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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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ALTERNATIVE MANAGEMENT OF COMMON PROPERTY RESOURCES UNDER FREE ACCESS AND PRIVATE OWNERSHIP

Martin L. Weitzman

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UNDER FREE ACCESS AND PRIVATE OWNERSHIP

by

Martin L. Weitzman*

1. Introduction

One case of an external economy is so often cited that it has become a classical example. This is the notorious situation where in a competitively organized industry no rent is imputed to a scarce fixed factor such as land, fishing grounds, or highways. As is well known, the resulting free access equilibrium is inefficient because what tends to get equated among alternative uses is the _average_ product of the variable factor instead of its _marginal_ product.

In this paper a formal model is developed which is used to characterize and compare the alternative allocations of resources which occur under conditions of free access and of private property ownership. It is shown that the better properties are overcrowded by communal ownership, so that this well known description is indeed an apt characterization of what is happening when there is free access.

It turns out there is a definite bound on the amount of inefficiency which can be introduced into a competitive situation when property is freely

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accessible. This bound has an interesting welfare interpretation. The variable factor will always be better off with (inefficient) free access rights than under (efficient) private ownership of property. Looked at another way, the social benefit of the efficiency which private ownership of common property induces is exceeded by the social cost of rental payments which must be made to ensure it.

2. The Economic Framework

Suppose there are n pieces of property. Each property is a fixed factor having the potential of producing income when the variable factor works with it.

The index i will stand for any integer from 1 to n. Let $x_i \geq 0$ be the amount of variable input (assumed uniform and quantified in equivalency or efficiency units) applied to the i^{th} property. This results in total product or output

$$y_i = f_i(x_i)$$

measured as a flow of income. The production functions are assumed to be non-negative with $f_i(0) = 0$.

For all $x_i > 0$ the average product of x_i is defined as

$$A_i(x_i) \equiv \frac{f_i(x_i)}{x_i} .$$

It is assumed that average products cannot increase with input,

$$0 < x \leq x' \implies A_i(x) \geq A_i(x') \ . \tag{1}$$

For completeness the average product of zero input is defined as the limit

$$A_i(0) \equiv \lim_{x \to 0} A_i(x) \ .$$

This limit must exist (although it might be infinite) because $A_i(x)$ is monotonic in x .

If $w \geq 0$ is the return to the variable factor, the latter is offered up in total amount $S(w) \geq 0$. The supply schedule $S(w)$ shows the willingness of variable input units to commit themselves to active work as a function of the return they receive. It reflects both alternative employment opportunities and the labor-leisure choice. We assume that the supply curve is upward sloping,

$$w \leq w' \implies S(w) \leq S(w') \ . \tag{2}$$

The above model will be used to approximate certain competitive economic activities which could be and often are operated on a common property basis. These might include fishing, hunting, grazing, wild crop gathering, raw materials extraction from common pools, mass transportation, queueing, etc.

For example, consider fishing. Property i might be the i^{th} lake. Variable input x_i could be the number of effective fishing units operating on lake i . Output y_i would be the total catch of fish on lake i , measured in dollars. The interpretation of (1) and (2) is obvious.

In the case of transportation, property i might be the i^{th} highway route connecting two given cities. Variable input x_i would be the number of effective units of trucking capacity hauling freight between one city and the other via route i. Output y_i is the daily income received from transporting freight out of one city and into the other along highway i. In this case A_i is the shipping revenue per truck day using highway i, which would presumably decline when the road starts to become congested and deliveries per truck day diminish. Here w is the competitive return to an effective unit of trucking capacity engaged in shipping freight between the one city and the other.

3. Free Access Equilibrium

One of the most ancient and basic forms of property management is communal ownership. The essence of this economic system is that the community denies to any group or individual the prerogative to block usage of communally owned property. There are no private or governmental property rights and therefore no institutional arrangement exists for collecting rents. As a result of free access, competitive variable input units can and will move freely to that property which offers them the highest product per unit. An external diseconomy is typically created because independent units of the variable factor ignore the effects of their actions on the average products of others in considering only the product *they* stand to gain or lose by a proposed change.

Free access competitive equilibrium thus allocates the variable factor so that its average product is equalized on all properties that are used.

This allocation system is denoted FA and entities in it are capped by a tilde.

$$\tilde{x}_i > 0 \implies \frac{f_i(\tilde{x}_i)}{\tilde{x}_i} = \tilde{w} \tag{3}$$

$$(\underline{FA}) \qquad \tilde{x}_i = 0 \implies A_i(0) \leq \tilde{w} \tag{4}$$

$$\sum_1^n \tilde{x}_i = S(\tilde{w}) \ .$$

We assume that there is an FA equilibrium. (The issue of existence is not of interest for its own sake in the present paper and anyway it is not difficult to give supplementary conditions which would ensure it.)

4. Private Ownership Equilibrium

The economic system of private ownership sanctions the property rights of a certain class (landlords or rentiers) who owns the property and determines its usage. Under perfect competition the variable factor can be hired at a common competitive price and self interested rentiers will hire that amount of variable input which maximizes their profits. Perfectly competitive private ownership equilibrium therefore equates the marginal product of the variable factor with its price on all properties in competitive use. Such an allocation system is denoted PO, and in it variables are capped by a circumflex.

$$f_i(\hat{x}_i) - \hat{w}\hat{x}_i = \max_{x \geq 0} f_i(x) - \hat{w}x \tag{5}$$

(PO)

$$\sum_1^n \hat{x}_i = S(\hat{w}) .$$

A solution of the above equations is assumed to exist with $\sum_1^n \hat{x}_i > 0$.

Note in PO that $\hat{w}$, the "marginal product" of x_i , is a tangent to $f_i(x_i)$ at $x_i = \hat{x}_i$. The competitive rental

$$R_i \equiv f_i(\hat{x}_i) - \hat{w}\hat{x} \geq 0$$

will be collected on property i .

As economists are fond of pointing out, PO is an efficient economic system. If it were not, for some set of $x_i' \geq 0$ with

$$\sum_1^n x_i' \leq \sum_1^n \hat{x}_i$$

we would have

$$\sum_1^n f_i(x_i') > \sum_1^n f_i(\hat{x}_i) .$$

This would mean that for some j

$$f_j(x_j') - \hat{w}x_j' > f_j(\hat{x}_j) - \hat{w}\hat{x}_j$$

which contradicts (5).

With PO it is conceptually equivalent to think of rentiers as hiring variable input to maximize profits or to envision them as charging efficiency tolls for the use of their property and then allowing the competitive variable factor to allocate itself with otherwise unimpaired access à la FA. In the latter arrangement variable inputs hire property at the going toll rate and then receive the net average product (after payment of tolls) which thus tends to get equalized on property in use. Equilibrium quantities and prices under both arrangements will be identical if the tolls are correctly reckoned. The proper efficiency toll for property i, denoted τ_i, would be defined as follows

$$\tau_i = \begin{cases} 0 & \text{if } \hat{x}_i = 0 \\ R_i/\hat{x}_i & \text{if } \hat{x}_i > 0 \,. \end{cases}$$

This is a competitive toll schedule because any other toll on property i would not yield greater revenue to that property.

There is even a way of envisioning PO in terms of producer cooperatives which take a lease on property at the competitive rental price and determine their membership size by maximizing the dividend of net income (after payment of rent) per variable factor member. The solution is the same as before.

It is also conceptually irrelevant to the determination of an optimal allocation whether PO is regarded as based on competitive private ownership of property or on efficiently organized government public ownership. The solutions in both cases ought to be the same, the only difference being in who gets the surplus--private rentiers or the government.

Thus, for the model building purposes of theoretically characterizing efficient allocation, who owns property and what factor is thought of as hiring the other in the economic system we are calling PO is somewhat arbitrary. Which arrangement is in fact to be employed would largely depend on institutional considerations and on tradition. For example we usually think of optimal highway management as a public ownership problem involving efficiency tolls.

5. Overcrowding under Free Access

A notion commonly held about FA is that in some sense property will be overcrowded by comparison with PO. In the present section this characterization of FA allocation is quantified and demonstrated to be true.

From Theorem 3 of the next section (which does not in any way depend on the results of this section for its proof)

$$\hat{w} \leq \tilde{w} . \tag{6}$$

Combining (6) with (2),

$$\sum_{1}^{n} \hat{x}_i \leq \sum_{1}^{n} \tilde{x}_i . \tag{7}$$

Inequalities (6) and (7) will be used in proving the two theorems of this section. Inequality (7) demonstrates by itself a form of overcrowding in FA since it shows that more of the variable factor is used in that system than in PO.

Property j is said to be competitive in PO if there is a strictly positive value of x_j' satisfying

$$f_j(x_j') - \hat{w}x_j' = \max_{x \geq 0} f_j(x) - \hat{w}x = f_j(\hat{x}_j) - \hat{w}\hat{x}_j .$$

Non-competitive properties will of course not be used in <u>PO</u> .

The following theorem shows that the collection of properties used in <u>FA</u> is a subset of the set of properties competitive in <u>PO</u> . If a piece of property cannot break even under <u>PO</u> it will definitely not be used under <u>FA</u> . The converse is not true--if there is any property j roughly on the margin with

$$\hat{w} < A_j(0) < \tilde{w}$$

it will be employed under <u>PO</u> but not under <u>FA</u> . Since more or the same variable factor is being spread out over less or the same property in <u>FA</u> as compared with <u>PO</u> , Theorem 1 (in the jargon of agricultural economics) implies that the aggregate land-labor ratio is lower or the same under <u>FA</u> .

<u>Theorem 1</u>: If $\tilde{x}_j > 0$, there is an $x_j' > 0$ such that

$$f_j(x_j') - \hat{w}x_j' = f_j(\hat{x}_j) - \hat{w}\hat{x}_j .$$

<u>Proof</u>: If $\hat{x}_j > 0$, merely set $x_j' = \hat{x}_j$. If $\hat{x}_j = 0$, we have from (3), (5) and (6) that

$$0 = f_j(\hat{x}_j) - \hat{w}\hat{x}_j = f_j(\tilde{x}_j) - \hat{w}\tilde{x}_j .$$

In this case set $x_j' = \tilde{x}_j$.

Theorem 1 demonstrates that <u>FA</u> equilibrium involves the overall crowding of competitive <u>PO</u> properties. But what about the allegation that _individual_ pieces of better quality property will be overused in _FA_ as compared with _PO_ ? The classical story is frequently told in terms of labor crowding and overworking the more fertile land when no rents or tolls are collected.

To investigate this proposition, we must start by defining what is to be meant by the quality of a given piece of property. A natural measure might be the efficiency toll which would be charged for user services to a unit of variable input in _PO_. The _average_ toll charged in _PO_ is

$$\bar{\tau} \equiv \frac{\sum_1^n \tau_i \hat{x}_i}{\sum_1^n \hat{x}_i} .$$

It would be natural to say that if $\tau_j > \bar{\tau}$ property j is in some sense of better than average quality. The following theorem shows that the notion of superior property being overcrowded in _FA_ can be given a rigorous justification.

Theorem 2: If $\tau_j > \bar{\tau}$, then $\tilde{x}_j > \hat{x}_j$.

Proof: Since $\tau_j > 0$, it follows that $\hat{x}_j > 0$. We then have

$$\frac{f_j(\hat{x}_j)}{\hat{x}_j} = \hat{w} + \tau_j > \hat{w} + \bar{\tau} = \frac{\hat{w} \sum_1^n \hat{x}_i + \sum_1^n R_i}{\sum_1^n \hat{x}_i} = \frac{\sum_1^n f_i(\hat{x}_i)}{\sum_1^n \hat{x}_i} .$$

Combining,

$$\frac{f_j(\hat{x}_j)}{\hat{x}_j} > \frac{\sum_1^n f_i(\hat{x}_i)}{\sum_1^n \hat{x}_i} . \tag{8}$$

Using (5) and summing over all properties,

$$\sum_1^n f_i(\hat{x}_i) - \hat{w} \sum_1^n \hat{x}_i \geq \sum_1^n f_i(\tilde{x}_i) - \hat{w} \sum_1^n \tilde{x}_i .$$

Employing (7), dividing the (non-negative) left hand side of the above inequality by (positive) $\sum_1^n \hat{x}_i$ and the right side by $\sum_1^n \tilde{x}_i$ yields

$$\frac{\sum_1^n f_i(\hat{x}_i)}{\sum_1^n \hat{x}_i} \geq \frac{\sum_1^n f_i(\tilde{x}_i)}{\sum_1^n \tilde{x}_i} .$$

Substituting from (3) into the right side of the above,

$$\frac{\sum_1^n f_i(\hat{x}_i)}{\sum_1^n \hat{x}_i} \geq \tilde{w} . \tag{9}$$

Combining (8) and (9),

$$\frac{f_j(\hat{x}_j)}{\hat{x}_j} > \tilde{w} .$$

It follows from (1) and (3) that

$$\tilde{x}_j > \hat{x}_j .$$

6. Efficiency and Distribution in the Two Systems

As was already noted, PO must be efficient whereas FA need not be. But this is only a partial consideration in deciding which system is "better." Suppose we take the side of the variable factor. Under which system is it better off? In theory, rental or toll income could be directly siphoned over to the variable factor by government policy, but in practice this is not often done. Putting aside for the moment such enlightened if unrealistic textbook transfer policies, it is not easy to see off hand under which allocation system the variable factor does better. In FA there is a smaller distribution pie than there otherwise might be, due to inefficiency. On the other hand the variable factor gets *all* of the pie instead of just a slice, as with PO . The following theorem shows that there is a definite limitation to the amount of inefficiency which can creep into a competitive common property activity under free access average product equalizing. The variable factor cannot be better off under PO than under FA .

Theorem 3: $\tilde{w} \geq \hat{w}$.

Proof: Suppose by contradiction that $\tilde{w} < \hat{w}$. From monotonicity of $S(w)$, $S(\tilde{w}) \leq S(\hat{w})$ or

$$\sum_1^n \tilde{x}_i \leq \sum_1^n \hat{x}_i .$$

Since $\sum_{1}^{n} \hat{x}_i > 0$, there must be at least one $\hat{x}_j > 0$ with $\tilde{x}_j \leq \hat{x}_j$. Because $R_j \geq 0$,

$$\hat{w} \leq A_j(\hat{x}_j) .$$

Thus,

$$\hat{w} \leq A_j(\hat{x}_j) \leq A_j(\tilde{x}_j) \leq \tilde{w} .$$

q.e.d.

By way of summarizing, there may be a good reason for property-less variable factor units to be against efficiency improving moves toward marginalism like the introduction of property rights or tolls unless they get a specific kickback in one form or another.

7. Cost Benefit Analysis of the Private Ownership System

There is a way of employing Theorem 3 to derive a statement about comparable social costs and benefits of the two systems of economic management treated in this paper. Since this exercise is undertaken for the unorthodox task of analyzing the worth of an entire system of resource management, special care is required to correctly specify social costs and benefits.

Suppose that private rentiers own all property in PO . The way the rest of society sees things, the rental income which a landlord receives must be regarded as a social cost of the PO system since less real goods and services in that amount are available to others. The benefit which a rentier provides is the rendering of efficient production, resulting in more goods and services than would otherwise be available. To non-owners of property, landlords are just intermediate producers of economic efficiency who cost their rental fees.

Taking as a point of departure FA equilibrium, the following question can be asked. Would it be in the present society's interest to change FA to PO by bringing in an outside ownership element in charge of managing the common property under consideration?

Implicit in this way of phrasing the question is a presumption that the return to ownership does not count on the benefit side *per se*. Such an assumption would *not* be true if the concept of a present sub-community of non-rentiers whose interests are at stake were enlarged to include potential property owners. It would also *not* be true if the government were to obtain and use the rental income in PO for projects benefiting the present sub-society.

The social *cost* of imposing a PO system with outside ownership, denoted C, would be

$$C = \sum_{1}^{n} R_i \ . \tag{10}$$

The social *benefit*, B, would be the increased output plus the value of variable input economized (variable input is evaluated at its worth in its best alternative employment as determined by its supply curve)

$$B = \sum_{1}^{n} f_i(\hat{x}_i) - \sum_{1}^{n} f_i(\tilde{x}_i) + \int_{\hat{w}}^{\tilde{w}} w \, dS(w) \ . \tag{11}$$

In the thought experiment of creating a PO system with outside ownership, C might be called the distribution effect and B the efficiency effect of private ownership. The following theorem shows that the distribution effect outweighs the efficiency effect, so that the present society enjoys higher national product under FA.

Theorem 4: $C \geq B$.

Proof: Integrating by parts,

$$\int_{\hat{w}}^{\tilde{w}} w\, dS(w) = \tilde{w} \sum_{1}^{n} \tilde{x}_i - \hat{w} \sum_{1}^{n} \hat{x}_i - \int_{\hat{w}}^{\tilde{w}} S(w)dw \ .$$

Substituting the above equation in (11) and simplifying,

$$C - B = \int_{\hat{w}}^{\tilde{w}} S(w)dw \ . \tag{12}$$

The right hand side of (12) is non-negative because $\tilde{w} \geq \hat{w}$.

q.e.d.

Thus, a transition from FA to PO with outside ownership would lower the contribution to national product of the present society by $C-B$. It would lower the present community's income by the loss of higher average product earnings

$$\int_{\hat{w}}^{\tilde{w}} S(w)dw \ .$$

From (12) losses on the product and income sides are identical, as they must be.