distribution of money to the two types of traders.

Assume that traders of type 1 hoard m then money assets, hoarding, spending and income are as follows (an example for $\beta = 2/3$ is also shown).

TABLE 1

	Type 1		Type 2	
Period 1				
Assets	γ	3/11	$1-\gamma$	8/11
Hoarding	m	1/11	---	----
Spending	$\gamma-m$	2/11	$1-\gamma$	8/11
Period 2				
Income	(1-m)/2	5/11	(1-m)/2	5/11
Assets	(1+m)/2	6/11	(1-m)/2	5/11
Hoarding	---	----	(1-m)/2	5/11
Spending	(1+m)/2	6/11	---	----
Period 3				
Income	(1+m)/4	3/11	(1+m)/4	3/11
Assets	(1+m)/4	3/11	(3-m)/4	8/11

If the system is in stationary state then

$$\frac{1+m}{4} = \gamma \ .$$

Furthermore if it gives the same distribution as the competitive equilibrium then

$$\frac{\gamma-m}{1-\gamma} = \frac{.5 - .45\beta}{.5 + .45\beta} ,$$

or

$$2\gamma = 1 - .9\beta + m(1 + .9\beta)$$

hence

$$\gamma = \frac{9\beta}{10 + 18\beta} \quad \text{and} \quad m = \frac{18\beta - 10}{18\beta + 10} \ .$$

Suppose $\beta = 2/3$ then $\gamma = 3/11$, $m = 1/11$ as is shown in Table 1.

A new and interesting case distinction appears when $\beta < 5/9$. For values in this range the noncooperative model of ownership manifests another strategic difference from the market without forced trading for money. We have assumed equal claims to ownership thus when $\beta = 5/9$ the value to each of their initial claims of 1/2 in each period is $1/2 + .9(5/9)1/2 = .75$ and $1/2 + .1(5/9) = .55$. When $\beta < 5/9$ the noncooperative model cannot be fully compared to the general equilibrium model with ownership claims of 1/2 each period as the traders of the first type may be required to accept less than their "competitive model no trade point."

Further discussions of monetary models are given in other related papers noted.[5]

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