COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 218

An Impossibility Result Concerning the Theory of Decision-Making

Gerald H. Kramer

February 18, 1967

An Impossibility Result Concerning the Theory of Decision-Making*

Gerald H. Kramer
University of Rochester

1. Introduction

1.1 The purpose of this study is to investigate abstractly the relationship between the theory of rational decision-making on the one hand, and the finite information-processing capacities possessed by real decision-makers on the other. In particular, we wish to establish formally that the behavioral implications of the former are incompatible with the limitations on behavior imposed by the latter.

By "rationality," we shall mean simply that the decision-making agent in question behaves as though he were capable of forming consistent preferences over the set of relevant possible states of affairs, and acts in accord with these preferences. This concept of rationality seems an indispensible component, at least at some level, of any comprehensive theory of political behavior. In formal attempts at a pure theory of politics, as exemplified by the works of Arrow (1963), Black (1958), Downs (1957), or Riker (1962), the rationality premise usually is made explicit and plays a fundamental role in the analysis. Even in more informal descriptive treatments of political processes, we can scarcely avoid speaking at times in terms of the interests, or aspirations, or deprivations of various agents in the

---

* The author is Associate Professor of Political Science at the University of Rochester. This research was originally undertaken at Massachusetts Institute of Technology under a grant from the Social Science Research Council, and subsequently revised during the author's stay at the Cowles Foundation for Research in Economics at Yale University.

system, and such terminology again suggests that we regard these agents as entities which possess definite interests or preferences, which they attempt to advance in their actions.

If the environment in which such a decision-maker must operate is sufficiently complex, his attempts to attain desired outcomes may be severely distorted, and indeed it may be necessary to extend or redefine our concept of rationality for some such situations. Such environmental complications can arise, for example, because of:

a) limitations on the power or resources at the disposal of the decision-maker; or

b) institutional or environmental factors which make uncertain the relations between means and ends; or

c) interactions with other agents, who are pursuing their own interests.

Constraints of the first type can in principle be incorporated into the analysis by suitable restrictions upon the decision-maker's choice set. In the second case, if probabilities can be associated with the various uncertainties, the rationality concept can be readily extended to cover the type of situation, of decision-making under risk. If no such probabilities can be assigned, or if the uncertainty arises from the third type of complication -- interaction with other agents -- then serious conceptual and theoretical problems arise. However, these problems, which form the subject matter of the theory of decision-making under certainty and the theory of games, will not be considered here. For our purposes, it will suffice to consider the case of a single decision-maker in the simplest of environments, in which there is no uncertainty and in which there is a one-to-one relationship between the desired outcomes and the alternative courses of action available to the decision-maker. Hence it will not be necessary to explicitly distinguish means from ends, and we can use the term "alternatives" to refer to either, without ambiguity.

1.2 There is another kind of constraint, however, arising from the limited information-processing capabilities of the decision-making agent, which poses difficulties of a more fundamental nature. Herbert Simon, especially, has argued that in many contexts the rationality premise is an unrealistic one on which to base a theory of decision-making, because it fails to take into account the limited computational capacities possessed by real decision-makers.[1] Clearly, if such limitations do exist, and if their existence affects the ability of the agent to pursue its goals, then the viability of the theory is indeed brought into serious question, since such internal constraints are not easily reconciled with a theory of overt behavior.

That such capacity limitations do exist, and are important, is generally taken to be self-evident, or in any event easily justified on the basis of casual observation of everyday experience. For example, to reduce a game of chess to normal form (that is, to enumerate the possible strategies), which is a necessary prerequisite for any game-theoretic type of analysis, would require some fantastic number of hours on a large, high-speed computer. Even at a more prosaic level, for a consumer to determine the "best" bundle of commodities compatible with a given budget constraint, as he is pictured as doing by the theory of consumer behavior, would require considerable time and expertise with a sliderule or desk calculator, given the packaging and pricing practices which typically prevail. Clearly the vast majority of chessplayers and consumers do not and indeed cannot perform such computational feats; hence, the argument goes, the rationality premise, which implicitly assumes that they can and do, is clearly unsatisfactory for a descriptive theory of decision-making.

Even if we grant the validity of these empirical observations, however, the argument itself is not necessarily conclusive. For the validity of the theory of decision-making does not directly depend on whether decision-makers really have consistent preference orderings, or actually perform complicated calculations; the theory asserts only that they behave as if they did. Even if real decision-makers in everyday environments cannot perform the calculations required for a comprehensive analysis of their problems, and instead rely on simple rules of thumb or other devices to guide their choices, it may nevertheless still be that these simpler rules lead them to behave as if they were acting rationally, at least to a reasonable approximation. The question of whether they are "really" rational or merely act as if they were, is surely unimportant practically and probably is meaningless epistemologically as well. So goes the counterargument.

1.3 In principle, the best way to resolve this question of the behavioral implications of information-processing limitations would be with a carefully designed series of experiments; in practice, it has proved very difficult to design really conclusive experiments, and the empirical evidence is ambiguous.

The purpose of the present study is to approach this question in a different, more formal fashion. Specifically, we shall consider a decision-maker as being some sort of finite information-processing device, or automaton. We shall then use a formal theory of such devices -- the theory of finite automata [2] -- to explore the question of whether such a device is capable of behaving rationally. Thus, we will not inquire into the specific decision rules, or program, used by any particular automaton, but rather, we shall ask whether there is any conceivable program for such a device which could lead to input-output behavior consistent with the theory of decision-making.

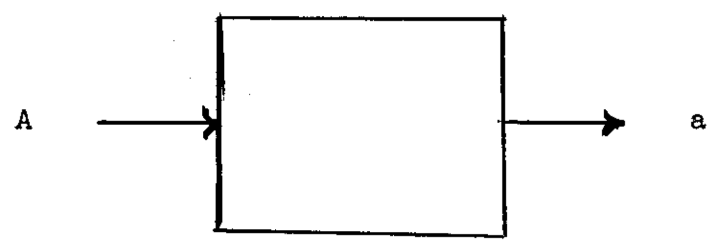
1.4 We can pose the question more formally in the following terms. We shall say that a decision-making entity behaves rationally just in case its behavior satisfies the following conditions: With respect to a given universal set $\mathcal{U}$ of alternatives $\{a, a', a'', \ldots\}$, there is a binary relation $\succsim$ (the preference-or-indifference relation) defined over $\mathcal{U}$, and a (partial) input-output function F (the decision function), where [3] $F : \mathcal{P}(\mathcal{U}) \to \mathcal{U}$, which satisfy:

i(a) for any $a, a' \in \mathcal{U}$, either $a \succsim a'$ or $a' \succsim a$ or both

(b) for any $a, a', a'' \in \mathcal{U}$, if $a \succsim a'$ and $a' \succsim a''$, then $a \succsim a''$

(c) for any $A \subset \mathcal{U}$, $F(A) \in \mathcal{A}$ where $a \in \mathcal{A}$ iff $a \in A$ and $a \succsim a'$ for all $a' \in A$.

Schematically, we consider the decision-maker as a kind of input-output device:



The input A to the device is a set of alternatives, and the output a is some member of A which is preferred or indifferent to all other available alternatives, according to some consistent preference ordering $\succsim$ over the set of all possible alternatives.

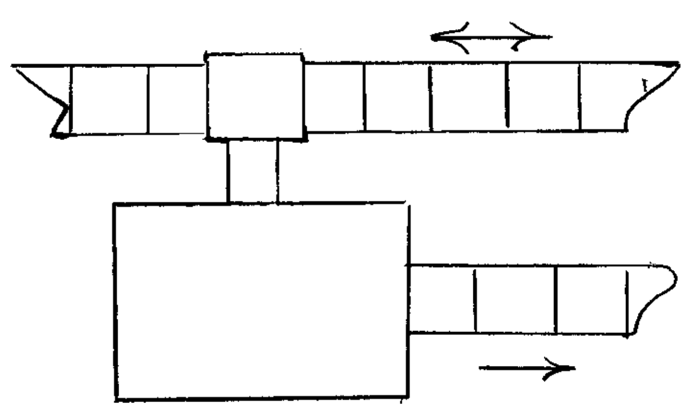
Suppose we now impose the additional condition, that the decision-making entity must be a finite device, which we will define more carefully below. The question which we wish to investigate is whether this additional condition leads to a contradiction; we shall show that, except for some degenerate special cases, such a contradiction does indeed arise.

2. Preliminaries

2.1 By a finite computing device, or finite automaton, [4/] we shall mean a finite apparatus which somehow can accept or receive a symbolic input (the problem) and can subsequently produce and communicate a symbolic output (the decision). The device may be mechanical, electrical, or may operate on some other principle; we shall require only that it be finite -- that is, of finite extension and composed of finitely many parts, each of which can take on only finitely many distinct configurations.

We can think of the symbolic input as being transcribed onto an arbitrarily long input tape. The tape is subdivided into spaces, and on each space an input symbol (a member of a fixed, finite input alphabet, Σ_I) is printed. The entire input is written on the tape in this fashion, by means of the <u>input language</u>, to be described below.

The device itself we can think of as being composed of three major components: A tape reader, to read the input tape; an output unit, for communicating the final output; and a central computing unit, which monitors and controls the tape reader, performs the appropriate computations, and eventually communicates its output by means of the output unit. We can think of the output as being printed, symbol by symbol, upon a one-way output tape, according to some fixed output language. Schematically, we have:



We assume time to be partitioned into discrete intervals, $t = 1, 2, \ldots$. The tape reader at any instant of time is positioned over some space on the input tape, sensing the symbol printed on that space; upon command from the computing unit it can move the input tape in either direction and stop it on some new space, whose contents it then senses and communicates to the computing unit. Thus the reader can "scan" the symbolic input in any required fashion. We wish to enable reader to anticipate the ends of the tape, to avoid having it run off the tape inadvertently while scanning the input. This we can do by requiring that every input tape begin with a special symbol "B" and end with another special symbol "E" , neither of these special symbols appearing elsewhere in the input. Thus an input string of k symbols will be transcribed onto an input tape $k + 2$ spaces long. The tape reader behaves as before, except that it can now avoid running off the tape, by always moving the tape to the left when it encounters the "B" , and to the right when it meets the "E" , on all except the terminal scan.

2.2 The central computing unit is composed of some finite number μ of distinct parts, each of which can take on, say, η_i distinct configurations or positions. When each part is in a specific position, the device as a whole is in an overall configuration, or <u>state</u> ; when some part changes its configuration, then the device as a whole is in a new state. Clearly the number of distinct states is bounded by $\prod_{i=1}^{\mu} \eta_i$, and is, therefore, finite.

At any time t , the device will be in some state s , with its reader sensing some particular input symbol σ . The device may then do any or all of the following: It may produce some output; it may change its internal configuration to some new state; or it may move the input tape a number of spaces in either direction. Hence, the behavior or the device can be completely described by a set of rules which specify the output, the new state, and the direction and distance that the input tape shall be moved, whenever the device is in a certain state reading a certain symbol. Without loss of generality we can restrict the outputs to single symbols, and we can require that the input tape be moved only one square at a time. Let S be the set of all states, and let Σ_I and Σ_O be the input and output alphabets, respectively. As noted above, Σ_I is augmented with the special symbols "B" and "E", to mark the beginnings and ends of tapes. We shall also want the device to be able to perform a part of its computation without giving any output, so we augment Σ_O with another special symbol "e", with the property that whenever the computing unit specifies adding an "e" to the output, this leaves the output unchanged -- i.e., amounts to giving no output. The rules which describe the operation of the device must specify, for each state s and input symbol σ for which the device does not "jam," a new state s', an output symbol σ', and an integer $m \in \{-1, 0, +1\}$ the latter meaning that the input tape shall be moved m spaces to the left (where $m = -1$ means 1 space to the right). Abstractly we can represent these rules by a finite set Q of <u>quintuples</u>, of the form $(s, \sigma, s', \sigma', m)$. To completely characterize the behavior of the device, the beginning and end of its computation must be described. We designate a certain member s_O of S as the <u>initial state</u>; the computation is begun with the device in s_O reading the initial "B" of the input tape. If there is a quintuple beginning with s_O, B then the device goes into the new state, gives the output, and moves the input tape as specified by that quintuple. If $m = 1$ on that quintuple, the device is now in a state s' reading the initial symbol of the input string, and it now invokes a new quintuple. The device continues to operate in this manner until one of four things happens: It might eventually run off the tape to the right; or it might run off to the left; or it might enter a state s reading a symbol σ such that there is no quintuple beginning with s, σ, in which case it simply halts or "jams"; or finally, it might go into an infinite "loop" and keep computing forever. For any string x of symbols in the input alphabet, we shall say that the device accepts the string x (or the input tape BxE) just in case, when the device is begun in s_o reading the "B" of the input tape BxE, it eventually does the first of these -- that is, runs off the right end of the tape. If the device accepts x and produces a string y of output symbols (possibly the null string, Λ) while doing so, then we shall say that y is an output, and that the device maps x into y. We can summarize the input-output behavior of the device with a function G, defined by

$$G(x) = y \text{ iff the device maps } x \text{ into } y,$$
$$\text{not defined otherwise.}$$

Hence, the domain of G is the set of all acceptable inputs, and its range is the set of possible outputs.

2.3 We must also characterize the input and output languages more precisely. By a language, generally, we mean a system (Σ, R_1, R_2), where Σ is a fixed, finite set of symbols (the alphabet); R_1 is a finite set of syntactic rules (the grammar), which govern the manner in which symbols can be combined into well-formed strings ("sentences"); and where R_2 is a set of semantic rules governing the use ("meaning") of various well-formed strings.[5/] In the case of a natural language, for example English, Σ might be the set of morphemes of English, R_1 a syntax or grammar of English, and R_2 would constitute a semantic theory of English. As an example of a formal language, take the usual notation of the theory of consumer behavior, where commodity bundles are represented by n-tuples of numbers $(x_1, x_2, \ldots, x_n)$. Let $\Sigma = \left\{ \underline{1}, \underline{2}, \ldots, \underline{9}, \underline{0}, \underline{,}\, , \underline{(}\, , \underline{)} \right\}$, and let R_1 be a set of simple recursive rules specifying how to combine digits to form numerals, and how to combine numerals, commas, and parentheses to represent sets of n-tuples. Finally R_2 would specify that the string representing an n-tuple $(x_1, x_2, \ldots, x_n)$ shall be interpreted as a commodity bundle of x_1 units of some designated first commodity, x_2 units of the second commodity, and so on.

2.4 We do not wish to inquire in detail into the structure of the particular input and output languages used by the device. However, it will be necessary to place certain restrictions on these languages, to ensure that the decision problem is not "solved" by simply transforming it into a linguistic problem. With respect to the input language L_I, for example, we clearly do not want the alternatives to be presented to the device in a way which depends upon their relative values; just as, in the case of experimentation with a decision-maker, we would not want the physical arrangement of the alternatives to reveal the solution. Hence, we shall require that the input language L_I satisfy the following condition, with respect to a given universal set $\mathcal{U}$ of alternatives:

ii(a) every finite set A of alternatives must be representable by a well-formed string.

(b) Every well-formed string must represent one and only one set.

(c) There exist substrings $w, y, z,$ and for every finite set A_i of alternatives there exists a substring x_i, such that the string $wx_{i_1} y x_{i_2} y x_{i_3} \ldots x_{i_k} z$ represents $A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_k}$, $k > 1$.

The first condition ensures that L_I will be rich enough to describe any set, while the second specifies that the descriptions shall be unambiguous. The third condition in effect requires that we be able to describe sets by lists, where the order of listing is inessential. Thus, suppose that A_1 is the set composed of objects a and b, while A_2 consists of object c; if we let w be the symbol {, y be , , z be }, and let x_1 and x_2 be a, b and c, then the usual notation of elementary set theory clearly satisfies ii(c), since the union of A_1 and A_2 is represented by {a, b, c} or by {c, a, b}. This listing condition is also satisfied by English, by means of the commutative connectors "and," "or," and it clearly is also satisfied by most other natural and formal languages of interest.

2.5 Trivial solutions can also arise from the choice of an output language. For example, the verbal responses "I choose alternative a_3," and "I choose that alternative which maximizes my utility," may in fact denote the same thing, but in the second case the computational burden of identifying the alternative in question is simply passed on to whoever must decode the device's output. We can circumvent this problem by requiring that the choices be indicated in a uniform manner; more precisely, we require that L_o satisfy:

iii (a) Every alternative shall be representable by some well-formed output string.

(b) Distinct well-formed output strings y, y' shall represent distinct alternatives a, a' .

Hence, we require the correspondences between output strings and alternatives be one-to-one.

2.6 A final degenerate type of solution we shall want to exclude is that arising from a trivial universal set of alternatives, or from a trivial preference structure. If the number of possible alternatives is limited, it would be possible in principle for a sufficiently large finite device (large relative to the number of alternatives) to simply memorize them, and behave consistently in this uninteresting way; hence we require that the number of alternatives be infinite. It will suffice for our purposes to consider a denumerably infinite universal set $\mathcal{U}$ of alternatives (that is, a set whose members can be put into a one-to-one correspondence with the positive integers). We also wish to preclude the uninteresting situation in which the decision-maker is indifferent to all the alternatives, or in which he has only a limited number of categories of preference; thus we shall also require that the number of preference categories be (denumerably) infinite. More formally, we require that the relation of indifference, $\sim$, which is an equivalence relation defined by $a \sim a'$

if and only if $a \succsim a'$ and also $a' \succsim a$, be of infinite index.

3. Proof of the Result

3.1 With respect to a given automaton, the set of all input tapes can be partitioned into finitely many classes $T_1, T_2, \dots T_m$, such that the members of each class are equivalent with respect to the transitions they produce in the device. More precisely, for any tape x, define the function [6/]

$$\tau_x \text{ (where } \tau_x : S \times \{-1, +1\} \longrightarrow [S \times \{-1, +1\}] \cup \{0\} \text{)}$$

as follows: To find $\tau_x(s, -1)$ put the device in state s reading the leftmost symbol of x, and let it begin computing. If it eventually runs off x to the right, in state s', then set $\tau_x(s, -1) = (s', +1)$; if it exits to the left in s'', set $\tau_x(s, -1) = (s'', -1)$; and if it does neither (that is, jams or goes into an infinite loop), then set $\tau_x(s, -1) = 0$. We obtain $\tau_x(s, +1)$ similarly, beginning the device on the rightmost symbol of x. Thus τ_x summarizes the transitional behavior of the device with respect to the tape x, and we can define such a function for each possible input tape. However, each such function has the same finite domain, $S \times \{-1, +1\}$, and its range must be a subset of the same finite set, $\times [S \ \{-1, +1\}] \cup \{0\}$. Hence, only finitely many distinct τ_x are possible (at most, $(2n + 1)^{2n}$, where n is the number of states), and the set of input tapes can therefore be partitioned into finitely many classes which are equivalent with respect to the transition function τ_x.

3.2 For any output string x, we define $\lambda(x)$ as the number of symbols in the string. We now wish to establish the following proposition, which relates an input tape $Bx_1x_2 \ldots x_kE$ to the length of the resulting output string $G(x_1x_2 \ldots x_k)$:

iv With respect to the automaton (S, s_o, Q) with associated equivalence classes $T_1, T_2, \ldots, T_m$, let $p_1, p_2, \ldots, p_k$ be a finite sequence of integers such that $1 \leq p_i \leq m$. Form an input tape $Bx_1x_2 \ldots x_kE$, such that the i^{th} segment x_i is a member of the p_i^{th} class, T_{p_i}. Then either:

(a) the device will fail to accept any such tape, or else

(b) the device will accept every such tape, and moreover there exists a sequence of functions $f_1, f_2, \ldots, f_k$ (where $f_i\colon\ T_{p_i} \longrightarrow \{0, 1, 2, \ldots\}$) such that

$$\lambda(G(x_1x_2 \ldots x_k)) = \sum_{i=1}^{k} f_i\,(x_i)$$ for each such tape.

More informally, from any tape accepted by the device we can form a new tape, by segmenting the original tape in any fashion and then changing some or all segments by substituting another member of the same equivalence class. Part (b) of the proposition assures us that this new tape will also be accepted, and that the effect (upon the length of the resulting output) of each substitution can be measured independently.

To establish the proposition, note that from the definition of τ, if $\tau_y = \tau_{y'}$ then $\tau_{wyz} = \tau_{wy'z}$. Thus, in particular, if we let

w be $Bx_1 x_2 \ldots x_{i-1}$ and z be $x_{i+1} \ldots x_k E$, and let x_i' be from the same class T_{p_i} as x_i, then $\tau_{wx_i z}(s_o, -1) = \tau_{wx_i' z}(s_o, -1)$, and either both tapes are rejected, or both are accepted. This establishes (a) and the initial part of (b). To establish the remainder of (b), consider the movement of the reading head across the boundary between wx_i and z: It will cross first to the right, then to the left, and so on until the computation terminates. Let s_{r_1}, s_{l_1}, s_{r_2}, s_{l_2}, ..., s_{r_k} be the sequence of states the device is in after each successive crossing. Clearly the output produced during the i^{th} passage through z is completely determined by the state s_{r_i} at the beginning of the passage and by the segment z itself, and does not depend on wx_i directly. The above sequence of states is defined by the recursive conditions:

$$1) \quad \tau_{wx_i}(s_o, -1) = (s_{r_1}, +1)$$

$$2) \quad \tau_z(s_{r_i}, -1) = (s_{l_i}, -1), \quad i < k$$

$$= (s', +1), \quad i = k$$

$$3) \quad \tau_{wx_i}(s_{l_i}, +1) = (s_{r_{i+1}}, +1), \quad i \leq k$$

If we now substitute another segment x_i' for x_i, such that $\tau_{x_i'} = \tau_{x_i}$, then (2) is obviously unaffected, and since $\tau_{wx_i} = \tau_{wx_i'}$, as noted above, conditions (1) and (3) are also unaffected. Hence, the sequence of states, and therefore the amount of output produced by the device while scanning the segment z, is unchanged by such a substitution. By an analogous argument, the amount of output produced while scanning w is also unchanged, and the only effect of the substitution is in the output produced while actually scanning the substituted segment, x_i'. Thus, if we define $f_i(x_i')$ as the amount of output produced while scanning x_i', for any $x_i' \in T_{p_i}$, $i = 1, \ldots, k$, then the remainder of (b) follows immediately.

3.3 We now return, finally, to the original question, of whether a finite device is capable of behaving rationally. An input-output device which behaves rationally must satisfy conditions (i), (ii), and (iii), while a finite device must satisfy (iv). We now show that these two sets of restrictions are inconsistent, so that a finite device satisfying proposition (iv) is indeed incapable of also satisfying the rationality conditions.

Let $A = \{a, a', a'', \ldots\}$ be a denumerably infinite set of alternatives such that no two of them are indifferent. Our assumptions concerning the universal set $\mathcal{U}$ and the preference ordering $\succsim$, in Section 2.6, guarantee the existence of such a set. From condition (ii c) on the input language for each member a of A there must exist a substring x which represents the singleton set $\{a\}$. Hence corresponding to A we have an infinite set $X = \{x, x', x'', \ldots\}$ of substrings.

If the decision-making device is finite, it has associated with it the equivalence classes $T_1, T_2, \ldots, T_m$. If we partition the set X of substrings into these classes, clearly at least one of the equivalence classes will contain infinitely many members of X. Let $X^* = \{x_1, x_2, \ldots, x_i, \ldots\}$ be this infinite subset of X, and let $A^* = \{a_1, a_2, \ldots, a_i, \ldots\}$ be the corresponding set of alternatives, where x_i corresponds to $\{a_i\}$ for all i.

Let a_i and a_j be any two distinct members of A^*. Since no two members of A (and therefore of A^*) are indifferent, one of these two must be strictly preferred to the other; let us suppose that a_j is the preferred one. From condition (ii c), there exist substrings w, y, z such that wx_iyx_jz and wx_jyx_iz both represent $\{a_i\} \cup \{a_j\} = \{a_i, a_j\}$, while wx_iyx_iz and wx_jyx_jz represent $\{a_i\} \cup \{a_i\} = \{a_i\}$ and $\{a_j\}$, respectively. If the device behaves rationally, in the sense of (i), then it must give an output corresponding to alternative a_j when given as input any description of the set $\{a_i, a_j\}$, or of the set $\{a_j\}$. In view of (iii) this alternative must be always represented by the same output string, so the device's input-output function must satisfy

$$G(wx_iyx_jz) = G(wx_jyx_iz) = G(wx_jyx_jz)$$

Let the length of this output be L; from proposition (iv), this length can be decomposed in three ways:

$$\begin{aligned} L = \lambda(G(wx_iyx_jz)) &= f_2(x_i) + f_4(x_j) + \beta \\ = \lambda(G(wx_jyx_iz)) &= f_2(x_j) + f_4(x_i) + \beta \\ = \lambda(G(wx_jyx_jz)) &= f_2(x_j) + f_4(x_j) + \beta \, , \end{aligned}$$

where $\beta = f_1(w) + f_3(y) + f_5(z)$. From the first and third lines, we have $f_2(x_i) = f_2(x_j)$, while from the second and third $f_4(x_i) = f_4(x_j)$. We obtain the same result, clearly, if we suppose that x_i is the preferred alternative, by interchanging x_i and x_j throughout. This implies that

$$\lambda(G(wx_i yx_j z) = f_2(x_i) + f_4(x_i) + \beta$$

$$= f_2(x_j) + f_4(x_j) + \beta$$

$$= \lambda(G(wx_j yx_j z)) = L .$$

Since this holds for all i and j, it follows that every input of the form $wx_i yx_i z$ causes the device to give an output of the same length, namely L.

However, the output alphabet is finite, say of size n, so there can be at most n^L distinct output strings of length L. Thus if we consider a sufficiently large number (larger than n^L) of different inputs of this form, there must be at least two such distinct inputs, say

$wx_{i^*} yx_{i^*} z$ and $wx_{j^*} yx_{j^*} z$, where $i^* \neq j^*$, for which the device gives precisely the same output. This is clearly a contradiction, however, since these two inputs represent disjoint sets of alternatives, namely $\{a_{i^*}\}$ and $\{a_{j^*}\}$, from which no common choice is possible.

REFERENCES

[1] Arrow, K. J., Social Choice and Individual Values, 2nd ed. New York: Wiley, 1963.

[2] Black, Duncan, The Theory of Committees and Elections, Cambridge: Cambridge University Press, 1958.

[3] Chomsky, Noam, Syntactic Structures. 's-Gravenhage: Mouton, 1957.

[4] Downs, Anthony, An Economic Theory of Democracy. New York: Harper, 1957.

[5] Krulee, G. K., D. J. Kruck, D. M. Landi and D. M. Manelski, "Natural language inputs for a problem-solving system." Behavioral Science, 1964, 9. Pp. 281-288.

[6] Rabin, M. O. and D. Scott, "Finite automata and their decision problems." IBM Journal of Research and Development, 1959, 3. Pp. 114-125.

[7] Riker, W. H., The Theory of Political Coalitions. New Haven: Yale University Press, 1962.

[8] Shepherdson, J. C., "The reduction of two-way automata to one-way automata." IBM Journal of Research and Development, 1959, 3. Pp. 198-200.

[9] Simon, H. A., Models of Man, New York: Wiley, 1957.

[10] ________, "Theories of decision-making in economics and behavioral science." The American Economic Review, 1959, 49. Pp. 253-283.

FOOTNOTES

1. See particularly Simon (1957), pp. 196ff., 241ff., and also Simon (1959).

2. See, for example, Rabin and Scott (1959).

3. Here $\mathcal{P}(\mathcal{U})$ is the power set, or set of all subsets, of $\mathcal{U}$. By the notation $F: A \rightarrow B$ we mean that F is a mapping from A into B, that is, a function with domain in A and range in B.

4. Cf. Rabin and Scott (1959), and Shepherdson (1959).

5. Cf. Chomsky (1957) and references cited there, and also Krulee _et.al._, (1964).

6. Cf. Shepherdson (1959). The cross-product $A \times B$ denotes the set of all ordered pairs $\langle a, b \rangle$, where $a \in A$ and $b \in B$.