COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 336

A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART VIII

TRANSACTIONS COSTS IN A MARKET ECONOMY

Martin Shubik

April 5, 1972

A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART VIII

TRANSACTIONS COSTS IN A MARKET ECONOMY

by

Martin Shubik*

*This research was supported by the Office of Naval Research. The research was also partially supported by a grant from the Ford Foundation.

TABLE OF CONTENTS

INTRODUCTION

In two previous papers [1] a closed economic market model was formulated as a noncooperative game. The first of these papers noted dealt with a market using a commodity money and the second dealt with a market using a fiat money.

The stress in this paper is on a closed market economy using commodity money where there are transactions costs present. This model can be extended to one dealing with fiat money. However it is easier to consider commodity money first. The extension to fiat money is discussed in a previous paper [1].

There are several different ways in which transactions costs can be modeled. They are basically different and have direct association with different forms of trading that are, or have been used in various societies. The most important differences concern payment of transactions costs in kind or in money and whether the market is a mechanism or governmental agency or whether traders or "middlemen" form a distinct type of economic agent.

2. THE NONCOOPERATIVE ECONOMY WITH COMMODITY MONEY

Suppose that there are n individuals in a closed economy trading in $m+1$ commodities. The preferences of the i^{th} individual can be represented by a utility function of the form $\varphi_i(x_1^i, x_2^i, \ldots, x_m^i, x_{m+1}^i)$ where x_j^i for $j = 1, 2, \ldots, m+1$ is the amount of the j^{th} commodity consumed by the i^{th} individual.

We assume that the initial endowment of the i^{th} individual is given by $(A_1^i, A_2^i, \ldots, A_m^i, A_{m+1}^i)$. From a utilitarian viewpoint none of the $m+1$ commodities is distinguished. They all appear in the utility function and have consumption value to the individual.

One of the commodities may be selected for use as a commodity money. For convenience in notation we may consider that it is the $m+1$ st commodity. The commodity money (although it is not distinguished from other commodities in terms of consumption) is distinguished from the others in terms of its strategic use, or its employment in trade.

A strategy for a trader i is a 2m-dimensional vector where when an individual wishes to sell a commodity j he offers an amount $q_j^i \leq A_j^i$. If he wants to buy a commodity k he offers an amount of money d_k^i for the commodity, where:

$$(1) \qquad \sum_{k=1}^{m} d_k^i \leq A_{m+1}^i \quad .$$

It is assumed that an individual enters the market as either a buyer or seller for any commodity he wishes to trade but does not simultaneously buy and sell the same commodity. Thus a typical strategy for a trader i might be as follows:

$$(q^i_1,\ 0;\ 0,\ d^i_2;\ 0,\ 0;\ q^i_4,\ 0;\ q^i_5,\ 0;\ \ldots;\ 0,\ d^i_m) \quad . \tag{2}$$

In the strategy illustrated above the trader i is offering quantities of products 1, 4 and 5 for sale, he is in the market to buy products 2 and m and he is inactive in the market for product 3. His final position in any commodity j he sells will be $A^i_j - q^i_j \geq 0$ (i.e. markets clear). His final position in any commodity k he buys will be $A^i_k + d^i_{k/p_k}$ where p_k is the price. (The possibility that $p_k = 0$ is discussed in 2.2 later). The trader's final position in commodity money is determined residually by the effect of the strategies on the market. It is:

$$x^i_{m+1} = A^i_{m+1} - \sum_{k=1}^{m} d^i_k + \sum_{j=1}^{m} p_j\, q^i_j \quad . \tag{3}$$

2.1 The Market Clearance Process and Prices

We may imagine that there exist m markets or "clearance posts" at which all traders deposit goods for sale and commodity money bid for purchase. Prices are determined "a la Cournot" i.e. they are the ratios of the total amount of commodity money bid or put up for the purchase of a good and the amount of the good offered for sale.

$$p_j = \sum_{s=1}^{n} d^s_j / \sum_{t=1}^{n} q^t_j \tag{4}$$

The method of bidding introduces a cash flow problem for each trader. He is paid in commodity money for all goods he sells but (in contrast with the purely competitive model) he cannot use his immediate income for immediate purchase.

These are both theoretical and applied reasons for using the type of strategies suggested here. The simple abstract purely theoretical justification for using a "quantity" strategy is simply that we know that we are able to define a quantity strategy open-market noncooperative oligopoly model. This is the Cournot model of oligopoly. Can we define the same type of model for a closed economic system? The answer given in a previous paper [2] and here is "yes we can" and this is the model.

Having defended the model as no less unrealistic than the Cournot model, we may now turn to a different consideration. Although it may be unreasonable to believe that an individual will allocate quantities of commodity money among the items he wishes to buy without knowing their price. This may only be the case when there are few individuals in the market. If there are many individuals in the market even though the sum of individual strategies determines price each individual knows that although he may have some influence on price the chances are small that it is large, hence as a first order approximation his strategy in naming a quantity of commodity money to purchase a good is approximately the same as specifying how much of the good he wishes to buy.

2.2 Inactive Markets: No Supply or Demand

In our description of the markets and the formation of price we must ask what happens when no product is offered, or no commodity money is offered for a commodity or both.

We have a problem in modeling. What happens to the stock of product I offer for sale if there is no bid for it? What happens to the commodity money I offer in a market if there are no goods for sale? There are several different conventions which can be used. Each has some basis in the activities of economic institutions. If you fail to sell your flowers or tomatoes you may get them back but they may have wilted or rotted by the end of the market period. Even if the goods are not perishable there is the inventory cost and other costs incidental to bringing goods to the market, offering them for sale and removing them.

If one makes an offer to buy a commodity, but no commodity is offered in return one keeps one's money however certain nuisance costs have been experienced.

Given the considerations above if an individual is the only one offering to sell or buy in a market, if there are any marketing costs whatsoever, he would be better off not being in the market. Two individuals, one on each side are not sufficient to achieve a noncooperative equilibrium in any active market. Given the price mechanism it will always pay the buyer to offer less money, holding the seller's strategy constant. Similarly it would pay the seller to offer less, holding the buyer's strategy constant.

One needs at least four traders to be active in any market for the existence of a noncooperative equilibrium with active trade. The four must consist of two traders on each side of the market. Each seller is needed to provide competition for the other and each buyer is needed to provide competition for the other.

A noncooperative equilibrium is defined in terms of stability in face of the actions of any single individual. Suppose there are n players in a game. Each player i has a set of strategies S_i . A particular strategy of a player i may be denoted by $s_i \in S_i$. Suppose that associated with each player i is a payoff function $P_i(s_1, s_2, s_3, \ldots, s_n)$. Each payoff is a function of n variables, the strategic choices of all individuals.

A set of strategies $(s_1^*, s_2^*, s_3^*, \ldots, s_n^*)$ form a noncooperative equilibrium if, holding the strategies of any set of $n-1$ players fixed the remaining player has no incentive to change his strategy. Specifically for all i

$$\max P_i(s_1^*, s_2^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*) \Longrightarrow s_i = s_i^* .$$

Given this definition, if by chance there are no active bids or offers in a market, the inactive market can form part of an equilibrium because no single individual can risk entering the market himself without others present. Thus in an economy trading in m commodities if there is at least one noncooperative equilibrium involving active trade in all commodities there will be as many as 2^m-1 other noncooperative equilibria with every combination of inactive markets.

If there is economic gain to be had by having a market be active we may suspect that a noncooperative equilibrium with a market inactive will be relatively unstable, i.e. a slight perturbation which gets two traders active on each side is all that is needed to destroy the equilibrium.

It is conjectured that if there is a competitive equilibrium solution to an economy which calls for all markets to be active then there will always exist at least one noncooperative equilibrium with all markets active that yields traders higher payoffs than any particular noncooperative equilibrium with some markets inactive.*

An example of this type of relationship among noncooperative equilibria is given by the following four person trading game. There are five commodities, each trader has the utility function:

$$U_i = \min(x_i, y_i) + \min(z_i, w_i) + m_i$$

Their initial endowments are for traders A, B, C and D respectively, (0, 2, 0, 2, 2), (0, 2, 0, 2, 2), (2, 0, 2, 0, 2) and (2, 0, 2, 0, 2). The last commodity is used as a commodity money.

There is a noncooperative equilibrium with all markets active yielding to all final holdings of (1, 1, 1, 1, 2) and a payoff of 4. However there are also equilibria with the first two or last two markets inactive yielding positions of:

(0, 2, 1, 1, 2), (0, 2, 1, 1, 2), (2, 0, 1, 1, 2), (2, 0, 1, 1, 2)

and (1, 1, 0, 2, 2), (1, 1, 0, 2, 2), (1, 1, 0, 0, 2), (1, 1, 2, 0, 2) respectively.

These each have a payoff of 3.

---

*We also require that there must be at least two buyers and sellers in every market at the competitive equilibrium.

In the example above owing to the extreme simplicity of its structure the competitive equilibrium solution coincides with the noncooperative equilibrium with all markets active.

When we include trading costs in our model and consider as feasible only distributions which can be attained after having paid these costs then it is possible that the Pareto optimal surface that is attainable may include many inactive markets.

2.3 An Aside on An Inactive Money Market

In this paper although our prime concern is with transactions costs in a market using a commodity money; the model can be extended to a multiperiod market where individuals trade using a paper money. Suppose that we consider the existence of two types of paper neither of which enters directly into the utility functions of the traders. The first is paper money which is used for the purchase of all real goods. The second is a "promise to pay" future fiat money (specifically to deliver money at the start of the next period).

Suppose we permit a market between present money and next period's money. Then there will be two noncooperative equilibria in the noncooperative game. One with an active money market (i.e. a banking system, lending, borrowing and a rate of interest) and the other without an active money market and hence no borrowing or lending or defined interest rate. Furthermore we may conjecture that both of these noncooperative equilibria (if all start with positive amounts of fiat money) will have the same value.

In a previous paper [3] it was noted that in a finite sequence of markets without exogenous uncertainty and without a futures market in any of the goods entering the trader's utility functions there could be two competitive equilibria, one with a money market and one without.

It is conjectured that for an infinite sequence of markets the loss of the final budget constraint destroys the competitive equilibrium with no money market. However in any finite model considering either the competitive equilibrium or noncooperative equilibrium solutions, in each case, both types of equilibria will exist.

3. ON MODELING TRANSACTIONS COSTS

In every economy resources are used in the act of trade. Going to market takes time. Bargaining in a barter economy takes time. Sorting, grading, classifying, transporting, describing, displaying, searching, accounting and information processing all use various types of resources.

Stockbrokers, real estate agents and auctioneers are usually paid in money. Frequently they obtain a percentage of the funds realized from a sale. In barter exchanges the transactions costs are implicitly paid by the traders in terms of their time and transportation, packaging and other resources.

In primitive (and not so primitive) economies some (or all) of the costs can be jointly imputed to other activities such as festivals and ceremonies associated with going to the market or fair. The markets provided important social events, and in some instances such as trade fairs and art auctions in a modern society they still do.

Setting aside the worth of the trading process itself as an economic good we are still confronted with several alternative ways of modeling the way in which the costs of transacting are paid for.

3.1 Transaction Expense Payments in Time, Kind, or Other Commodities.

Intuitively it would appear to be more satisfactory to consider an economy in which fiat money is used and in which all payments are made in terms of this money. In this paper however the choice has been made to consider trade using a commodity money because in this manner not only is it possible to formulate a one period noncooperative game model of the economy with a mechanism for deducing price, it also is easier to avoid the dangers of confusing the monetary and resource utilization in carrying out transactions.

Model 1

The first model involves transactions costs in kind. An individual i who sells a quantity i of commodity j is only able to deliver an amount $f_j^i(q_j^i)$ where $0 < f_j^i(q_j^i) \leq q_j^i$. This is tantamount to suffering, breakage, theft and other inventory losses on the way to market. An individual who buys suffers no losses in the commodity money he offers for purchase. Thus the strategy for an individual is identical to that described in (2). Price formation however is somewhat different. Instead of (3) we obtain:

$$p_j = \sum_{s=1}^{n} d_j^s \Big/ \sum_{t=1}^{n} f_t(q_j^t) \tag{5}$$

Using the formulation it is easy to see that the Pareto optimal surface is fixed when there are no transaction costs (i.e. when $f_j^i(x_j^i) = x_j^i$). However when there are transactions costs the feasible set of distributions (and hence the Pareto optimal surface) will depend upon the initial distributions. The amount that any individual i will obtain of the j^{th} good he buys is given by:

$$x_j^i = \frac{d_j^i}{\sum_{s=1}^{n} d_j^s} \sum_{t=1}^{n} f_j^t(q_j^t) \tag{6}$$

The noncooperative game is described in detail in 4.1.

This model is not conservative in goods in the sense that the wastage, breakage etc. disappears from the system. Thus implicitly free disposal has been assumed.

3.2 Some Problems in Modeling

In a modern monetary economy many of the costs incurred in trade are paid for in money. These money payments frequently represent rewards for trading, retailing, distribution or information services. Real estate brokers, stockbrokers auctioneers and others are paid commissions that are usually a percentage of the monetary take. Economic advisors, advertising agencies, racing sheet tipsters, consultants, lawyers, accountants and other experts are paid fees, honoraria and wages or sell their "informational product" in the market.

Transportation firms, truckers, movers, packagers, distributers and retailers are paid in money for their services. The distinction between their contribution and the contribution of the brokers and advisers is that their product is easier to measure in physical terms than that of the others. The major inputs provided in the carrying out of trade are:

(1) Well defined and easy to measure commodities
(2) labor
(3) large fixed plant and organizational structures
(4) hard to measure professional skills and information patterns.

Examples of these inputs help to provide insights into the modeling problems. Paper products, bottles, crates, gasoline, trucks and display cases are consumed in large quantities and their consumption can be described relatively easily in a formal model.

The time of clerks, dispatchers, truck-drivers, warehousemen, stevedore crews, packers, postmen and many others is consumed in trade and exchange. It is probably not too bad an approximation to consider these inputs as measured in man-hours of a standard quality.

The marketing, transporting, retailing and information systems in most modern economies involve a large overhead of fixed plant, institutions, bodies of law and custom. The railroad tracks, roads and telephone cables and switchboards provide examples of fixed plant. The stock exchange, the financial community, retailer associations and the informal but vital network of communication that exists among professionals such as lawyers, accountants and other consultants are examples of a delicate linkage of knowledge for searching and evaluating that are critical to the functioning of markets, and which take many years to build up. The laws and customs of trade provide another example of overheads. These include special languages, codes and symbols such as the hand signals at auctions, and customs such as closing a contract by a handshake which may not carry legal force, but which if violated places a trader beyond the pale.

Information is a good for which the attempts of economists and other social scientists to produce an adequate measure has as yet failed. Two promising starts have been made by Marshak and Radner [4], and in the treatment of information sets in the description of the extensive form of a game [5]. However even with this work no adequate model of the role of the specialist, expert or insider has yet been incorporated into general equilibrium analysis. In a less formal context than general equilibrium Hirschleifer has considered the role of information [6].

3.3 Transaction Expense Payments in Money

In general transactions expense payments are made in money as joint payments to agents when buying houses stock or many other items; or as transportation, handling and installation charges when buying items such as consumer durables; or as hidden factor payments for marketing, grading, inventorying and transportation reflected in the difference between factory price and final retail price.

In a modern economy the individual consumer does have a certain amount of leeway in paying his transactions costs in money or kind. Bargain hunting enables an individual to pay partially in terms of his own time* and expertise. In some instances among the poor, in areas where well developed markets do not exist, in times of hyperinflation or to avoid taxation even barter may be resorted to. Barter in general is far less impersonal than money-for-goods exchange in a market. Furthermore it is probably reasonable to assume that individuals can more easily make the evaluation of worth of a good for a money than one good in terms of another without having money as an intermediary.

For our second model we will make the simple assumption that transactions expenses are paid for in money. As we consider a commodity money we might assume it to be gold or silver. With some extra work and generalization we could consider a paper fiat money [7].

Among the properties usually associated with a commodity money are that it should be durable, easy to transport, inventory and identify. Furthermore divisibility of the monetary unit should be sufficient to make small purchases [8]. The best money should be completely durable. As a good first approximation

---

*Although for some individuals the <u>process</u> of bargain hunting is a consumer good in and of itself and much pleasure may be achieved from search, haggling and bargaining.

we may assume that money is conserved in the system.

If we assume that transactions expenses are paid for in money several definitional problems are encountered before we can usefully define the strategies for the traders. First, by a mere redefinition of the concept of a consumer good we may wipe out transactions costs as a separate problem replacing it by a trade and production problem that differs from the basic model of Debreu [9] only in the possible presence of nonconvex production conditions. The redefinition of consumer good involves defining the good <u>in situ</u> and operational at the place of use, such as flowers-in-the-vase at home, or a dishwasher installed for operation where you want it. These are extremely different goods from flowers in the field or flowers at the market or from a dishwasher at the factory, the distributor or the retailer.

By redefining consumer goods in the manner noted above all aspects of transactions are subsumed in the productive process (the nonconvexity may be brought about by increasing returns in transactions caused by better utilization of large fixed plant and information processing).

The redefinition of a consumer good to distinguish between "a good in the market" and "a good in the home" while certainly logically sound tends to obscure the study of transactions costs and their special properties. Hence this is not the only approach we adopt. However the point has been made that <u>transactions activities</u> are part of a <u>production process</u>.

In order to do full justice to treating transactions as a production process we really need a dynamic model of the economy. The general equilibrium model as usually presented is only pseudo-dynamic, time is distinguished by adding a date subscript to commodities but all conditions are solved for simultaneously. Furthermore as the general equilibrium model is nonstrategic the knowledge that the inventories of producers are high or low is of no interest to anyone*.

A way of capturing at least a crude first approximation of the dynamics of transactions activities as production in a strategic game is as follows.

Let there be $m + s + 1$ commodities all of which appear in the utility functions of the traders. The first m commodities are final consumer goods. The next s commodities have intrinsic utility but also are used in those production processes usually described as transactions activities. Items such as bags, bottles, wrapping paper, cookbooks, connoisseur magazines etc. are among those in the second category. The last item is a commodity money, gold or silver coin for instance.

We assume that the transactions technology is given and fixed. Furthermore(and this is an implicit way to handle dynamics) the traders have previously selected and hence have produced enough transaction activity input goods that trade can take place. Part of the strategy by a trader i is to offer an amount of money d_j^i to purchase a commodity j (where $j = 1, 2, \ldots, m + s$). The condition on expenditures is that:

---

*From a game theoretic viewpoint the general equilibrium analysis presents the economy modeled as a "one-shot" game in normal form. Furthermore as all individuals are constrained to maximize as though prices are given the solution concept removes any strategic freedom from the individuals.

$$(7) \qquad \sum_{j=1}^{m+s} d_j^i \leq A_{m+s+1}^i \quad .$$

The initial endowment of each trader i is $A^i = (A_1^i, A_2^i, \ldots, A_{m+s+1}^i)$.

The total strategy of a trader i consists of offering to sell quantities of some of the first $m+s$ commodities as well as monetary offers to buy quantities of other of the first $m+s$ commodities.

If a trader i offers to sell q_j^i units of commodity j he must also supply from his stock a supply of commodities z_k^i where $k = m+1, \ldots, s$ sufficient to run the transactions activities so that the delivery of q_j^i is feasible.

This model provides essentially the same construct as Model 1. The basic difference between the two is that here the production technology of carrying out trade must be specified and an offer to sell involves providing as a joint product for a single price the services needed to get the prime item for sale into the hands of the purchaser.

Model 3

We could adopt the attitude that the buyer pays directly for the transactions expenses. This can be modeled by assuming that when he wishes to spend d_j^i on an item j he must offer $f_j^i(d_j^i) \geq d_j^i$ where only d_j^i is applied to its purchase. The rest is "taxed away" in order to cover transactions expenses. In this case the computation of price is still as in (3). The difference between this model and Model 2 is reflected in the budget constraints

$$(8) \qquad \sum_{s=1}^{m} f^i(d^i_m) \leq A^i_{m+1}$$

The constraint in (8) must be contrasted with (1). In the first model the goods were "taxed" or diminished on the supply side of the market. Here money is taxed on the demand side.

In Model 1 goods "evaporate". The system is not conservative in goods. We can justify this in terms of dynamics and production. The real resources have been used in carrying out trade. This justification was made more explicit in Model 2. In Model 3 the system is not conservative in money. This appears to be less reasonable than Model 1. In order to account for this gap we must appeal to dynamics or have the extra money paid to a distinguished group of traders or brokers.

3.4 On Different Economic Actors in Trade

The traditional microeconomic approach to general equilibrium is not only static it makes use of only one, two or three actors. They are the consumer, the producer and occasionally "Nature" or "the Market". An example which uses all three is given by Arrow and Debreu [10]. The Edgeworth analysis of bilateral monopoly in its original form [11] had employers on one side of the market and laborers on the other. This model can be formulated completely symmetrically with only one type of actor termed a "trader".

All models are abstractions with varying degrees of aggregation. The aggregation selected will depend upon the questions to be answered and the technical problems of analysis. It is easy to confuse the specification of too simple a model with having avoided institutional detail and having modeled at a higher level of generality. For example a model of trade which specifies neither the nature of markets nor distinguishes brokers, retailers or other traders from producers or consumers is not more general than one which does. It has merely aggregated differently.

The general equilibrium model is virtually useless for the study of the structure of markets and the role of money and financial institutions without adding considerable relevant detail. The sort of detail required is that information is not free, trust is not universal, consumers tend to sell very few commodities and buy many; all individuals have the same amount of time each day; bankers, brokers, retailers and wholesalers are different from producers. Their roles in evaluating credit, in evaluation of and search for new opportunities, in setting up information services, in providing focal points and markets cannot be studied in models which make no assumptions which distinguish them from consumers or producers. The market simulation of Balderston and Hoggatt [12] serves as an interesting example of a study of search procedures.

It is my contention that in the same way that consideration of production made it useful to distinguish producers from traders so will consideration of the technology of exchange, information processing and credit call for the distinction of other actors.

4. TRADE WITH TRANSACTIONS COSTS

In this section we return to the model presented in 3.1. It is suggested that it is useful to consider two major types of models involving transactions costs. The first cover the relatively well defined aspects of technology of trade. These include the inputs of time, storage, transportation and communication. The second cover the less well defined features of trade such as evaluation, the granting of credit, insurance and the construction of financial instruments and institutions.

4.1 The Existence of Equilibrium with Transactions Costs

Model 1 is offered as a model adequate to portray the well defined production features of the technology of trade. Because it fails to deal with the cost of information processing and the problems of search evaluation and trust in more than a superficial manner, there is no gain in distinguishing more than one actor. He is a trader. The model is presented in somewhat more detail than in 3.1.

There are n traders $i = 1, 2, \ldots, n$.
There are $m + 1$ goods $j = 1, \ldots, m + 1$.
The initial endowment of trader i is $(A_1^i, \ldots, A_{m+1}^i)$

where $A_j^i \geq 0$ for $j = 1, \ldots, m+1$.

Each trader i has a utility function $\varphi_i(x_1^i, x_2^i, \ldots, x_{m+1}^i)$.

The $m+1^{st}$ commodity serves as a commodity money. A strategy by a trader i is a 2m dimensional vector consisting of offers to buy or sell in the m markets. A trader is constrained so that he cannot both buy and sell in the same market simultaneously.

When a trader i offers an amount q_j^i of the j^{th} good only $f_j^i(q_j^i)$ reaches the market. The remainder is lost thus the price p_j is shown in (5) and the amount x_j^i obtained by a trader is given by (6).

The noncooperative game can be described as:

$$\underset{d_j^i,\ q_j^i}{\text{Maximize}}\ \varphi_i(\{\delta_1^i(A_1^i - q_1^i) + (1 - \delta_1^i)(A_1^i + x_1^i)\},\ \{\delta_2^i(A_2^i - q_2^i)(A_2^i - x_2^i)\},$$

$$\ldots,\ \{A_{m+1}^i + \sum_{k=1}^{m} p_k\ f_k^i(q_k^i) - \sum_{k=1}^{m} d_k^i\})\ .$$

for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$

$$\text{where}\quad \delta_j^i = \begin{cases} 0 & \text{if } x_j^i \geq 0 \text{ and } q_j^i = 0 \\ 1 & \text{"}\quad q_j^i > 0\ , \end{cases}$$

$$\text{and}\quad \sum_{j=1}^{m} d_j^i \leq A_{m+1}^i$$

If the f_k^i are concave functions the existence and convergence conditions for the noncooperative equilibrium are the same as those for the noncooperative game without transactions costs. Here however one must take care in defining Pareto optimality as there in some sense exists two prices for each commodity. There is the price p_j paid by a consumer and the lower price $\frac{p_j}{q_j^i}\ f_j^i(q_j^i)$ received by the seller. This is similar to the results of Foley [13] and Hahn [14].

4.2 Nonconvexity of Transactions Costs

In the model above we note that $f_j^i(q_j^i) \leq q_j^i$ (otherwise we would create output with no input). Thus even if the f_j^i are nonconcave they are bounded as is shown in Figure 1. When we consider communication and transportation

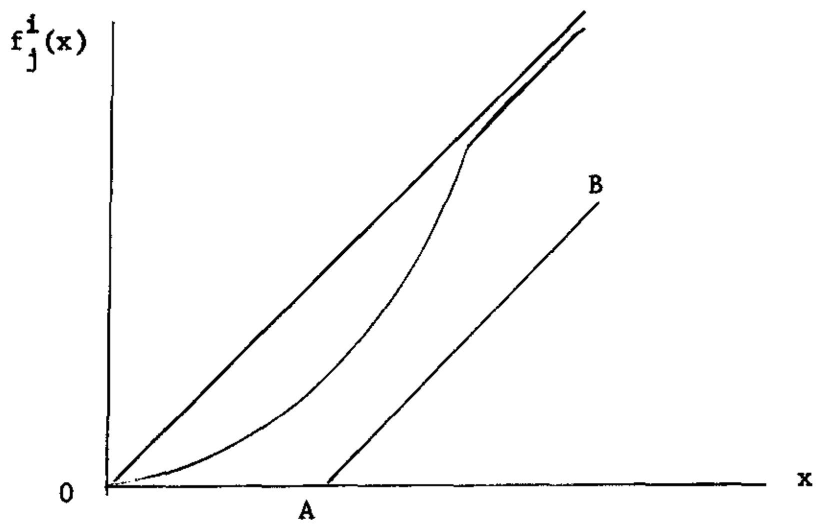

Figure 1

systems high setup costs such as shown by OAB might be expected. In a mass economy this type of transaction cost will cause increasingly little nonconvexity as numbers increase.

4.3 Pareto Optimality, Feasibility and Strategy Spaces

Pareto optimality must be defined so that the Pareto optimal set is feasible. When there are no transactions costs the Pareto optimal set is independent of the initial distribution of goods. When there are costs this is no longer the case. If we view the delivery of goods to the individual as a part of the production process then statements that a certain method of trade is not Pareto optimal can only have meaning in reference to another trading technology. Comparing an economy with trading institutions and costs to trade with the "ideal economy" with no costs to trade is about as meaningful as comparing an economy with production and production costs with an ideal economy with costless production.

5. ON ASKING THE APPROPRIATE QUESTIONS

In the last few years there has been a considerable upswing of interest in the theory of money. In particular it appears that money as an intermediary in trade is intimately related to the technology of transactions and hence should be considered in terms of its relationship to transactions costs.

On the surface the connection between money and transactions technology seems to be direct and clear. We should take some blue chips called "money" have everyone trade in these blue chips and compare this to barter [15]. Exercises in this type of combinatorics are extremely difficult and are certainly useful in increasing our knowledge of the fine structure of certain limited forms of trade. However there are two major considerations which may take precedence over approaches of this variety as a means for explaining the existence of monetary economies. The first calls for a careful exploration of what is meant by "money"; the second calls for care to be taken in distinguishing between short and long term costs and opportunities in trading technologies.

For example, individuals in the short run in a modern economy rarely have a meaningful choice between using money or barter for more than a few of their trades.

5.1 Money and Financial Institutions

It is extremely difficult and of dubious value to define money in vacuo without specifying all the rules of the monetary system and the market and financial institutions. This is not merely an appeal for more economic history and institutionalism it is at the very basis of the mathematical modeling of economies with a financial structure. In a previous paper [16] it was shown that an economy with a money market cannot be well-defined without specifying the bankruptcy laws.

The invention of new financial instruments and the creation of new market structures is to the technology of exchange what the invention of new machines and the development of scientific institutions is to production technology.

Some of the appropriate questions to ask of an economy with markets and financial institutions are:

(1) How many individuals and what conditions are needed for casual barter to be replaced by barter concentrated at fixed market places?

(2) What are the conditions for a commodity money to appear? What is the smallest group of individuals who have used a commodity money?

(3) When do specialized traders and brokers appear?

(4) When will trading in warehouse receipts start to take place? When will banks evolve?

(5) What are the necessary conditions for the appearance of a fiat money? What has been the minimum size of a society with the functioning of a successful independent fiat money?

(6) What are the minimal conditions for the evolution of a credit system?

(7) When will formal insurance contracts first appear?

(8) When will bankruptcy laws be formulated?

(9) What are the conditions for the emergence of a money market and rate of interest?

(10) When will forward contracts for trading in commodities come into being?

(11) What are the conditions under which stocks, bonds and like forms of ownership and debt will appear?

(12) What is the minimum size of a society that will support special information services such as credit agencies and expert evaluators.

These are only a few of the more obvious questions when we wish to consider the structure of markets and financial institutions relevant to transactions.

At the most abstract and simultaneously most institutional level "money" is defined by all of the rules of the game pertaining to operations of the physical entity or bookkeeping cyphers called money in various societies.

Perhaps it would lessen confusion and improve research and communication if we used a more neutral set of terms such as M1, M2, M3, ... to stand for "monies" manifesting only a few of the many properties it has in a modern economy.

5.2 Institutions, Trade and the Long and Short Run

The general equilibrium system presents a stable view of the world. Even if we were able to recast it in terms of dynamic programs much more economic modeling would be called for before we could portray the long run features of financial and market institutions. Even if we are unable to do so we can nevertheless recognize that the de facto presence of large scale facilities, organizations, institutions, networks of communication, and customs makes certain questions which might have a great appeal a priori in a noninstitutional world, hardly important in a world with institutions. Thus we might experiment in a large groups laboratory with message changes under a host of conditions to find out if barter or the use of coins or the use of a double entry bookkeeping credit system is most efficient. This however tells us little about the stability or efficiency of actual systems which may have come into being through the compounding of thousands of special circumstances and for thousands of special reasons. Once they are in being however they take on lives of their own.

Given that an individual enters an economy with functioning markets and financial institutions does he have major economic choices concerning his means of trade? This is an empirical question to which the answer appears to be no in general. The markets and financial infrastructure are a vast public good or externality which except in periods of social or political breakdown (such as revolution or defeat in war) impose a pattern of doing business on the individual that he has little choice. Although during a hyperinflation we may consider an individual bartering rather than buying a package of cigarettes, it is difficult to believe that in normal times he will be able to buy cigarettes or pay for a subway ride by barter. Once the system of trade is in existence it takes on the aspects of a multiperson noncooperative equilibrium for all concerned. How it came into being may be irrelevant to determining its stability.

5.3 Concluding Remarks

Transactions costs as a manifestation of expenses incurred in the productive acitivity of getting goods to their ultimate consumer can be handled in a relatively straightforward manner in a formal model of a closed economy, as was suggested in 4.1 and elsewhere.

Those aspects of transactions and trade which depend upon lack of knowledge, search and the evaluation of information have scarcely been dealt with.

Financial institutions and various forms of money and near monies play an extremely important role in the technology of trading. There is not a single problem involving some simple easy-to-define substance called "money", but a class of related problems calling for the study of the costs and productivity of different financial instruments and institutions.

The study of financial instruments and institutions can take place at a highly abstract level. However this calls for an enlarging or enriching of the type of model that has been built to study general equilibrium in a closed nonmonetary noninstitutional economy.

It is my belief that an extremely fruitful approach is via the theory of games, specifically using the concept of a noncooperative equilibrium applied to a dynamic game. The study must make direct and vital use of the phenomenon of mass markets, i.e. the numbers of individuals in the markets should play an important role in determining the outcome.

By using this approach the discipline imposed by being required to completely specify the rules of the game, the strategy sets and payoffs for each player under all circumstances are sufficient to require the mathematical description of basic financial instruments agents and institutions such as different types of banks, insurance companies, bankruptcy laws, stocks, bonds, brokers, factors, etc. The description and explanation of the nature of the basic or elemental forms of these financial phenomena appears to be a necessary preliminary to the full understanding of the economics of transactions.

REFERENCES

[1] Shubik, M., "The Missing Degree of Freedom: Commodity Money and Oligopoly in a General Equilibrium Model," A Theory of Money and Financial Institutions, Part III, Cowles Foundation Discussion Paper No. 324, December 1971; "Fiat Money and Noncooperative Equilibrium in a Closed Economy," op. cit. Part IV, Cowles Foundation Discussion Paper No. 330, February 1972.

[2] Shubik, M., op.cit. Part III, Cowles Foundation Discussion Paper No. 324, December 1971.

[3] Shubik, M., "The Rate of Interest on Fiat Money in a Closed Economy," op.cit Part V, Cowles Fcundation Discussion Paper, March 1972.

[4] Marshak, S., and Radner R., Economic Theory of Teams, Yale 1972.

[5] Von Neuman, S., and Morgenstern, O., Theory of Games and Economic Behavior, Princeton, 1944, Ch. 10.

[6] Hirshleifer, J., "Foundations of the Theory of Speculation, Information, Risk and Markets," Western Management Science Institute, W.P. 182, September 1971.

[7] See Shubik, M., op.cit. Part IV.

[8] For a good discussion see: Einzig, P., Primitive Money, London: Eyre and Spotteswoode, 1948.

[9] Debreu, G., The Theory of Value, Wile, 1959.

[10] Arrow, K.S., and Debreu, G., "Existence of an Equilibrium for a Competitive Economy," Econometrica, July 1954, pp. 265-290.

[11] Edgeworth, F.Y., Mathematical Psychics, London: Kegan Paul, 1881.

[12] Balderston, F.E., and Hoggatt, A.C., Simulation of Market Processes, Institute of Business and Economic Research, Berkeley, 1962.

[13] Foley, D.K., "Economic Equilibrium with Costly Marketing," Journal of Economic Theory, 2, 1970, pp. 276-291.

[14] Hahn, F.H., "Equilibrium with Transactions Costs," Econometrica, 39, 1971, pp. 417-440.

[15] Ostroy, J.M., and Starr, R.M., "Money and Decentralized Exchange," forthcoming

[16] Shapley, L.S. and Shubik, M. "The Rate of Interest, Noncooperative Equilibrium and Bankruptcy," A Theory of Money and Financial Institutions, Part VI, Cowles Foundation Discussion Paper , April 1972.