COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 347

INFORMATION, DUOPOLY AND COMPETITIVE MARKETS

A SENSITIVITY ANALYSIS

Martin Shubik

November 29, 1972

INFORMATION, DUOPOLY AND COMPETITIVE MARKETS

A SENSITIVITY ANALYSIS*

by

Martin Shubik

1. Introduction

Implicit in the discussion of markets and competition are differing conditions on information as the number of competitors changes. In particular the market with many competitors has long been considered as an entity with relatively low levels of information. People react to the average market price and aggregate inventories. Personalities disappear each individual adjusts to the impersonal market that supplies him aggregate information.

In contrast with the many person market, duopolistic behavior, bilateral monopoly bargaining and oligopolistic behavior in general appears to depend upon detailed information. The other individuals in the market cannot be safely regarded as part of an anonymous aggregate. Specific information about individuals may be of importance.

This paper is devoted to exploring the variations of information conditions for an extremely simple set of duopoly models. An interpretation of the sensitivity of solutions to changes in information is given and the effect of increasing the number of competitors is considered.

*The research was supported by the Office of Naval Research. The research was also partially supported by a grant from the Ford Foundation.

1.1. Information, Communication and Knowledge

Before setting up a formal model of a market it is important to distinguish several different features encountered in the characterizing of communication in markets.

The state of general or long term knowledge of the actors must be specified. The problems of interpretation and coding must be made explicit. The structure of the communication system (both formal and informal) must be described. And the information conditions (in the sense of the theory of games) must be given.

Long term knowledge in most markets consists of two parts. They are "knowing the business" and "knowing the people in the business." Many formal economic models start from the assumption that businessmen know the business. This is tantamount to "knowing the rules of the game." In businesses where success rests on expertise and discernment the assumption of equal knowledge may be poor; elsewhere however we may claim that it serves as a crude approximation.

In oligopolistic markets long term knowledge of people plays an important role. The knowledge that A is honest, but stupid; B is cooperative but untrustworthy; C is competitive; D is irrational, and so forth may serve as a guide to the type of solution that emerges from that market. We return to this point in Section 4.

Human communication at best is imperfect. When language is used the possibility for many interpretations or misinterpretations is great. Many deals in the modern business world involve highly complex technical and legal features. Success may hinge on the correct interpretation of a clause in a bargain. The problems of interpretation and coding involve the technical communication problem of how to optimally transmit a message and the semantic problem of how to control for the possibility of misinterpretation. We do not concern ourselves further in this paper with this type of communication problem.

The structure of the communication network varies considerably from market to market and varies in like markets in different countries. People in the same trade tend to know each other. Both formal and informal communication networks play a part in facilitating trade. In some instances however certain forms of communication are expressly forbidden. Hence proof of private discussions among competitors concerning prices or written communications suggesting prices could be used as evidence in antitrust proceedings in some countries. In auctions, bidding procedures are highly formalized and in many other markets the role of custom is important.

Different stockmarkets follow somewhat different rules of trade. Most markets <u>are</u> institutions or have institutions intimately associated with them. If this is the case we must ask how sensitive is the formation of price to the institutional details and the idiosyncracies of individual markets? An answer to this question can be sketched and is given in 3.2.

In the formal models presented here conditions concerning both formal and informal communication are part of the specification of the model.

Given the communication system, there remains the task of specifying the state of information among the competitors.

1.2. Markets and Models

It is my contention that the outcome in an oligopolistic, oligopsonistic or "thin market" does depend in detail upon personality factors, specific aspects of the communication system and the specifics of the distribution of information. Some of these in turn may depend upon customs, institutions and technology. For example the availability of information on production and levels of inventories may depend upon the length of production and size of the objects being produced. Financial information that is supplied to the public is highly dependent on law and custom.

Without specifying an oligopolistic model in institutional, technological and behavioristic detail it is unlikely that any operationally useful predictions of oligopolistic behavior can be made. Simple reaction function models in the style of Stackelberg[1/], difference equation models or behavioral models[2/] can and have been constructed on a more or less *ad hoc* basis. But a general economic dynamics for oligopolistic behavior without the specification of detail appears to be a mirage.

When numbers are large in a market the costs of communication and information fast obliterate the fine structure of competition or collusion. Aggregates and stereotypes must be dealt with. The methods of trade may be elaborate, but they must be unchanging or at best slowly changing. In other words given that individuals know the *system* the amount of information required to keep it going is low.

Individuals have a limited capacity for detail and this capacity is much diminished if details are rapidly changing. Elaborate strategies are most plausible when the field of competition involves two or three powerful actors. Otherwise, in general, the costs of elaborate plans are high and their value may attenuate with the growth in the number of competitors.

1.3. On Sensitivity Analysis

It is easy to be seduced by the plausible but not particularly operational argument that everything is a special case and must be treated as such. Although in the study of oligopolistic markets there is no substitute for knowing your business, this does not mean that we must retreat to mere institutional description with little or no analysis or ability to generalize. It means that the generalizations and the analysis must be based on, to some extent a different (or at least a more detailed) description of individual firms and markets than is customary in economic theory. The major difference comes in the description of information and communication.

In contrast with the descriptive and institutional approach to the study of markets and competition there are the analytical methods of economic theory. The mathematical economics approach to oligopoly has been typified by the works of Cournot[3/], Edgeworth[4/], Hotelling[5/] and more recently, the theory of games[6/].

An even more striking development of economic theory has been the work on general equilibrium and the theory of price[7/]. It is dangerously easy however to mistake the highly limited results of the mathematical analysis of general equilibrium theory for having a level of generality and independence from institutional detail which they do not possess. Information conditions and the role of numbers have been virtually ignored or treated obliquely. Perfect information and unlimited futures contracts have been explicitly assumed in some of the advanced mathematical treatments of general equilibrium[8/]. The work of Roy Radner[9/] provides an exception where nonsymmetric information conditions have been considered.

A natural question to ask of any model of human activities is how sensitive is it to slight changes in assumptions or variations in parameters. In particular two important types of sensitivity analysis for models of markets are: (1) sensitivity with respect to the number of competitors, and (2) sensitivity with respect to changes in the information conditions.

The need to understand the role of information conditions is especially important if microeconomic price theory is to be given any interpretation in terms of dynamics. It is my contention that the concept of a general equilibrium price system may be interpreted as one among a class of equally plausible solutions, once information and communication conditions have been specified. This statement cannot be fully developed in this article, however the key to it lies in the interpretation of the meaning of a strategy or economic plan for an individual in terms of his information, communication possibilities and his data processing capabilities.

2. Duopoly Models and Moves

In order to give substance to the somewhat general remarks made in Section 1 a relatively detailed investigation of an extremely simple duopoly model is carried out.

2.1. The Single Move Models

The well known single strategic variable models of duopoly are the production or quantity model of Cournot[10/] and the price models of Betrand[11/] and Edgeworth[12/]. One may add "realistic" features such as product differentiation (as did Chamberlin[13/] and others). However these models remain appropriately robust under such a change.

In many texts the Cournot equilibrium is given a dynamic interpretation in terms of reaction functions. Similarly the Edgeworth cycle[14/] can be interpreted in terms of a dynamic process. As information conditions can become extremely complex very quickly, furthermore as it is easy to proliferate ad hoc dynamic models with little technological, institutional or behavioral basis, we restrict ourselves to at most one or two moves per competitor. This restriction, though highly limiting enables us to carry out a sensitivity analysis with respect to information and to illustrate the role of communication.

The Cournot Model with Different Information

When we talk about the competitors moving simultaneously we do not mean that they necessarily act at the same moment but that each must act without information concerning the behavior of the other. The two simple game tree diagrams shown in Figure 1 are representations of two games that are strategically equivalent. For ease in exposition suppose that two competitors must each select either a low or high level of production. P_1 stands for the first competitor and P_2 for the second. In Figure 1a the first competitor is assumed to actually set his production before the second. But the second is not informed of this, as is indicated by the information set containing both vertices at which the second must select his production. In Figure 1b the order of the moves is reversed, but as the information conditions are unchanged, no strategic difference has been made. If we wish we could consider a third representation of this game to cover the (highly unlikely) possibility that the competitors actually move at the

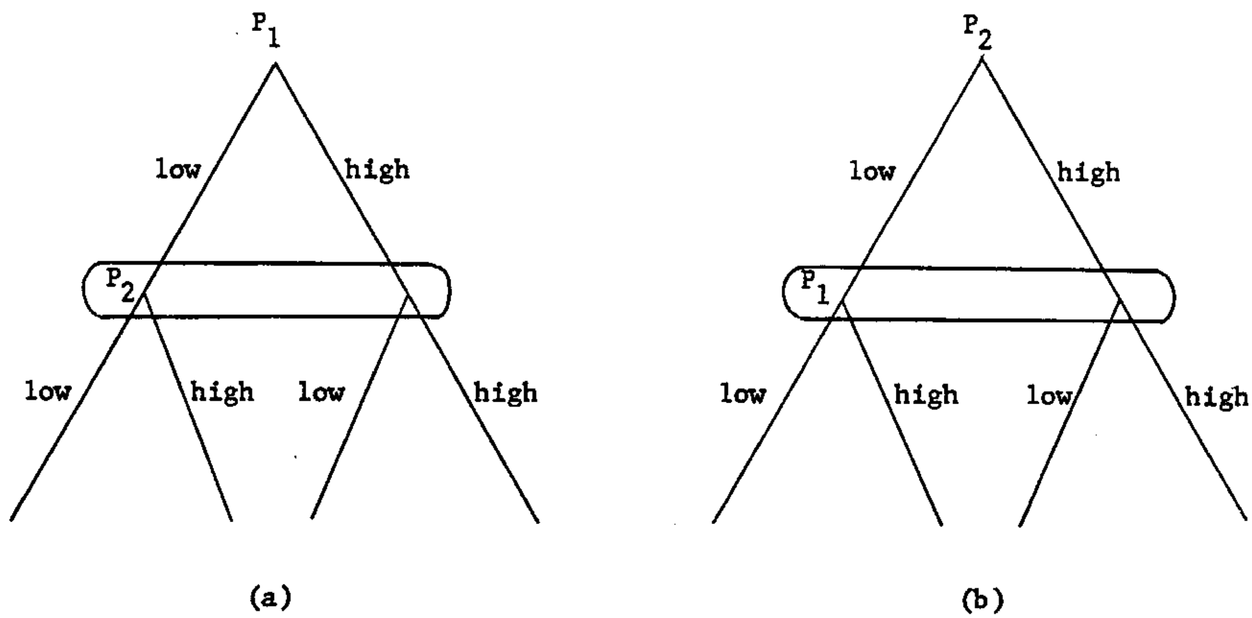


FIGURE 1

same split second. The symbol $P_{1,2}$ indicates that both move together, thus the game shown in Figure 2 is also strategically equivalent to both of those shown in Figure 1.

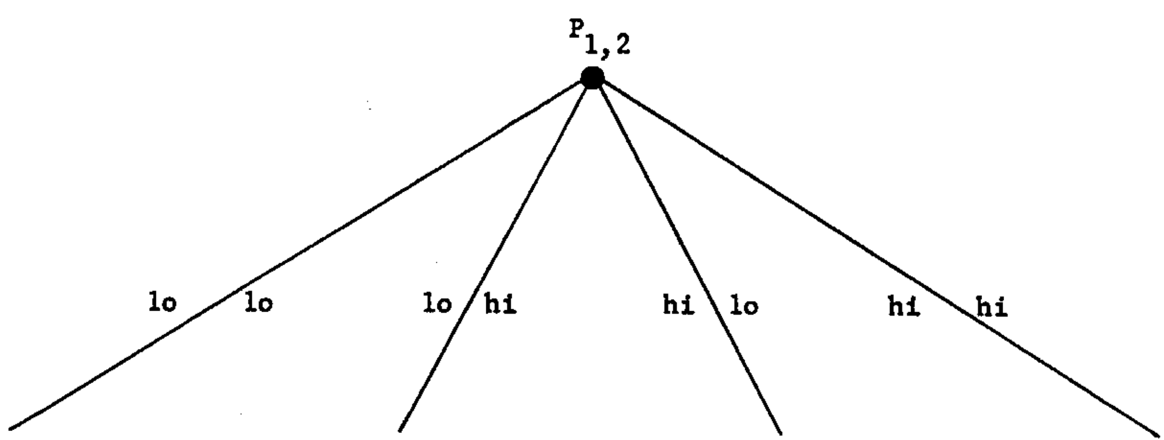
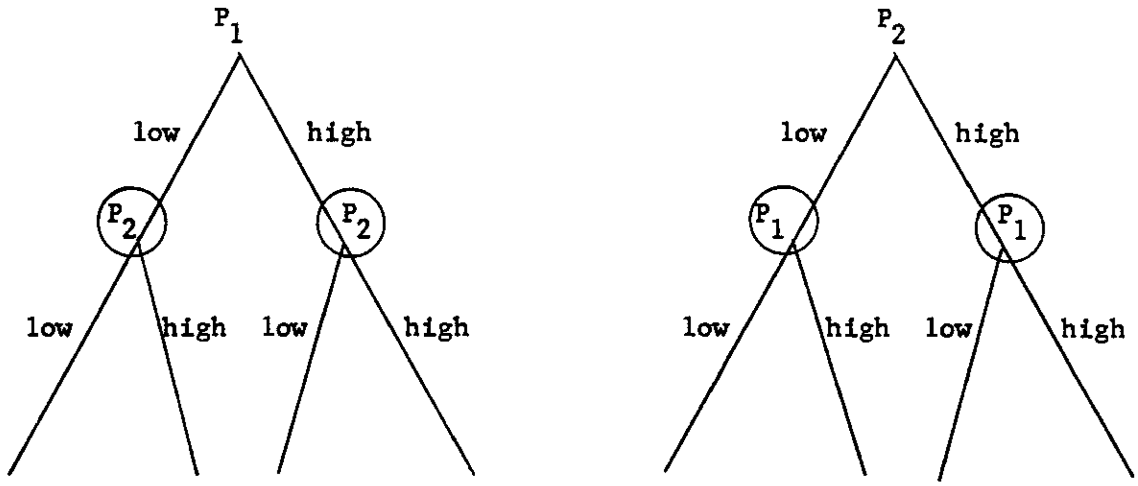


FIGURE 2

There are two other possibilities which are not strategically equivalent. Those are shown in Figure 3. In 3a the first competitor moves first and the other is informed prior to making his move. In 3b the situation is reversed.



(a) (b)

FIGURE 3

Depending upon technology and the formal and informal communication systems there is the possibility of an information leak. If the individuals act independently sometimes one will move first and sometimes the other will move first. Furthermore there may on occasion be information leaks. These possibilities are illustrated in Figure 4 where P_0 , the first to move may be regarded as "Nature" who selects from four alternatives with probabilities of p_1 , p_2 , p_3 , and p_4 (where $\sum_{i=1}^{4} p_i = 1$) . It can be seen that this game encompasses all of the others previously described as special cases. For example if $p_1 = 1$ this is equivalent to the situation in Figure 3a.

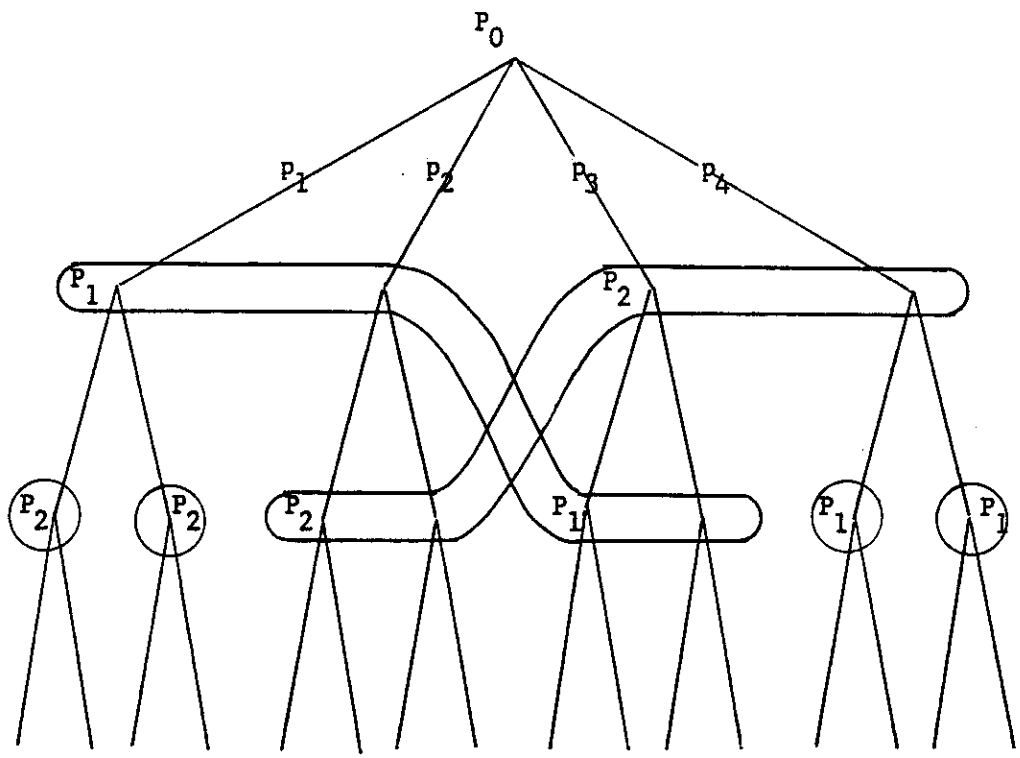


FIGURE 4

The Cournot model presupposes no direct discussion or communication among the competitors. Their actual moves are their communication and their messages are essentially sent via the market.

The solution concept used is that of the noncooperative equilibrium 15/. Suppose that the strategy for firm i is to name a production level q_i. Let the payoff (or net revenue) to firm i be $R_i(q_1, q_2)$. Then a pair of productions (q_1^*, q_2^*) form an equilibrium point if:

$$\max_{q_1} R_1(q_1, q_2^*) \text{ implies } q_1 = q_1^*$$

and

$$\max_{q_2} R_2(q_1^*, q_2) \text{ implies } q_2 = q_2^* .$$

The mathematician may be satisfied with the non-cooperative equilibrium point purely from the point of view of statics and existence. In other words he may be content with showing that for certain classes of markets the noncooperative equilibrium point always exists without stating anything about the behavior that brings it about.

The economist may call for more of an explanation than mere existence. The specification of reaction functions and dynamic models represent an attempt to further the explanation. Experiments with duopoly games indicate that the noncooperative equilibrium is by no means always the best predictor.[16/] Much appears to depend upon the briefing and the state of the long term knowledge of the players and upon the nature of their communication.

Setting aside many of the difficulties with the interpretation of the noncooperative equilibrium we examine how sensitive it is to information conditions by solving for all four models described above. We consider the market with symmetric quadratic payoffs for purposes of illustration.

(1) Let $R_i(q_1, q_2) = q_i(a - b(q_1 + q_2))$ where $a, b > 0$

then for the usual Cournot model given by Figures 1a and 1b with (informationally) simultaneous moves we have:

(2) $$\frac{\partial R_i}{\partial q_i} = 0 \text{ which gives:}$$

(3) $$\tilde{q}_1 = \tilde{q}_2 = \frac{1}{3}\left(\frac{a}{b}\right) \text{ and } R_1 = R_2 = \frac{a^2}{qb} .$$

We now consider the games illustrated in Figure 3. Only one needs to be solved as the solution for the other can be derived by changing the names of the players.

A strategy and move for P_1 who is assumed to move first (see Figure 3a) are equivalent. He has no contingencies to plan for. This is no longer the case for P_2, he is informed of q_1 before he selects q_2 thus although the actual move he makes is given by a production level q_2, his strategy is a function $f_2(q_1)$. In this game an equilibrium point is defined by the strategy pair (q_1, f_2).

One natural way of to select f_2 is to consider that P_2 maximizes his revenue given that he knows q_1. This gives

(4) $$\frac{dR_2}{dq_2} = 0 = a - b(q_2 + q_1) - bq_2 \quad \text{hence}$$

(5) $$q_2 = f_2(q_1) = \frac{a - bq_1}{2b} .$$

If this is the case P_1 should select q_1 to maximize

(6) $$R_1 = \frac{q_1}{2}(a - bq_1) \quad \text{hence} \quad q_1 = \frac{1}{2}\left(\frac{a}{b}\right) .$$

Thus equilibrium is given by

(7) $$q_1 = \frac{1}{2}\left(\frac{a}{b}\right), \quad q_2 = \frac{1}{4}\left(\frac{a}{b}\right) \quad \text{and} \quad R_1 = \frac{a^2}{8b}, \quad R_2 = \frac{a^2}{16b} .$$

Under the general definition of equilibrium point, the above is by no means the only equilibrium in this game. A different strategy for P_2 which could comprise part of an equilibrium pair if $f_2^*(q_1)$ described as:

(8) $$\text{"} q_2 = \frac{1}{2}\left(\frac{a}{b}\right) \quad \text{if} \quad q_1 = 0$$
$$q_2 = \left(\frac{a}{b}\right) \quad \text{if} \quad q_1 > 0 \text{."}$$

The equilibrium pair of strategies is $(0, f_2^*)$ which gives rise to productions of:

$$(9) \qquad q_1 = 0 , \quad q_2 = \frac{1}{2}\left(\frac{a}{b}\right) \text{ and profits } R_1 = 0 , \quad R_2 = \frac{a^2}{4b} .$$

The strategy f_2^* is essentially a threat and in order to even start to be plausible must be communicated in some manner to P_1 . If communication beyond the market is ruled out then it is reasonable to rule out this type of equilibrium. The other equilibrium does not really require communication for it to be reasonable in the sense that each individual maximizes in a noncontingent manner using only the information he has on hand. This point is enlarged later.

Returning to the threat strategy we may observe that several new difficulties in the modeling of the market are introduced along with this type of strategy. In particular the ability to carry it out depends upon the capacity of the threatner's plant. Thus we would require:

(a) P_2 has enough capacity for the whole marjet,

(b) P_1 knows the size of P_2's capacity,

(c) P_2 can communicate his intentions before P_1 selects his production,

(d) P_1 receives the message, interprets it correctly and believes it.

The belief may depend upon P_2's knowledge of P_1's costs (in this example they are zero). The higher they are, the more P_1 damages himself in carrying out a threat. The belief may also depend upon P_2's knowledge of previous history and his personal knowledge of P_1 .

Contrasting (5) with (8) we see that the former needs none of the communication and belief structure required for the latter. In particular, it belongs to a special class of equilibria which we may call <u>perfect equilibria</u>. A perfect equilibrium has the property that not only is it an equilibrium in a game considered as a whole, but it is also an equilibrium in every subgame of that game. A simple example illustrates this. The equilibrium described in (8) and (9) is not a perfect equilibrium. The equilibrium described in (5), (6) and (7) is. In the threat equilibrium if P_1 does not take P_2 seriously and actually produces some amount $q_1 > 0$ it does not pay P_2 to carry out his threat given that P_1 has acted. This contrasts with the equilibrium described in (5), (6) and (7). No matter what P_1 has done it virtually always pays P_2 to carry out his intended strategy.

We now turn to the model illustrated by Figure 4. Suppose that all of the p_i are equal hence any $p_i = 1/4$. There are four cases, but as is indicated by the information sets each player can only make two distinctions prior to moving. If he is told the move of the other he knows that he is in a situation akin to Figure 3a for the second player. If he is not given information then he knows only that he is in one of two simultaneous move games or that after he has selected his production the other player will be informed prior to making his selection.

An equilibrium strategy for P_1 is as follows:

If P_1 is informed of q_2 select $\hat{q}_1 = \frac{a - bq_2}{2b}$.

If P_2 is not informed of q_2 set* $q_1 = \frac{5}{14}\frac{a}{b}$.

---

*P_1 tries to maximize $2[q_1(a - b(q_1 + q_2))] + q_1\left(a - b\left(q_1 + \left(\frac{a - bq_1}{2b}\right)\right)\right)$, assuming symmetry we have $2a - 4bq_1 - 2bq_1 + \frac{a}{2} - bq_1 = 0$ hence $q_1 = \frac{5}{14}\frac{a}{b}$.

Thus the symmetric equilibrium is given by:

$$(10) \qquad \hat{q}_1 = \hat{q}_2 = \frac{9a}{28b} \quad \text{and} \quad q_1 = q_2 = \frac{5a}{14b}$$

with expected profits of

$$R_1 = R_2 = \left[\frac{1}{4}\left(\frac{9}{28}\right)^2 + \frac{1}{4}\,\frac{90}{(28)^2} + \frac{1}{2}\,\frac{80}{(28)^2}\right]\frac{a^2}{b} = \frac{331a^2}{3136b} .$$

The effect of symmetric uncertainty is to slightly increase the overall amount offered to the market. However the full comparison among different solutions is given in Table 1 below.

	q_1	q_2	q_1+q_2	P_1	P_2	P_1+P_2
Collusion	--	--	1/2	--	--	1
Duopoly P_1 first, with threats and communication	0	1/2	1/2	0	1	1
Duopoly P_1 first, perfect equilibrium	1/2	1/4	3/4	1/2	1/4	.75
Duopoly simultaneous	1/3	1/3	2/3	.444	.444	.888
Duopoly Random	9/28, 10/28	9/28, 10/28	39/56	.413	.413	.816
Duopoly Efficient	1/2	1/2	1	0	0	0

TABLE 1

In Table 1 profits and production have been normalized so that they are in the range from 0 to 1. This makes it easier to discuss the magnitudes of the three effects of numbers, information and communication.

We see immediately that the effect of fewness in numbers is considerable. In the range from the socially efficient profit of 0 to the monopoly profit of 1 the least duopoly model has a profit of .75 (this is in contrast with models which have price as the decision variable).

Limiting ourselves to variation of information, but no communication of strategies or discussion among players the variation in individual player profits is from .25 to .5 and in joint profits from .75 to .888. In this instance the ability to transmit the information is valuable to the player doing so.

An important distinction must be made between a general precommitment and information concerning a move. A general precommitment is a strategy or parts of a strategy with contingent clauses such as "If you do x I will do $f(x)$." Depending upon one's model of human behavior we may attach various degrees of belief to whether the contingencies stated will actually be followed.* In contrast with this, information concerning a move is information concerning an act that has taken place. In these models the acts are the selection of levels of production. Communication of strategies is in contrast to information concerning moves as are words to deeds.

---

*In the construction of game models we may decide to model verbal statements as actual moves in the game, in which case the degree of belief must be specified in the rules. Frequently words and the communication system are left out of the game.

When we include communication with the sending of threats the advantage to having more information now goes to P_2 _if_ his threats are believed.

We note that once threats are included the players must be able to judge both the _ability_ and the _intention_ to carry them out. Thus in the case of duopoly, costs and capacities must be known.

Price Models with Different Information

When price is taken as the strategic variable two major case distinctions must be made. They are when each firm has enough capacity to supply the whole market, and when this condition does not hold. In the first instance (corresponding to Figures 1 and 2) with simultaneous moves the efficient and the duopoly noncooperative solutions are the same (the Bertrand solution).

$$(11) \qquad p_1 = p_2 = 0 \ , \quad R_1 = R_2 = 0 \quad \text{and} \quad q_1 = q_2 = 1/2 \ .$$

As soon as one has to move first the increase in information improves the payoffs to both. This can be seen by a tedious but straightforward calculation of the _minoran game_ and has been done elsewhere.[17/]

When capacity limitations are such that not every firm can supply the market by itself then it has been shown that no pure strategy equilibrium may exist[18/]. Edgeworth first illustrated this. Surprisingly if uncertainty concerning information leaks is high enough a pure strategy may be restored. A related phenomenon where lack of knowledge of the overall market size helps to stabilize behavior has been studied elsewhere[19/].

2.2. Duopoly, Communication and Information

Based upon the analysis in 2.1 it is suggested that for even an extremely simple one move per player duopoly model, the results are highly sensitive to the information conditions.

As is well known, furthermore the results are highly sensitive to whether price or production are the decision variables.

A new dimension of difficulty is introduced when communication between the players is considered. Equilibria whose stability depend upon the belief that threats will be carried out appear in profusion.

When there are only two or three competitors in a market the chances for information leaks are large. Individuals can easily identify each other and know what information they are after. Furthermore the chances are high that informal or formal communication channels between any two identifiable individuals exist. The dossier that General Motors has on Ford is large. Furthermore the management of General Motors may well know the management of Ford, even to the point of predicting probable reactions to policies.

An abstract, noninstitutional static oligopoly theory such as that of Cournot, Edgeworth, Hotelling or Shubik cannot do more than point out the bounds on oligopolistic behavior, suggest various solution concepts and investigate the sensitivity of the models to different types of change. They are at least sufficiently well defined that simple experiments can and to some extent have been run to see if there is any relationship between behavior in the laboratory and predictions based upon the various theories.

The quasi-dynamic models of Chamberlin, Stackelberg, Fellner[20/] and others are less formally defined than those noted above, but they do not contain a more complete or better institutional, technological or socio-psychological basis than do the more mathematical models. As such their value in casting light on oligopolistic competition is extremely limited. There appears to be no substitute for providing a detailed description of any oligopolistic market in order to determine the critical moves, information patterns, time lags and communication networks that have a considerable influence on the outcome.

The observations noted above are by no means meant to imply a pessimistic bias. When the participants in a market are few in number and large in size it is reasonable and economic to do a detailed study of the market and to construct a detailed _ad hoc_ model along with applying any of our theorizing.

When the number of participants in a market is large in general it would be too expensive and difficult to maintain and usefully use detailed dossiers on each. However it is argued in Section 3 that when competitors are many the model of the market is relatively _insensitive_ to variations in the information conditions. Thus detailed modeling of the information state is not necessary. This is not necessarily true for the communication system. Codes of behavior, group or aggregate threats may still play a role.

2.3. Market Mechanisms and Multimove Models

Markets differ considerably; thus fur, tobacco and some other commodity markets have fixed times of year for assembly, specific rules and signs for bidding and a professional group of buyers who inspect the lots in detail before the auction. Stock markets vary somewhat from each other. As computer communications improve the institutional power of clubs such as the New York Stock Exchange is lessened. Essentially there is a network of brokers connected by extremely fast communication serving millions of customers who belong to highly differentiated classes when viewed in terms of financial needs, knowledge of what they are buying, size and anonymity. A small customer trading a few shares of A.T. and T. is known to virtually no one. He may pick a broker randomly and open an account just to trade those shares. A large insurance company is known to the financial community. It may easily arrange a "third market" transaction with a large fund.

It is of interest to note that the stockmarket permits price, price-quantity or quantity strategies. Thus an order may be:

buy all you can get at 65

or buy 200 at 65

or buy 300 at market.

A wheat farmer at planting time uses a quantity strategy (with an exogenous random component), although if he can inventory, then his strategy is more complex. (If he can trade in futures contracts or hedge, it is even more complex.) Sealed bid methods are used for oil leases, large machinery and systems contracts. Dutch auctions are used for tulip bulbs and elsewhere. The customer in many parts of the modern world takes current price as given in most anonymous mass standardized markets. This does not hold in bargaining for a house or for large or special items where face-to-face offers and counter-offers may take place.

It is my contention that the differences in information patterns, strategic variables and institutional detail tend to wipe out for mass markets. However communication networks be they reflected in codes of behavior, "business ethics," formal or informal cartels can play an important role in markets even with many participants.

This contention rests upon considering different classes of strategies and equilibria. In particular we wish to consider general (fully historical) strategies and state strategies (which depend only upon the state that an individual perceives, not the history of how he arrived at that state).

3. Information, Communication and Many Firms in the Market

3.1. The Noncooperative Market, Numbers and Information

Since the time of Cournot it has been known that (under the appropriate conditions[21/]) as the number of competitors is increased in a market with quantity competition and simultaneous moves the noncooperative equilibrium outcome approaches that of the efficient point or competitive equilibrium. Using the same simple example as in (1) where however for the n person market we have:

$$p = a - \frac{2b}{n} q \quad \text{and} \quad q = \sum_{i=1}^{n} q_i \; ; \tag{12}$$

then

$$R_i = q_i\left(a - \frac{2b}{n} q\right) \quad \text{hence at the equilibrium} \tag{13}$$

$$q_i = \frac{n}{n+1} \frac{a}{2b} \; . \tag{14}$$

For n large this approaches $a/2b$ and $R_i = 0$. An illustration of an extensive form of this game is shown in Figure 5. The total number of information sets is n. This is the least amount of information possible for this n player n move game.

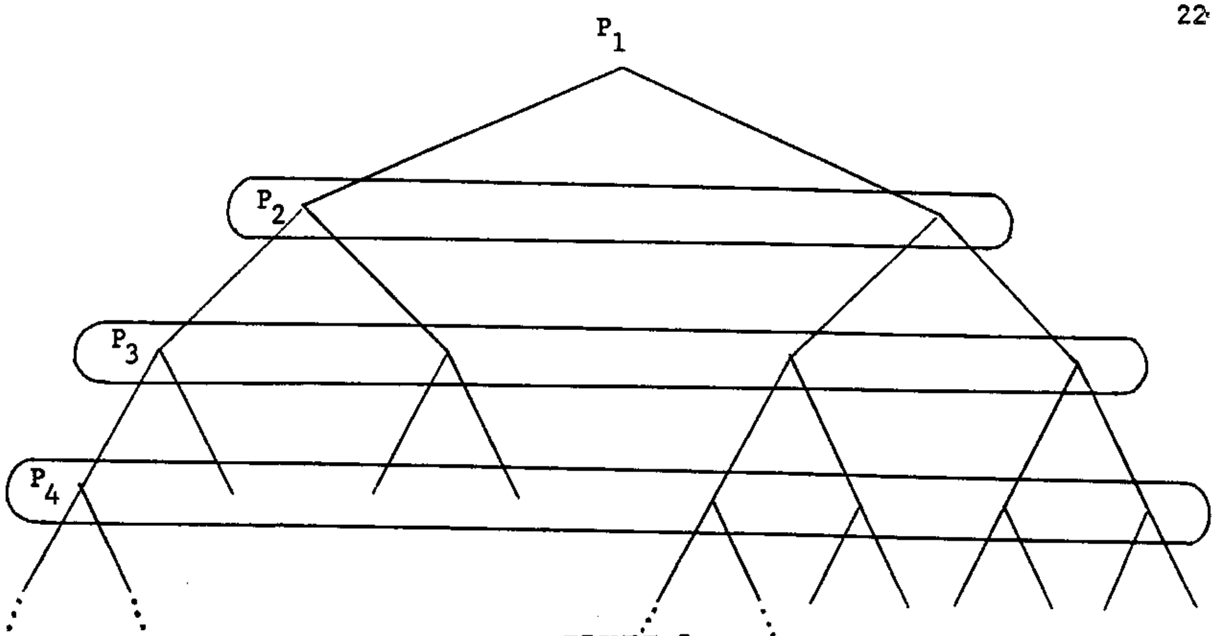


FIGURE 5

Suppose that for each player k levels of production can be distinguished. Then there will be $n!$ games which have the largest amount of information possible. The total number of information sets in each of these games is

$$1 + k + k^2 + \dots k^n = \frac{k^{n+1} - 1}{k-1} .$$

As there are games with perfect information we may solve for a perfect equilibrium point. This is relatively straightforward with each player acting as monopolist for the market remaining when he moves. Thus if the order of moves is 1, 2, 3, ..., n . The levels of production will be:

$$q_1 = \frac{1}{2}\left(\frac{an}{2b}\right), \dots q_n = \frac{1}{2^n}\left(\frac{an}{2b}\right) .$$

As n becomes large $q = \sum_{i=1}^{n} q_i$ approaches $an/2b$ which is the competitive equilibrium output, however profits (although approaching zero) are split in proportion $1 : \frac{1}{2} : \frac{1}{4} : \dots$.

An intuitively unsatisfactory aspect of this solution is that it requires that the firms have unlimited capacity. The first firm must be prepared to supply half of the market for any size of n . Suppose instead we assume that each has a capacity k . Then if $k \geq \frac{a}{2b}$ as n becomes large total production still approaches $\frac{an}{2b}$.

If all of the perfect information games are equally probable then the expected production from any firm is q/n or $a/2b$. The expected outputs and individual profits approach the competitive level.

Even a brief contemplation of the informational variations of the n person n move game leads one to conclude that an exhaustive investigation is not a promising approach and that some form of aggregation is called for. For example in the n moves game with perfect information the strategies for the m^{th} player are $k^{k^{(m-1)}}$ in number. If we try to extend our two player two alternatives game shown in Figure 4 to three players unless we have some reasonably natural way for aggregating information the difficulties and the profusion of cases are considerable.

A reasonable extension of the two person case to n persons can be made based upon the observation that in this game (in contrast with many others) there is a natural metric on moves. They are quantities of the same commodity and can be added. Thus we may consider an n person game where each has a probability p_1 that he belongs to a set of players (whose size he is told) that is informed about the moves of some of the others before they move. The information comes in the form of an aggregate figure on production together with the number of producers. Each has a probability p_2 that he moves in secrecy from others and has no information about others. And there is a probability p_3 $(p_1 + p_2 + p_3 = 1)$ that the player's move is part of the (aggregate) information supplied to part of the market. This model is a natural aggregate modification and extension of the game shown in Figure 4.*

*The event with probability p_1^n must be interpreted that all n are informed that the set of players moving first is empty.

One important feature of the aggregation of information is the swift increase in the anonymity of the players. Their actions are swallowed up in aggregate statistics. This means immediately that threats cannot be meaningfully directed against individuals as their acts cannot be identified. Furthermore if the market is a mass mechanism the acts of one individual effect all others, not selected targets[22/].

3.2. The Noncooperative Market and Communication

Given the type of aggregation suggested above in 3.1 we can provide a different interpretation of the game in terms of a control process. An individual recognizes two general types of states:

(a) he is informed about the acts of some others before he moves (probability p_1),

(b) he is not informed about the acts of others before he moves, but knows that some may be informed about his acts before they move.

The decision rule followed is to maximize expected payoff on the assumption that other players in the same information state will do likewise. This is an application of a principle of external symmetry; i.e. if the players do not have detailed socio-psychological knowledge of each other or other specified differentiating evidence (such as knowledge of large differences in size or costs) they assume similar behavior in similar circumstances.

The above approach converts the n person game into n parallel 1 person control processes with no communication among the players (beyond the specification of the information patterns given by the rules of the game).

Stress has been laid upon the need to aggregate information and to consider limits on the amount of communication owing to costs. Nevertheless there is a great difference between short term and long term communication costs. Institutions, customs and codes of behavior, professional societies and long standing laws all provide devices for communicating aggregate threat strategies among a group.

It may be expensive to keep an extensive dossier on members of a trade or profession. It is not necessarily prohibitive to sample to identify violators of a code. It is extremely expensive for an individual firm to keep detailed individual statistics on his competitors is they are many. It is relatively easy to pay for a trade association to maintain these figures.

Trade associations, technical societies, jointly used brokers or other information services provide the means to convey aggregate plausible threat strategies to a large group of individuals. Much trade is carried on at the quasi-anonymous level in such a way that if affairs go smoothly, little information is required; if someone violates the code, he can be identified and punished with a high probability. Payment by check provides an example. In many instances once the check has been honored the payor's name is forgotten by the recipient.

Credit rating agencies, membership committees, ethical practises groups and in some places cartel boards, fair trade laws and escrow requirements are all manifestations of devices for policing the level of competition.

In order to model quasi-anonymous markets we must include at least a first order approximation of the information handling and communication institutions serving the markets. The threat and the plausibility of threat is transmitted to individuals via these jointly visible institutions.

In a mass market the use of detailed historical strategies with directed threats by individuals against specific individuals is too expensive or infeasible due to costs of information gathering, costs of computation and difficulties in communication. Some aggregate threat behavior is feasible via the institutions of the market. In mass economic markets the initial description of behavior that is both tractable and has some empirical plausibility is that of the noncooperative behavior based upon a relatively parsimonious state description which implies virtually no communication outside of the rules of the game. This description may be adequate for some purposes of economic analysis. However there are undoubtedly some markets in which a more detailed description is called for in which more states are identified and more history is used.

4. Concluding Remarks

4.1. Institutions, Organizations and Information

Institutions and organizations especially when viewed primarily in their economic roles are the carriers of process. They are the coding, communicating and information processing machines of the economy.

The variable costs in running efficient information and communication networks are extremely small in comparison with the costs of network building. Organized markets and their ancillary institutions provide important external economies to individual competitors. In large markets it is the history of the market and the aggregate forces of the market which provide much of the information to an individual about the other participants.

There is no such thing as institution-free economic theorizing. One can make the implicit or explicit assumption that everyone is completely informed about everything and that computation and communication are instantaneous and free. However to argue that in a complex set of models the one in which a host of parameters have been assigned extreme values is somehow more institution-free than the others is an argument without basis. Parsimony and simplicity may dictate this as a first approach. However it is argued here that when numbers are few this approach will be inadequate and when numbers are many communication and information are intimately related to the institutions designed to coordinate mass markets.

The convergence of outcomes and the approach of individual behavior to the norms of the competitive equilibrium is one low information and communication state outcome, but not necessarily the only one.

4.2. Solutions, Sociology, Economics and Communication

The contract curve of Edgeworth[23/], the bargaining point of Zeuthen[24/] and several of the solutions in the theory games such as the core[25/] or value[26/] are cooperative solutions where it is assumed that all parties are free to communicate and bargain costlessly outside of the actual game. They are ruthless abstractions which are of use answering questions about the upper bounds to the possibilities of distribution of resources. They have all of the weaknesses of discussions based on treating all points on the Pareto optimal surface as equally costlessly attainable regardless of initial conditions. They are completely static.

In a world in which process is costly and the flow of information is heavily determined by both technology and the structure of institutions the

the costs and limits of processes are not merely a "friction" or a minor correction on the grand static theory, they are at the central core of the problem.

A static theory cannot be made dynamic without an implicit or explicit introduction of institutions as the carriers of process. The economist likes to ignore uncomfortable subjects such as the different norms of cooperation among groups, the customs concerning tipping, or bidding or doing business. However both historical and experimental evidence points to the importance of institutional and sociological forces in an economic dynamics.

When viewed from the viewpoint of microeconomic theory the institutions represent the long term data processing and communication devices. Sociological conditions such as accepted codes of behavior provide the confidence levels and plausibility of plans and threats. Detailed technological considerations such as the length of an industrial process or the speed of a computer network influence the immediate flows of information.

When we contemplate a dynamic system the facile separation into cooperative and noncooperative theories that can be made for statics can no longer be made. Communication and enforcement mechanisms must be modeled as part of the game. They form an integral part of the process.

Perhaps one of the major confusions concerning both the power and limitations of game theory rests on the relationship between statics and dynamics. With great care von Neumann and Morgenstern[27/] stressed that as a <u>first step</u> the theory they presented was <u>static</u>. The import of this statement is that just like in much of microeconomic theory communication and enforcement mechanisms are <u>not</u> modeled explicitly. Schelling[28/] laid stress on the role of communication and the nature of commitment. His interests were however aimed at developing a theory to explain and illustrate a dynamics of bargaining. This was not at cross purposes with the developments in the theory of games it was merely a call for a theory to take into account dynamics as well as statics. To this day work towards this goal has been extremely limited.

It is my belief that the development of a dynamic theory of games that has any applied value must incorporate more institutional and technological detailed than is customary in say, microeconomic theorizing. Furthermore the prospects for the development of a <u>mass participant</u> model are far better and less dependent upon sociopsychological and other behavior detail than the development of a dynamics for few participants. For the latter (which includes political, diplomatic and economic bargaining) the details of communication are more important and the enforcement mechanisms are less developed than for the former.

In many mass economic markets the use of escrow agents, the pledging of assets and signing of contracts are highly organized activities. If enough assets have been put up as security, strangers do <u>not need</u> to trust each other. Their promises and contingent plans become completely plausible owing to the losses any violator would suffer. Financial punishment or jail sentences for contract violators provide much of the enforcement mechanism in a stable society.

The institutions and social and legal rules which cover most dealings for say, householder mortgages or small bank loans are well developed and usually can be clearly enforced. This is not the case for international bargaining. Furthermore even in large labor negotiations or deals among few large firms the economic and legal institutions do not necessarily provide commitment enforcement mechanisms. "Trust" among many can be depersonalized, socialized, institutionalized or made unnecessary. In contrast when competition takes place among the few and the large the outside enforcement agencies may not suffice. Thus the need for the examination of the formation of trust and expectations concerning threats and promises is called for. This in turn implies a need for a detailed knowledge of the state of communication, technological factors influencing moves and countermoves as well as the history of previous behavior and social and socio-psychological information on the participants.

FOOTNOTES

1. Stackelberg, H. von, Marktform und Gleichgewicht, Berlin: Julius Springer, 1934.

2. See for instance the models in: Chamberlain, E. H., The Theory of Monopolistic Competition, Cambridge: Harvard University Press (6th ed.), 1950.

3. Cournot, A. A., Researches into the Mathematical Principles of the Theory of Wealth, New York: MacMillan, 1897.

4. Edgeworth, F. Y., Papers Relating to Political Economy, I, London: Macmillan, 1925.

5. Hotelling, H., "Stability in Competition," The Economic Journal, XXXXI, March 1929, p. 41.

6. See for instances: Shubik, M., Strategy and Market Structure, New York: John Wiley and Sons, Inc., 1959.

7. Debreu, G., Theory of Value, An Axiomatic Analysis of Economic Equilibrium, Cowles Foundation Monograph 17, New York: John Wiley and Sons, Inc., 1959.

8. Arrow, K. J., "Le Rôle des valeurs boursienes pour la reportetion la meilleure des risques," Econometrie, Paris, C.N.R.S., 41-48, 1953.

9. Radner, R., "Equilibrium under Uncertainty," Econometrica, 36 (1968), 31-58.

10. Cournot, A. A., op.cit.

11. Bertrand, J., "Theorie mathematique de la richesse sociale" (review), Journal des Savants, Paris: September 1883, 499-508.

12. Edgeworth, F. Y., op.cit.

13. Chamberlain, E. H., op.cit.

14. See Shubik, M., op.cit., Ch. 5.

15. Nash, J. F., Jr., "Non-Cooperative Games," Annals of Mathematics, LIV, September, 1951.

16. Shubik, M., Wolf, G., and Eisenberg, H., "An Experimental Oligopoly Business Game, Part I, Some Experiments," General Systems (in press). Hoggatt, A. C., "An Experimental Business Game," Behavioral Science, 4, 1959, 192-203. Stern, D. H., "Some Notes on Oligopoly Theory and Experiments," in M. Shubik (ed.), Essays in Mathematical Economics (In Honor of Oskar Morgenstern), Princeton: Princeton University Press, 1967, 255-281.

17. Shapley, L. S. and Shubik, M., Competition, Welfare and the Theory of Games, Part III (manuscript in process).

18. See Shubik, M., Strategy and Market Structure, Ch. 5.

19. Levitan, R. E. and Shubik, M., "Price Variation Duopoly with Differentiated Products and Random Demand," Journal of Economic Theory, March 1971, 23-39.

20. Fellner, W., Competition among the Few, New York: Knopf, 1949.

21. Wald, A., "On Some Systems of Equations of Mathematical Economics," Econometrica, 19, October 1951.

22. Shubik, M., "Pecuniary Externalities: A Game Theoretic Analysis, American Economic Review, Vol. 61, No. 4, September 1971, 713-718.

23. Edgeworth, F. Y., Mathematical Psychics, London: Kegan Paul, 1881, 20-25.

24. Zeuthen, F., Problems of Monopoly and Economic Welfare, London: Routledge and Sons, Ltd., 1930.

25. Shapley, L.S. and Shubik, M., Competition Welfare and the Theory of Games (manuscript in process), Ch. 6.

26. Shapley, L. S., and Shubik, M., op.cit., Ch. 7.

27. von Neumann, J. and Morgenstern, O., Theory of Games and Economic Behavior, Princeton: Princeton University Press, 1947.

28. Schelling, T. C., The Strategy of Conflict, Cambridge: Harvard University Press, 1960.