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GAME THEORY AND POLITICAL SCIENCE

Martin Shubik

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by

Martin Shubik

There are several topics in political science to which the application of game theory is prima facie appealing. They are voting, the study of power, diplomacy, negotiation and bargaining behavior, coalition formation among political groups and logrolling.

This brief survey is by no means intended to be comprehensive but is intended to serve as a guide to some of the more immediate applications of game theory to political science.

1. Voting and Group Preferences

There is already a considerable body of literature using game theory or techniques closely related to game theory in application to political science. The relationship of individual preferences to group preferences and the role of different mechanisms for political choice such as voting is of interest to the political scientist.

Much of the work on voting has left out the strategic aspects of voting and concentrated on the "aggregation" of individual preferences via the vote.

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The assumptions usually made are that:

1. Individuals know their own preferences and these are fixed.
2. They know and are able to evaluate all alternatives.
3. The "rules of the game" are known and understood by all.
4. Each individual is rational and suffers no information overload or computational problems in decisionmaking.
5. It is possible to consider the social choice problem in a static context; i.e., a static model serves as a reasonable approximation of a real world social choice process such as an election.

A partial list of writings includes Arrow, Blau, Black, Buchanan, Coleman, Condorcet, DeMeyer and Plott, Dodgson (Lewis Carroll), Fishburn, Garman and Kamien, Goodman and Markowitz, Guilbaud, Hildreth, Inada, Kramer, Ledyard, May, Murakami, Nanson, Plott, Rae, Riker, Rothenberg, Sen, Tullock, Vickrey and others.*

The Condorcet voting paradox and the Arrow "General Possibility Theorem" are key examples of this approach. Various conditions on preference structures such as "single peaked preferences" have been investigated by Black and others.

The properties of different voting methods and different assumptions concerning the measurability and comparability of intensity of individual preferences have been considered. Thus majority voting, weighted majority voting, various rank ordering methods and rules for eliminating candidates and other schemes have been studied.

*Arrow (1951, 1963), Blau (1957), Black (1958), Buchanan (1954), Coleman (1966), Condorcet (1785), DeMeyer and Plott (1970), Dodgson (Lewis Carroll) (1873), Fishburn (1969), Garman and Kamien (1968), Goodman and Markowitz (1952), Guilbaud (1952), Hildreth (1953), Inada (1969), Kramer (1972), Ledyard (1968), May (1952), Murakami (1966), Nanson (1882), Plott (1967), Rae (1969), Riker (1961), Rothenburg (1961), Sen (1970), Tullock (1967), Vickrey (1960).

Strategic Choice

A different but highly related approach to problems of political choice takes into account the strategic aspect of voting.

Explicit assumptions must be made about not only what the individual knows about his own preferences but what he knows of the preferences of others. In most of the work it has either been explicitly assumed that individuals are informed of all preferences or the information conditions considered have not been fully specified.

Studies of strategic voting may be divided into those using cooperative, noncooperative or other solution concepts and can be further divided into those with a formal structure (such as political parties) assumed or those in which only individuals are considered in a setting with no institutional details specified explicitly. Cooperative solutions are considered in 2.

A clear and concise application of the noncooperative equilibrium solution to strategic voting has been made by Farquharson* where he defines the conditions for sincere, straightforward or sophisticated voting.

The theory of competition among political parties offers another important area of application for the theory of games. Some starts have been made, as is reflected in the works of Downs, Chapman, Frey, Shubik and others.** These are primarily based upon analogies between the economics

*Farquharson (1969).

**Downs (1957), Chapman (1967), Frey (1968), Shubik (1968a).

of oligopolistic competition and noncooperative party struggles. The tendency has been to apply some form of noncooperative or mechanistic solution to the models. The classic work of Dahl* describing political participation serves as a guide for those who wish to model different actors in political competition.

Some of the questions that can be examined by noncooperative game models are: How are the differences between economic and political competition manifested? What political mechanisms guarantee a Pareto optimal distribution of resources, and under what conditions?***

Although, to our knowledge, satisfactory models have not yet been built, it appears that it might be worth while to construct game theoretic models of the political process in which power groups such as unions, large industry and possibly several other organizations are distinguished as well as the parties and the individual voters. With such models both cooperative and noncooperative solution concepts would merit examination.

Possibly the most important aspect of the application of game theory methods to economic analysis has been in the providing of formal mathematical models to study the effect of large numbers of participants in the economy.*** A well-known feature of the economic system is that mass markets exhibit highly different characteristics than do markets with few participants. It is my belief that the study of game theoretic models of politics with

*Dahl (1961).

**Shubik (1970).

***See Shubik (1968b).

large numbers of participants could cast some light on the role of numbers in the political process and help to clarify some of the problems inherent in mass democracy. A small number of works containing formal mathematical models of voting processes where numbers of participants is a key variable already exist. They deal primarily with extensions of the voters' paradox* of Condorcet or with simple games.**

Even with the simpler models of voting, if some of the assumptions are relaxed, extremely difficult problems are encountered. For example, if individuals do not know each other's preferences then in their strategic behavior they must evaluate the worth and consequences of deliberate misinformation. In disclosing one's stand on tax bills or appropriations this becomes important. The work of Harsanyi, Maschler, and others has begun to deal with the strategic aspects of incomplete information.*** This work has not been directed at political science in particular hence any applications would call for more specific modeling.

It is important to stress that the game theory applications to voting and group preferences noted above, in general, tend to play down the non-strategic aspects of human behavior. After all, many people vote more out of habit or affiliation than as the result of a carefully planned strategy. Even though this may be the case, game theoretic and behavioral approaches are not incompatible. The problems of application lie as much or more in

*DeMeyer and Plott (1970), Garman and Kamien (1968).

**Shapley (1962).

***Harsanyi (1968a, 1968b, 1968c), Maschler (1966).

the model building than in the analysis. There is every indication, however that a considerable amount of ad hoc modeling is required to reflect the intermix of habit, nonstrategic, but conscious behavior and strategic cooperative and noncooperative behavior that describe the actors in the political arena.

2. Coalitions and Bargaining

Much of the writing on voting has dealt with a single vote. A special class of game, the simple game serves as a good representation of the single vote in isolation. However, frequently it is unreasonable to consider isolated issues. Coleman, Tullock, Wilson and others* have attempted to incorporate logrolling and the trading of votes into game theoretic models.

In designing new legislatures, or attempting to obtain an a priori feeling for how the voting power of individuals might change with changes in the formal aspect of the voting structure Shapley, Shapley and Shubik, Mann and Shapley and others** have applied the value solution to obtain a measure of the importance of an individual.

Riker, Luce and Rogow, Leiserson and others have gone beyond simple voting to consider the nature of coalition formation. There are several basic difficulties in modeling which must be overcome in extensions of this variety. Thus when Riker considers minimal winning coalitions, the

*Coleman (1970), Tullock (1971), Wilson (1969).

**Mann and Shapley (1964), Shapley (1953), Shapley and Shubik (1954).

more formally oriented game theorist might have difficulty in interlinking this quasi-dynamic discussion with the specific role played by minimal winning coalitions in simple games.

Logrolling

In many political processes the choice is not one among a set of alternatives, but to decide how many of a set of motions will be passed. Suppose that there are n individuals and m motions. The individuals each have a vote on each motion. The final outcome achieved by society is one of 2^m states. As the individuals may have differing preferences for the outcomes they may be in a position to trade votes among themselves, thus the votes serve as a limited type of currency for making sidepayments.

Among those who have investigated the trading of votes or "logrolling" (as this procedure has been called) are, Bentley, Buchanan and Tullock, Coleman and Wilson.*

Given the properties of a voting method and preferences, several considerably different models may be specified. Three models are: (1) the voting process viewed as a market with prices for votes; (2) voting as a noncooperative game; and (3) voting and logrolling as an explicit cooperative game.

We may wish to suppress the strategic aspects of individual choice and regard the votes of the individuals much in the same manner as the initial endowments of commodities are treated in models of an exchange

*Bentley (1949), Buchanan and Tullock (1962), Coleman (1966), and Wilson (1969).

economy. Assume that there is a price given for a vote on every issue. The initial vote endowment of each can be evaluated at the given prices hence a budget constraint can be calculated for each individual for buying and selling votes. It remains to establish the conditions for which an efficient price system will exist.

If we wish to consider the strategic aspects of logrolling we might attempt to extend the noncooperative analysis suggested by Farquharson.* There are, however, some difficult problems in modeling vote-trading as a noncooperative game. A more natural approach is to consider logrolling as a cooperative game.

At least three cooperative solution concepts appear to be applicable to logrolling. They are the core, the value and the bargaining set. The existence of the core is undoubtedly a necessary condition for the existence of a market for votes. An example of this type of relationship is given in the work of Klevorick and Kramer as well as Wilson.** Wilson has also considered the bargaining set.

A key aspect of the discussion on logrolling that is the relationship between the utility space and the commodity space or outcome space. The existence of a stable price system in an economy depends explicitly upon properties of both the outcome and utility spaces. When considering political issues, the possibility for the existence of a market for votes will depend delicately

*Farquharson (1969).

**Klevorick and Kramer (1972), Wilson (1968).

upon the characterization both of the strategies and outcomes. In general, unless strong assumptions are made we may expect that an efficient price system for votes does not exist. This observation also holds true of many discussions in welfare economics. The relationship between money side-payments, or other forms of transfer and the actual attainable set of outcomes is frequently not made clear.

The applications of game theory to problems involving bargaining and coalition formation split into those with a primary emphasis on statics and combinatorics and those which stress dynamics and process. The former tend to be parsimonious in the introduction of new variables, and as free as possible of personality and institutional factors; the latter are frequently far more descriptive and behavioristic.

The most direct and institution free application of cooperative game theory to voting comes in the calculation of the value for simple games, where a simple game reflects the strategic structure of a voting procedure applied to a single issue.

When our concern is directed towards voting on a single issue, many of the difficulties concerning the nature of preferences are not germane. When, furthermore, we assume that the voting process can be represented by a simple game we have implicitly accepted a set of assumptions that limit the immediate applicability of any results yielded by the analysis. In particular (as is the case with all cooperative game theoretic solutions and with the mechanistic approaches of Arrow, Black, Plott, Rae and others), the costs, details and dynamics of the voting process are assumed away.

Thrown away with most formal game theoretic and non-game theoretic models of voting are important features such as the power and role of chairman. The assumption of external symmetry* is a powerful and important factor in most mathematical model building. For many purposes of analysis is it valuable, but the price paid is high. Before one can interpret any results obtained from a theory which makes such an assumption, in terms of actual political processes the sensitivity of the results to a variation of this assumption must be considered.

Even given the enormous simplifications implicit in the study of voting by means of simple game models several interesting questions can be posed and answered. They concern the design of voting bodies and methods. Work on this topic includes that of Shapley and Shubik, with an example of the Security Council of the United Nations; Mann and Shapley on the Electoral College; Riker and Shapley, Riker, Banzhaf, Nozick** and others. Several of these papers have dealt with the problem of weighted voting and its relationship to the principle of "one-man, one-vote" in districts of unequal size.

A step towards relaxing the assumptions made in applying game theory methods to models representing voting processes can be made without constructing a completely dynamic theory. This has been done by introducing

*I.e., the assumption that individual personal characteristics do not matter and each individual is considered to be identical with any other individual.

**Shapley and Shubik (1954), Mann and Shapley (1964), Riker and Shapley (1965), Riker (1959), Shapley (1962), Banzhaf (1965, 1966), Nozick (1968).

some structure on coalitions. Luce, Luce and Rogow, Riker, Leiserson, Riker and Ordeshook* and others have followed this approach. In particular, Luce** has suggested a solution different to that of the value and Riker*** has stressed the importance of forming "minimal winning coalitions." This principle is in contradistinction to the analysis of Downs**** which suggests that political parties attempt to maximize their majorities. Riker argues that the payoffs to extra individuals not vital to forming a winning coalition may not be worth making.

The contribution of Riker is expressly based upon a game theoretic approach to political coalitions, as contrasted with Downs who provides an economic market model. Nevertheless Riker's proposal of a minimal size for winning coalitions (called his "size principle") is heavily based upon his description of a dynamics of coalition formation. The political scientist may find this description both fascinating and fruitful; nevertheless from the viewpoint of mathematical modeling it is difficult to formalize the appropriate game, characteristic function and solution concept that are being suggested.

The approach of Luce and Rogow***** to the modeling of coalitions is somewhat more formal than that of Riker and simultaneously more static.

*Luce (1954), Luce and Rogow (1956), Riker (1962, 1966), Leiserson (1968, 1970), Riker and Ordeshook (1968).

**Luce (1954).

***Riker (1962).

****Downs (1957).

*****Luce and Rogow (1956).

They assume a priori that there are limitations on coalition formation that are given as part of the description of the problem. The solution concept they use is that of Ψ -stability proposed by Luce.

On Limit Solutions

Although the techniques for considering games with masses of players are relatively well developed and have been applied with considerable success to economics, such an application has scarcely taken place in political science. I suspect that both for cooperative and noncooperative solutions the study of limiting properties of game theoretic models of political phenomena may prove to be fruitful. In particular the behavior of noncooperative equilibria, the value, the core and the bargaining set appear to merit investigation.*

Bargaining

The literature on bargaining is considerable and varied. Several different domains of interest can be distinguished. Labor-management bargaining is almost a subject in itself. The economics of haggling and market bargaining between traders is an allied but somewhat different topic. The boundaries between economics and politics melt when international trade bargaining is considered. There is a large (and in general non-game theoretic) literature on the social-psychology of bargaining. In political science most of the literature on bargaining that has made use of game theoretic concepts is concerned with party politics or with international negotiations.

*Shubik (1973).

The work in political science has to some extent been influenced by the studies of bargaining in economics or by economists and game theorists extending various mixtures of economic analysis and game theoretic reasoning to political problems. Included in this group are Aumann and Maschler, Bishop, Boulding, Chamberlain, Cross, Edgeworth, Ellsberg, Harsanyi, Pen, Schelling, Shubik, Zeuthen and many others.*

Among those who have noted the use of game theory for analogies to international bargaining and conflict situations are Boulding, Morton Deutsch, Ellsberg, Harsanyi, Iklé, Mitgaard, Rapoport, Sawyer and Guetzkow, Schelling, Shubik, Wohlstetter** and many others.

Many of the problems faced in attempting to apply game theoretic reasoning to bargaining and negotiation are substantive. They involve the modeling of process. A brief survey such as this cannot do justice to a description of the difficulties in modeling. The reader is referred to Iklé's "How Nations Negotiate" for a perceptive description of many aspects of the process of international negotiation.*** A useful bibliographic note on literature relevant to international negotiation is also supplied by Iklé. "The Economics of Bargaining" by John Cross**** provides the political scientist as well as the economist with static and process models of bargaining where stress is laid upon economic features such as the cost of the process. Frequently a "taxi meter" is running while bargaining is taking place.

*Aumann and Maschler (1964), Bishop (1963), Boulding (1962), Chamberlain (1951), Cross (1969), Edgeworth (1881), Ellsberg (1956), Harsanyi (1956, 1962), Pen (1952), Schelling (1960), Shubik (1952), Zeuthen (1930).

**Boulding (1962), M. Deutsch (1961), Ellsberg (1961), Harsanyi (1965), Iklé (1964), Mitgaard (1970), Rapoport (1960, 1964), Sawyer and Guetzkow (1965), Schelling (1960), Shubik (1963, 1968), Wohlstetter (1964).

***Iklé (1964).

****Cross (1969).

Rapoport's* "Fights, Games and Debates" stresses modeling problems in the application of game theoretic reasoning. Schelling's** Strategy of Conflict provides many provocative simple games and analogies to international conflict situations. However there is a considerable danger in being misled if one tries to push analogies between extremely simple games and international bargaining too far. It is important to realize not only the power but the limitations of simple game models as a didactic device.

3. Power

One of the key concerns of political science is the study of power. In the work on the various versions of a value solution the relationship between these solutions and the concept of power has been noted. The immediate game theoretic basis for the investigation of power is given in the writings of Harsanyi, Nash, Selten, Shapley and Shapley and Shubik.*** A paradoxical aspect of value solutions is that they were primarily motivated by a concern for fairness and equitable division. The relationship between fair division and power comes in the way in which threats enter into a consideration of how to evaluate the no-bargain point between any two coalitions. In essence, the status quo or no-bargain point may be determined by the power of the bargainers. The fair division procedure is applied using this power-determined initial point as a basis for the settlement.

*Rapoport (1960).

**Schelling (1960).

***Harsanyi (1962a, 1962b), Nash (1950, 1953), Selten (1964), Shapley (1953), Shapley and Shubik (1954).

The various value solutions are essentially static and undoubtedly fail to portray adequately the interplay of power and influence in a dynamic system. Nevertheless they provide clear well-defined concepts which can be subjected to scrutiny and compared with the theories of power proposed by political scientists and other social scientists. A brief set of references to writings on power which can fruitfully be contrasted with the game theoretic value solutions include the works of Dahl, Lasswell, March, Simon and many others.*

Possibly one of the most comprehensive attempts to evaluate and reconcile the many approaches to the concept of power, including the game theoretic approaches, is "The Descriptive Analysis of Power" by J. Nagel.** This together with the papers of Harsanyi would serve as sufficient guides to those who wish to pursue the investigation of the concept of power further.

*Axelrod (1970), Baldwin (1971), Barber (1966), Blau (1964), Brams (1968), Champlin (1971), Coleman (1966, 1970), Dahl (1961), Goldhamer and Shils (1939), Lasswell and Kaplan (1950), March (1955, 1957, 1966), Nagel (1968, 1972), Parsons (1957), Riker (1964), Simon (1953), Wagner (1969).

**Nagel (1972).

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