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ON DURBIN'S TEST FOR SERIAL CORRELATION IN DISTRIBUTED LAG MODELS

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by

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Introduction

In a recent paper Durbin [1] has proposed two tests for testing serial correlation in distributed lag models. The present paper compares the relative performance of these tests and the likelihood ratio (L.R.) test in samples of sizes 32 and 76. The main conclusions of the paper are that both the tests proposed by Durbin do equally well and that they compare favorably with the L.R. test except for some parameter values. The paper presents some clues as to when the Durbin tests are likely to lead to wrong conclusions. The plan of the paper is as follows: Section II describes the tests proposed by Durbin and the L.R. test. Section III describes the sampling experiments conducted and the results. Section IV presents the conclusions and the results for practical work.

II. The Tests Used

Consider the model

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$$y_t = \alpha y_{t-1} + \beta x_t + u_t \tag{1}$$

$$u_t = \rho u_{t-1} + e_t \tag{2}$$

where e_t are $IN(0, \sigma_e^2)$. $|\alpha| < 1$, $|\rho| < 1$.

The first test proposed by Durbin: Test I is as follows:

Estimate (1) by ordinary least squares (OLS). From the computed residuals, calculate the first order serial correlation $\hat{\rho}$. Let $\hat{v}(a)$ be the estimate of the variance of a the least squares estimate of α . Define

$$h = \hat{\rho}\sqrt{\frac{T}{1 - T\hat{v}(a)}}$$

where T is the sample size. Use h as a standard normal deviate to test the hypothesis $\rho = 0$. If $T\hat{v}(a) > 1$ then the test is not applicable. In this case Durbin suggests fitting (1) assuming $\rho \neq 0$ or to test the hypothesis $\rho = 0$ by Test II described below or the L.R. test.

The second test: Test II proposed by Durbin is as follows: From the least squares estimation of (1) compute the residuals $\hat{u}_t$. Then regress $\hat{u}_t$ on $\hat{u}_{t-1}$, y_{t-1} and x_t . The test for $\rho = 0$ is carried out by testing the significance of the coefficient of $\hat{u}_{t-1}$ by ordinary least squares procedures.

Finally, the L.R. test is carried out as follows: Let s_1^2 be the residual sum of squares from the least squares estimation of (1). Define

$$y_t(\rho) = y_t - \rho y_{t-1}$$

$$x_t(\rho) = x_t - \rho x_{t-1}$$

and let $s_2^2(\rho)$ be the residual sum of squares from a regression of $y_t(\rho)$ on $y_{t-1}(\rho)$ and $x_t(\rho)$. Let $s_2^2 = \underset{\rho}{\text{Min}}\ s_2^2(\rho)$.

Then $\lambda = -T \text{Log}_e(s_2^2/s_1^2)$ has a χ^2 distribution degrees of freedom 1 under the hypothesis $\rho = 0$.[1]

As Durbin shows, all the three tests are asymptotically equivalent.

III. Design of the Sampling Experiments

We generated samples of sizes 32 and 76 for the model given by equations (1) and (2) for the following parameter values:

$\beta = 1.0$

$\alpha = .2, .4, .6, .8$

$\rho = 0.0, .1, .2, .3, .4, .5, .6, .7, .8, .9$

e_t were $IN(0, \sigma_e^2)$, u_0 was set $= 0$, and y_0 was set $= 200.0$. The x_t series was postwar quarterly GNP data for the U.S. in constant dollars starting from 1947 I. For the sample size 32, we used GNP data from 1947 to 1954. For the sample size 76 we used the years 1947-1965. Let σ_x^2 be the variance of the x-series. For sample size 32, σ_x was 38.1. We considered two values for σ_e : 8.0 and 20.0. These were chosen to correspond to two signal-noise ratios. For sample size 76, σ_x was 83.7 we considered $\sigma_e = 40.0$.

In each case 100 samples were generated.

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[1]A multiplicative factor in the likelihood function of $(1 - \rho^2)^{2/T}$ has been omitted. This should increase the power of the L.R. test.

For Durbin's Test I we tabulate the number of cases where the test is inapplicable because $T\hat{v}(a) > 1$. We also tabulate, for the cases the test is applicable, the proportion of cases in which the hypothesis of zero serial correlation is rejected. We chose $h > 1.645$ as the region of rejection.

For Durbin's Test II, we define t = (Estimate of the coefficient of $\hat{u}_{t-1}$)/(S.E. of the coefficient of $\hat{u}_{t-1}$). We again chose $t > 1.645$ as the rejection region.

For the L.R. test, we computed the maximum of the likelihood in the case $\rho \neq 0$ by a search procedure varying ρ at intervals of 0.1. We chose $\lambda > 2.706$ as the region of rejection.

Tables I, II and III present the results of the sampling experiments. R denotes the proportion of cases in which Durbin's test is inapplicable because $T\hat{v}(a) > 1$. $\hat{\alpha}$ and $\hat{\beta}$ denote the means of the OLS estimates of α and β respectively. The other columns indicate the proportions of cases in which the null hypothesis $\rho = 0$ is rejected.

One of the puzzling points in the above Tables is that except for $\alpha = .2$, the L.R. test had less power than Durbin's tests. The omitted factor mentioned in Footnote 1 actually works in favor of the L.R. test and hence cannot explain this.

IV. Conclusions

The following are the conclusions we draw from our results:

(1) Both the Durbin tests perform equally well for high values of α and they compare favorably with the L.R. test. As for Durbin's Test I, the

TABLE I: 32 Observations, $\sigma_e = 20.0$

	ρ	R	$\hat{\alpha}$	$\hat{\beta}$	Durbin I	Durbin II	L.R. Test
α = .2	0.0	39	.136	1.078	.180	.050	.070
	0.1	28	.240	.949	.167	.090	.110
	0.2	22	.321	.851	.192	.090	.150
	0.3	15	.404	.749	.294	.180	.230
	0.4	12	.429	.720	.375	.280	.310
	0.5	6	.556	.560	.394	.300	.360
	0.6	3	.602	.504	.309	.280	.410
	0.7	1	.675	.413	.424	.340	.480
	0.8	1	.756	.316	.364	.310	.480
	0.9	0	.810	.249	.430	.380	.530
α = .4	0.0	6	.343	1.093	.074	.050	.090
	0.1	4	.399	1.002	.229	.170	.180
	0.2	3	.423	.962	.278	.200	.240
	0.3	2	.510	.821	.337	.260	.280
	0.4	2	.557	.745	.398	.340	.420
	0.5	1	.622	.637	.535	.510	.410
	0.6	1	.677	.547	.566	.540	.540
	0.7	0	.735	.453	.690	.620	.660
	0.8	0	.779	.381	.740	.710	.740
	0.9	0	.823	.309	.850	.830	.800
α = .6	0.0	0	.591	1.021	.010	.010	.110
	0.1	0	.587	1.033	.070	.050	.170
	0.2	0	.610	.978	.170	.170	.170
	0.3	0	.621	.951	.370	.340	.270
	0.4	0	.625	.941	.580	.560	.480
	0.5	0	.667	.839	.640	.670	.550
	0.6	0	.669	.833	.830	.830	.750
	0.7	0	.713	.731	.910	.900	.800
	0.8	0	.762	.618	.910	.910	.890
	0.9	0	.785	.556	.980	.970	.950
α = .8	0.0	0	.799	1.004	.030	.040	.030
	0.1	0	.798	1.009	.070	.060	.070
	0.2	0	.800	.999	.160	.160	.090
	0.3	0	.803	.987	.280	.280	.160
	0.4	0	.802	.991	.450	.470	.210
	0.5	0	.803	.990	.600	.640	.440
	0.6	0	.802	.987	.840	.840	.620
	0.7	0	.806	.975	.900	.900	.670
	0.8	0	.814	.945	.970	.980	.920
	0.9	0	.811	.941	1.000	1.000	.890

TABLE II: 32 Observations, $\sigma_e = 8.0$

	ρ	R	$\hat{\alpha}$	$\hat{\beta}$	Durbin I	Durbin II	L.R. Test
α = .2	0.0	1	.159	1.050	.101	.060	.090
	0.1	1	.232	.960	.172	.100	.180
	0.2	4	.274	.908	.177	.120	.200
	0.3	1	.332	.837	.313	.240	.350
	0.4	1	.333	.837	.525	.390	.500
	0.5	1	.443	.700	.525	.430	.480
	0.6	2	.482	.652	.531	.500	.620
	0.7	0	.549	.569	.670	.640	.650
	0.8	2	.641	.457	.622	.600	.720
	0.9	1	.701	.383	.707	.660	.840
α = .4	0.0	0	.377	1.038	.010	.020	.030
	0.1	0	.386	1.024	.013	.110	.130
	0.2	0	.392	1.013	.230	.200	.200
	0.3	0	.453	.913	.350	.300	.180
	0.4	0	.459	.904	.460	.430	.330
	0.5	0	.503	.831	.670	.640	.430
	0.6	0	.542	.768	.750	.730	.550
	0.7	0	.582	.703	.870	.840	.690
	0.8	0	.637	.613	.880	.860	.790
	0.9	0	.687	.530	.880	.890	.820
α = .6	0.0	0	.601	.997	.010	.000	.030
	0.1	0	.597	1.009	.050	.050	.060
	0.2	0	.604	.990	.130	.120	.050
	0.3	0	.604	.990	.340	.330	.110
	0.4	0	.600	1.000	.550	.530	.210
	0.5	0	.615	.963	.680	.680	.360
	0.6	0	.612	.970	.830	.840	.480
	0.7	0	.632	.925	.920	.910	.580
	0.8	0	.652	.879	.970	.970	.790
	0.9	0	.653	.872	.980	.980	.870
α = .8	0.0	0	.800	1.000	.040	.040	.010
	0.1	0	.800	1.000	.070	.060	.010
	0.2	0	.800	1.000	.180	.180	.040
	0.3	0	.801	.994	.290	.270	.060
	0.4	0	.801	.996	.440	.450	.100
	0.5	0	.801	.998	.620	.620	.140
	0.6	0	.800	1.000	.840	.830	.320
	0.7	0	.801	.996	.900	.900	.460
	0.8	0	.802	.994	.970	.980	.790
	0.9	0	.800	.998	1.000	1.000	.850

TABLE III: 76 Observations, $\sigma_e = 40.0$

	ρ	R	$\hat{\alpha}$	$\hat{\beta}$	Durbin I	Durbin II	L.R. Test
α = .2	0.0	27	.178	1.026	.137	.070	.040
	0.1	12	.279	.904	.204	.090	.030
	0.2	10	.363	.799	.189	.160	.080
	0.3	4	.426	.724	.229	.150	.110
	0.4	0	.509	.620	.210	.170	.200
	0.5	0	.593	.517	.370	.330	.350
	0.6	0	.675	.414	.360	.330	.500
	0.7	0	.757	.312	.460	.440	.460
	0.8	0	.820	.234	.560	.550	.660
	0.9	0	.894	.146	.630	.610	.700
α = .4	0.0	3	.376	1.038	.186	.130	.080
	0.1	1	.441	.936	.253	.190	.120
	0.2	1	.499	.839	.293	.240	.230
	0.3	0	.592	.684	.400	.390	.390
	0.4	0	.645	.599	.540	.520	.570
	0.5	0	.717	.480	.670	.660	.670
	0.6	0	.760	.410	.770	.770	.710
	0.7	0	.820	.311	.870	.860	.840
	0.8	0	.871	.230	.930	.930	.890
	0.9	0	.921	.144	.950	.950	.960
α = .6	0.0	0	.582	1.043	.060	.050	.130
	0.1	0	.618	.957	.200	.170	.260
	0.2	0	.645	.888	.320	.300	.370
	0.3	0	.704	.749	.610	.570	.640
	0.4	0	.752	.630	.840	.840	.800
	0.5	0	.786	.548	.910	.910	.830
	0.6	0	.823	.457	.950	.950	.890
	0.7	0	.863	.363	.980	.980	.970
	0.8	0	.903	.262	.980	.980	1.000
	0.9	0	.944	.166	1.000	1.000	1.000
α = .8	0.0	0	.793	1.032	.040	.040	.020
	0.1	0	.797	1.012	.220	.220	.200
	0.2	0	.806	.971	.350	.350	.230
	0.3	0	.807	.965	.750	.740	.500
	0.4	0	.825	.883	.940	.950	.850
	0.5	0	.836	.835	.990	.990	.930
	0.6	0	.843	.801	.990	.990	.990
	0.7	0	.868	.684	1.000	1.000	.980
	0.8	0	.893	.569	1.000	1.000	1.000
	0.9	0	.937	.359	1.000	1.000	1.000

number of cases it is inapplicable is high only for small values of α and high values of σ_e/σ_x .

(2) The power of Durbin's Test I as well as Test II is very low for low values of α and high values of σ_e/σ_x . But for low values of α, even the L.R. test does not seem to do much better. For instance, for $\sigma_e = 20.0$, $T = 32$, $\alpha = .2$, even when $\rho = .9$, the null hypothesis that $\rho = 0$ is rejected only in 53% of the cases when the significance level used is 5%.

(3) The cases where Durbin's tests performed badly were also the cases for which the average R^2's from the OLS estimation of equation (1) were all less than .90, whereas when the average R^2's were greater than .95 the test showed good power.

It is possible that these results are sensitive to changes in the structure of the x-series.[2] However, since the x-series chosen by us has the characteristics of many of the series commonly encountered in Econometric work, the results we have obtained are expected to be of general applicability. We suggest applying Durbin's test only in those cases where the R^2's in the OLS estimation of (1) are high (say $> .90$). As for the cases where the test fails to be applicable because $T\hat{v}(a) > 1$, our results shown that this is more likely to happen when $\rho = 0$ than when $\rho \neq 0$. It does not appear that one would get much additional mileage by using the L.R. test in these cases. Nor is Durbin's Test II of any help. Since the interest is not in tests for serial correlation <u>perse</u> but in the estimation of the

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[2]Note that the x-series we have taken has a trend.

parameters (α, β) , one might as well go ahead and obtain the ML estimates of α and β on the assumption that $\rho \neq 0$. As remarked by Durbin [1] the purpose of the simple tests is precisely to avoid the computational burden of calculating the ML estimates.

APPENDIX

The x-series chosen was the following:

t	x_t	t	x_t	t	x_t	t	x_t
1	306.4	20	388.7	39	444.5	58	492.8
2	309.0	21	391.4	40	450.3	59	501.5
3	309.6	22	389.6	41	453.4	60	511.7
4	314.5	23	393.9	42	453.2	61	519.5
5	317.1	24	405.3	43	455.2	62	527.7
6	322.9	25	412.1	44	448.2	63	533.4
7	325.8	26	416.4	45	437.5	64	538.3
8	328.7	27	413.7	46	439.5	65	541.2
9	324.5	28	408.8	47	450.7	66	546.0
10	322.5	29	402.9	48	461.6	67	554.7
11	326.1	30	402.1	49	468.6	68	562.1
12	322.3	31	407.2	50	479.9	69	569.7
13	339.6	32	415.7	51	475.0	70	578.1
14	348.5	33	428.0	52	480.4	71	585.0
15	362.8	34	435.4	53	490.2	72	587.2
16	370.1	35	442.1	54	489.7	73	600.3
17	374.8	36	446.4	55	487.3	74	607.8
18	381.5	37	443.6	56	483.7	75	618.2
19	388.7	38	445.6	57	482.6	76	631.2

REFERENCE

[1] Durbin, J. "Testing for Serial Correlation in Least-Squares Regression when Some of the Regressors are Lagged Dependent Variables," Econometrica (forthcoming).