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A TIME SERIES ANALYSIS OF INTEREST RATES

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A TIME SERIES ANALYSIS OF INTEREST RATES [1]

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In 1953 the Federal Open Market Committee concluded that it was desirable to confine its open market transactions to short term securities, preferably Treasury bills. This policy was announced to the public in a speech by Chairman Martin, "The Transition to Free Markets," and has subsequently been called the "bills-only" policy. The 1953 Annual Report revealed that the "bills-only" policy was adopted after a special study of the Government securities market by an Ad-Hoc Subcommittee of the Federal Open Market Committee and that there was considerable difference of opinion between the majority view and those (like Mr. Sproul) who questioned the wisdom of accepting the Ad-Hoc Subcommittee recommendations.[2]

This policy imposed several restrictions on open market operations. Federal Reserve operations were to be limited to those necessary for supplying or withdrawing reserve funds and the authorities would not ordinarily intervene to prevent fluctuations in prices or yields of Government securities. On the other hand, the policy did permit actions needed "to correct a disorderly situation in the Government securities

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1 I would like to acknowledge financial assistance from the National Science Foundation which facilitated the research for this paper.

2 The Federal Reserve System has objected to the designation "bills-only." See Young and Yager: "The Economics of Bills Preferably" Quarterly Journal of Economics, LXXIV, August 1960. In practice almost all open market operations from late 1952 until 1960 were in bills.

market." In changing the volume of reserves by means of open market purchases and sales, the authorities would deal in short term securities, and preferably Treasury bills. Finally Treasury refundings would not be supported by open market operations in "rights," when-issued securities, or securities comparable in term to the new issues offered by the Treasury.

With very few exceptions, the Federal Reserve followed these principles, and open market transactions in Treasury Certificates, notes and bonds were virtually eliminated.[3] Except for 1955, and in 1958 when the market approached disorderly conditions, the Federal Reserve held its operations in Certificates, notes and bonds to something like 1% of its total open market transactions.[4] The policy was discarded in early 1961 when we were confronted with a balance of payments deficit at a time when the economy was operating at close to recession levels.

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3 The exceptions are: a purchase of $167 M of Certificates in 1955 to facilitate a large scale Treasury refunding; a sale of Certificates and notes in 1957 because the Federal Reserve was running low on bills; and a $1,265 M purchase of Certificates, notes and bonds in 1958 to correct the first disorderly market situation that developed since the bills-only policy was adopted. See S. H. Axilrod and J. Krummack: "Federal Reserve Security Transactions 1954-1963," Federal Reserve Bulletin, July 1964.

4 The bills-only policy attracted considerable discussion. An interpretation of the policy is given in D. I. Fand and I. C. Scott's: "The Federal Reserve Systems' "Bills Only" Policy: A Suggested Interpretation," Journal of Business, XXXI, January 1958. D. S. Ahearn: Federal Reserve Policy Reappraised 1951-1959 (Columbia University Press, 1963), pp. 47-121, discusses the policy and provides a fairly complete bibliography. For the Federal's justification of "bills-only" see W. Riefler: "Open Market Operations in Long Term Securities," Federal Reserve Bulletin, November 1958, and Young and Yager, Op cit.

A. <u>The Bills-Only Policy and the Term Structure of Interest Rates</u>

Riefler in his article attempting to rationalize the bills-only policy pointed out that an open market action influences the demand-supply relationships in the market by changing the volume of reserves available to the commercial banks, and by changing the volume of securities on the market. He follows this up with the observation that "Enduring effects either on short-term or long-term interest rates differ very little as between operations in bills and operations in bonds, either of which changes the volume of reserves available to the banks." These two propositions taken together lead to the conclusion that "the major effect of direct operations in long-term securities on basic supply-demand relationships would come from the fact that reserves were supplied or withdrawn, not the fact that long-term securities were purchased or sold."[5]

The rationale for bills-only rests on two substantive propositions: It assumes (1) that interest rates generally move together so that factors which cause short term rates to move up or down will typically have the same effect on long rates; and (2) that the significance of an open market

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5 In addition Riefler also argues that Federal Reserve Operations, especially operations in long-term securities, affect market expectations. And the consequences of mistaken expectations are most serious when they concern the long-term markets. This problem is minimized when the system changes the reserve position of banks through operations that cause little change in expectations, particularly unjustified expectations with respect to long-term yields. Open market operations confined to short-term securities meet this criterion.

operation derives from its effect on bank reserves and not from any effect it may have on the relative supplies of market held debt. If these assumptions are valid they raise questions for the theory and practice of debt management in general, and more specifically for a policy such as "Operation Nudge" adopted in 1961 to cope with our balance of payments problem. If changes in the maturity distribution of the debt have very little effect on the structure of interest rates, it would rule out some approaches to debt management, and if interest rates typically do move together, it is difficult to see how one could justify a policy of "Operation Nudge" which is explicitly designed to affect the structure of interest rates.[6]

The question of whether bills-only is or is not an appropriate policy for the central bank requires for its answer a theory of the term structure of interest rates and, in particular, a quantitative analysis of the extent it may be affected by changes in the supplies of long and short debt. Perhaps because of this connection the bills-only controversy did seem to bring forth a number of studies of the expectations hypothesis and the effect of changes in relative supplies. At first there was a wide spectrum of views; some studies supporting

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6 Debt management may be defined as discretionary changes in the average maturity of the publicly held marketable debt. A number of different views may be found in the literature as to the purposes of debt lengthening and shortening. Some would use it primarily as a weapon for counter cyclical stabilization; others to minimize the interest cost of the debt, to help bring about a desirable debt structure, or to achieve a combination of these objectives. For a discussion of these views see W. L. Smith: Debt Management in the United States, (Washington, 1960) and D. I. Fand: "The Problem of Public Debt Management," Southwestern Social Science Quarterly, XLI, March 1961. For a thorough analysis of this problem see J. Tobin: "An Essay on the Principles of Debt Management" in Fiscal and Debt Management Policies (Prentice Hall, 1963).

the view that short and long debt are imperfect substitutes and that the markets are segmented; and other studies supporting the opposite view that short and long debt are perfect substitutes. In the more recent work the disagreements have narrowed somewhat and the bulk of these studies seem to support the view that short and long debt are close but imperfect substitutes and that relative supplies should affect relative yields.[7]

There is however still considerable uncertainty as to the quantitative effect of changes in relative supply. An appraisal prepared by Okun for the Commission on Money and Credit concludes that the effect is probably small.[8] On the other hand, Okun's study covered the period 1946-1959 and we shall suggest some reasons why his results may need modification when applied to a later period. And more recent studies suggest a somewhat greater role for changes in relative supplies.

Although this theoretical and empirical literature has greatly clarified the issues, the term structure ideas implicit in the bills-only view of central banking have not been refuted or validated. It would therefore seem desirable to analyze our experience when we

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7 For a brief summary of present views and references to the studies of Conard, Culbertson, Kessel, Luckett, Malkiel, Meiselman, and Wood, see Appendix I.

8 See A. M. Okun: "Monetary Policy, Debt Management and Interest Rates: A Quantitative Appraisal," in Stabilization Policies (Prentice Hall, 1963).

followed this approach to central banking. In this paper we shall study the behavior of interest rates in this period with particular emphasis on the extent to which long rates and short rates behaved as predicted by the bills-only theory.

B. The Behavior of Interest Rates

Before we present the results of our statistical analysis we shall briefly review the main movements of interest rates for the period we are investigating, April 1953 - December 1960. The short term series is obtained from data on the average price of weekly offerings of Treasury bills while the long term series is an average of Treasury bond yields. Prior to April 1953 Treasury bond yields were computed on the basis of an average of closing bid and asked quotations; since then Treasury bond yields are based on closing bid quotations. To achieve greater comparability in the data, we start with April 1953; and since the bills-only policy was changed in early 1961, we limit our study to the 93 months April 1953 - December 1960.[9]

There are several features in the movements of the two series that are worth noting. Short term rates drop substantially in recession periods (1953-1954, 1957-1958, 1960-1961) and rise again in the boom, and the turning points in the series conform fairly well to the fluctuations in aggregate output. In addition to the main cyclical movement, there appears to be a shorter (3-4 month) cycle in

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9 For details see Treasury Bulletin, January 1961, pp. 56-59.

the short term series. This may be observed in most months except when short term rates are rising or falling sharply. The long term series may incorporate a cyclical pattern but its conformity to the fluctuations in aggregate output is not so clear; long rates in the early months of the recovery in 1958 were above the previous peak rates of the 1957 boom; and the drop in long rates in the three recessions was substantially less than the drop in short rates and may have impeded the subsequent recovery. Finally, the long term series seems, at most turning points, to lead the short term series.

An examination of interest rate movements in this period raises a number of questions: Do the movements in the short term series reflect the main cyclical movements in the post-war period? Does the timing, amplitude and duration of the movements in short term rates correspond fairly closely to the pattern of the post-war cycles? Or are there seasonal, and perhaps even secular, components? Do the movements in long term rates correspond to the shorter term (3-4 years) cyclical movements in aggregate output? Or do they reflect primarily longer term secular forces? If there are such secular forces affecting long rates, to what extent are long rates relatively independent of short rates and lead a life of their own, and to what extent are they influenced by movements in short term rates? Perhaps the most important question and the one that motivates this paper, does the behavior of interest rates in these years support the bills-only view of central banking that operations in short term markets can achieve, within limits, the desired degree of change in long term rates?

1. The periodicities of short and long rates [10]

The hypothesis that short rates are influencing long rates suggests that we should find the same kinds of cycles, or periodicities, in the two series. Consequently, if we find several recognizable cycles in short rates and no such evidence in long term rates this would constitute some evidence against the hypothesis that short rates are influencing long rates. We shall therefore attempt to isolate the cyclical movements of the short term and long term interest rate series. To carry out this test we calculate the covariogram (or correlogram) and the empirical spectral density function (or smoothed periodogram) for both series. [11]

If we define C_S to be the autocovariance coefficient of lag S, and $r_S = C_S/C_O$ the autocorrelation coefficient of lag S, the covariogram(correlogram) associates the successive values C_S or (r_S) as a function of S.[12] The spectral density, $f(w)$, assigns, for each

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10 I would like to thank Prof. Peter Whittle for his helpful comments during the preparation of this section; Prof. Jon Cunnyngham was kind enough to run my data through a program using a somewhat different smoothing procedure which he has developed.

11 The term "cycle" as used here is not intended to mean a definite period which repeats itself regularly. What we have in mind can be more accurately described as a periodic tendency of the type first distinguished and explained by G. U. Yule in "On a Method Investigating Periodicities in Disturbed Series with Special Reference to Wolfer's Sunspot Numbers," Phil. Trans. A., 226, 1927.
This distinction can be made more precise by a consideration of the spectral density: a definite period would correspond to a concentration of energy at a particular frequency, while a periodic tendency corresponds to a broad maximum at that frequency. In the language of Burns and Mitchell, this would be a "recurrent but not periodic" movement.

12 The Correlogram is a covariogram normalized so that $r_O = 1$, and we therefore use the two terms interchangeably.

value of w , a weighted sum of C_s where the weights are a function of the angle w . The covariogram and the empirical spectral density function (periodogram) for both the short and long term interest rates are shown in Charts 1 - 3.[13]

A study of the correlograms provides some evidence of the underlying process that is generating the series. For example, the correlogram of a moving average process will terminate at a finite lag; the correlogram of a strictly periodic process will repeat itself indefinitely; the correlogram of a purely random series will be zero for all non-zero lags; while the correlogram of an autoregressive series will consist of damped harmonics.[14]

Although the correlogram and the periodogram are mathematically related, it may still be possible to learn more about the process by studying both. The correlogram is useful in suggesting the number of periods (harmonics) that may be incorporated in a body of data, but is not so reliable for dating any particular harmonic. This is

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[13] If we have N observations and M , the maximum lag, is equal to N , $f(w)$ is an unsmoothed periodogram. In our case $M = 60$ while $N = 93$; the graphs of $f(w)$ are therefore smoothed periodograms. The definitions of, and the methods used to calculate C_s , R_s , and $f(w)$, are given in Appendix II.

[14] See M. G. Kendall: Contributions to the Study of Oscillatory Time Series, (Cambridge, 1946) p. 6 and Chapter 1, and H. Wold: Demand Analysis, (New York, 1953), Chapter 10, pp. 159-166.
On the other hand, the observed correlogram derived from the estimated autocorrelation coefficients is subject to sampling fluctuation and may differ considerably from the true correlogram. See E. J. Hannan: Time Series Analysis, (London, 1960), Chapter 2, pp. 28-51.

much better accomplished by the spectral density function if the periods are sufficiently small relative to the length of the series.[15]

If there is more than one harmonic in a series they will be difficult to separate in correlograms which reflect their joint influence; they may be more readily separated in the periodogram where each harmonic will produce a local maximum at a particular frequency. Since our test is designed to determine whether the two series exhibit similar cyclical movements, we thought it desirable to calculate both the correlogram and the spectral density function.

Evidence from the correlogram:

The correlogram of short term interest rates $C_s(X)$ appears to exhibit several harmonics and there is also some evidence of dampening. This would suggest an autoregressive process.[16] As we study the correlogram we observe that there are peaks at $S = 28$-29, $S = 48$, troughs at $S = 19$ and $S = 60$. The trough at $S = 60$ is particularly steep and abrupt. We also observe a very slight change in curvature at $S = 4$. We note that a 40 month cycle will have its second trough at $S = 60$, while the harmonic of 24 months will have its third trough at $S = 60$. These troughs and peaks may be rationalized

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[15] A rule that is often given is that if there are N observations, and T is the maximum period, it is desirable to have a series long enough so that $N > 5T$.

[16] The autoregressive model covers a wide range of stationary processes. It seems to be an appropriate model for an economic time series especially when dynamic considerations and new information affect the movement in the series. Also, the estimation procedures are simple. See the discussion of the autoregressive process in R. J. Hannan: "Recent Advances in Statistics," Economic Record, 1957.

by assuming two harmonics: one with a period of 40 months; the second with a period of 24 months. The trough at $S = 19$ provides some further evidence for assuming a cycle of approximately 40 months. The 40 month cycle corresponds roughly to the general pattern of cyclical movements in this period.[17]

The correlogram of long rates $C_s(Y)$ is strongly concave to the origin. It is clearly not convex as would be the case if the Y series was generated by a first order autoregressive scheme and its correlogram followed a path of exponential decay. This is strong evidence against the hypothesis that the Y series contains no harmonics at all. We adopt the next simplest hypothesis that the data contain just one harmonic. The estimate of its period is much less certain since we can follow the course of $C_s(Y)$ for approximately only 1/4 of a full cycle. Since $C_s(Y)$ crosses the axis between $S = 50$ and $S = 60$ it would suggest a period somewhat between 200 and 240 months. This would imply a cycle of long rates of approximately 16-20 years.

The most striking fact about the correlogram of long rates is that the evidence of the X influence in the Y correlogram is so slight as to raise very serious doubts that short term rates affect long rates.

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17 On this view, we need to imagine a peak at $S = 40$ which is being obliterated by the downward phase of the 24 month cycle which reaches its trough at $S = 36$. The peak at $S = 48$ would correspond to a peak at $S = 40$ displaced by superposition of the two harmonics.

Evidence from Spectral Density Function

The periodogram of both the short term and long term interest rates are smoothed and this is especially marked at low frequencies. Period discrimination at low values of W is consequently poor.[18]

The graph of the spectral density function of short term rates $f_x(w)$ has peaks at $w = 8° - 9°$, $w = 24°-30°$ and at $w = 90°$. (There may be two peaks in the region $w = 24°-30°$). From the relation $WT = 2\pi$, we note that the peak at $9°$ corresponds to a 40 month cycle, that the peak between $24°-30°$ corresponds to a period between 15 months and 12 months, or possibly two periods, and the peak at $9°$ corresponds to a 4 month cycle.

The graph of the spectral density of the long rate, $f_y(w)$, contains some evidence of very slight peaks at $w = 25°$ and $w = 30°$. If these peaks are not artifacts they would tend to confirm a less definite separation in the local maxima of $f_x(w)$ near $w = 24°-25°$ and at $w = 28° - 29°$.

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18 The smoothing procedure we used is due to Bartlett. See M. S. Bartlett: "Smoothing Periodograms from Time Series with Continuous Spectra," Nature CLXI (1948) and "Periodogram Analysis and Continuous Spectra," Biometrika XXXVII (1950). More recently a number of alternative smoothing procedures have been suggested. See R. S. Blackman and J. W. Tukey: The Measurement of Power Spectra, New York 1958, J. Cunnyngham: The Spectral Analysis of Economic Time Series, Bureau of the Census, 1963, C. W. J. Granger: Spectral Analysis of Economic Time Series, Princeton, 1964, E. J. Hannan: Time Series Analysis, London, 1960, and M. Nerlove: "Spectral Analysis of Seasonal Adjustment Procedures," Econometrica, XXXII, July 1964.

It should be noted that virtually all the energy in the Y spectrum is in the lower frequencies which is consistent with the hypothesis of a fairly long cycle.

The evidence just presented seems to suggest that short rates incorporate several cycles.[19] The evidence for a 40 month cycle is quite good while the evidence for the other 3 cycles, and especially for the 24 month cycle, is much less firm. Nevertheless there is more evidence to support the hypothesis of 3-4 cycles than an alternative hypothesis that there is only one cycle.

The evidence for a long cycle in long term rates is strong and we estimate its period at between 16 and 20 years. This would presumably correspond to long term movements in population and housing. Evidence for the other cycles is quite weak.

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19 This evidence is subject to a number of qualifications. We cannot take account of the sampling fluctuations of the calculated correlogram or the periodogram. Also there is considerable uncertainty as to the appropriate smoothing procedures in calculating the periodogram. And finally, further modifications may be necessary if the process generating these data is not covariance stationary, if $C(X_t, X_{t+s})$ is a function of both the lag S and the starting point t.

T A B L E 1

SUMMARY OF EVIDENCE OF CYCLES IN THE SHORT AND LONG TERM INTEREST RATE SERIES

	$C_s(X)$	$f_x(w)$	$C_s(Y)$	$f_y(w)$
Short term Series				
40 months	good	strong	[1]	
24 months	good		[1]	
12-15 months		strong		[2]
4 months	slight	strong		
Long term Series				
200-240 months			good	strong[3]
40 months			very slight	minute
24 months			very slight	minute
12-15 months				
4 months				

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1 It is possible to detect a very slight change in curvature in the Y correlogram at $S = 24\text{-}25$ and at $S = 40$. This might be viewed as providing some additional evidence for the existence of these cycles in short term rates.

2 There is a very slight change at $w = 25°$ and at $w = 30°$ in $f_y(w)$. This may be viewed as providing some evidence of a less definite separation in the local maxima of $f_x(w)$ near $w = 24°\text{-}25°$ and $w = 28°\text{-}29°$.

3 Virtually all energy in low frequencies.

2. Some simple models to predict long rates

In this section we shall summarize three experiments with simple, low order, models which attempt to predict long rates. In these models we are particularly interested in the extent to which we may predict long rates by using our knowledge of the past behavior of short rates. These predictions of long rates are compared with other predictions which are derived by using the past history of long rates or by using both short and long rates.

I. The autoregression hypothesis (Model I)

We shall first investigate the hypothesis that the current long rate is more or less determined by its own past history, and that it is relatively immune to the influence of short rates. Some support for the autoregression hypothesis may be derived from the correlograms which appear to contain damped harmonics. In addition, these regressions provide us with a criterion by which to judge the performance of X as a predictor of Y. We consider the following four autoregressions:[20]

$$\sum_{s=0}^{p} A_{ps} Y_{t-s} = \epsilon_{pt} \qquad \left| \begin{array}{l} A_{p0} = 1 \\ p = 1, 2, 3, 4 \\ \epsilon_{pt} = N(0, \sigma^2) \end{array} \right.$$

where the Y's are deviations from their means.

II. The regression of Y on lagged X (Model II)

In this model we attempt to predict Y by using past data on X.

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[20] In these models we assume that the disturbances are independent and normally distributed with zero mean and common variance. This assumption is more restrictive than is necessary.

Predictions of Y were based on the following three models:

$$Y_t = \sum_{s=1}^{p} A_{ps} X_{t-s} + \epsilon_{pt} \; ; \quad p = 1, 2, 3$$

where both X and Y are deviations from their means. While the reduction in variance is significant in all three cases, none is dramatic and the reduction is much less than the Y autoregression of the same order. For this reason it did not appear necessary to extend this model; it is clear that knowledge of past Y is far more useful in predicting current Y than is knowledge of past X .

III. The regression of Y on lagged X and Y (Model III)

Lagged Y is a much better predictor of Y than is lagged X (indeed, it is some 5-10 times better). It is still possible that X does provide some supplementary information. To investigate this possibility we set up a regression using as our predictors Y lagged one period with X lagged from one to eight periods. Since this may be viewed as a distributed lag model, we are, in effect, assuming that the influence of lagged X on Y is distributed over a number of periods.[21] The models considered are:

$$Y_t = A_p Y_{t-1} + b_p X_{t-p} + \epsilon_{pt} \qquad p = 1, 2, 3, \ldots 8$$

where both Y and X are deviations from their means.

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[21] Since A_p , the estimated coefficient of Y_{t-1} is close to unity in almost all the regressions, we may interpret this model as one in which lagged X is affecting the rate of change (first difference) in Y .

The results of our three experiments shown in Table 2 may be briefly summarized. Short rates have some value in predicting long rates, but the reductions in variance are not dramatic. Moreover, the reduction is much less significant than those associated with the Y autoregression of the same order; knowledge of past Y is far more useful in predicting current Y than is knowledge of past X .

In Model III where we are using a simple distributed lag setup, we find that X does not seem to provide any additional, or supplementary, information. On the other hand, the fact that when we use lagged X as regressors in Model II we do reduce the variance somewhat, suggests that we cannot rule out the possibility that there may be a dependence of Y on X . It is therefore possible that short rates have an effect on long rates but that this effect is neither a simple nor a direct one. In the next two sections we summarize our attempts to find evidence of a more complex relation between short and long rates.

TABLE 2

RESIDUAL VARIANCES V_p FOR THE THREE MODELS [1, 2]

P	MODEL I	MODEL II	MODEL III
1	0.0662	1.0927	0.0662
2	0.0594	0.5138	0.0659
3	0.0565	0.3170	0.0655
4	0.0526		0.0640
5			0.0630
6			0.0637
7			0.0642
8			0.0642

1 All the variances in the table have been multiplied by 10.

2 A quick way to tell whether a reduction in variance is significant is to use the following approximation. If there are N observations and P constants are fitted, the residual variance is V_p. It can be shown that $(N-P)\ (\frac{\Delta v}{v}) \approx X^2(1)$. For example, in Model I if we move from $P = 1$ to $P = 2$, we find that $(N-P)\ (\frac{\Delta v}{v}) = 10.3$ while $X^2(1)_{.05} = 3.84$.

3. Do short rates affect the rate of change or acceleration of long rates?

Our experiments in the three low order models indicate that short term rates are not particularly useful in predicting long rates. On the other hand, short rates may still affect the course of long rates, even though their direct influence, in the sense of predictors in a regression of low order, is small. For example, the course of a pendulum over time may be decisively influenced by a small force; similarly here if short rates affect either the rate of change (first differences) or the rate of acceleration (second differences) in long rates, they may have an important, and perhaps even decisive, influence on the movements in long rates and yet this would not show up in a simple regression model.[22]

To test this hypothesis we examine the correlations between X and first and second differences in Y. These correlations provide evidence on the extent to which short rates may affect the rate of change or acceleration of long rates. For example, if X is, indeed, affecting the rate of change of Y we should expect to find a lag S for which $r_s(1)\ (X,\Delta Y)$ is high. Similarly, if X affects the rate of acceleration of Y we should expect to find a lag S for which $r_s^{(2)}\ (X,\Delta^2 Y)$ is high. In fact, as is indicated in Table 3, the maximum correlations for $r_s^{(1)}(X,\Delta Y)$ and $r_s^{(2)}\ (X,\Delta^2 Y)$ are -.39 and .08 respectively, both of which are considerably lower than

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22 There are additional reasons for undertaking this test. The coefficient of Y_{t-1} in the distributed lag model (III) are close to unity, suggesting that there may be some relation between X and first differences in Y. Also, we are using low order models whereas the several harmonics in the X series suggest that we are dealing with a higher order process.

$r_o\,(X,Y) = /81$. This result does not tend to bear out the hypothesis that X is affecting the first or second difference in Y .

TABLE 3

THE LARGEST VALUE OF THE OBSERVED CORRELATION COEFFICIENT [1]

1. $r_o^o = .81, \quad S = 0$.

2. $r_{55}^{(1)} = .39, \quad S = 55$.

3. $r_{59}^{(2)} = .08, \quad S = 59$.

[1] The correlation coefficient is defined as follows:

$$r_s^{(i)} = \frac{\mathrm{Cov}\,(X_t, \Delta^i\, Y_{t+s})}{\sqrt{\mathrm{Var}(X_t)\ \mathrm{Var}(\Delta^i\, Y_{t+s})}}$$

For example, for i = 2 we have

$$r_s^{(2)} = \frac{c_s(X,Y) - 2\,c_{s-1}(X,Y) + c_{s-2}(X,Y)}{\sqrt{c_o(X)\ [6c_o(Y) - 8c_1(Y) + 2c_2(Y)]}}$$

and for $i = 0$,

$$r_s^{(0)} = \frac{c_s(X,Y)}{\sqrt{c_o(X)\ c_o(Y)}}$$

4. Evidence from the Higher Order Models

The evidence from the low order models summarized in the previous section suggests that while lagged X does help reduce the variance in Y , it does not do as well as lagged Y in predicting current Y . Additional experiments that we have performed suggest that lagged Y does perhaps slightly better in predicting current X than does past X in predicting current Y . The simple low order models do not provide an understanding of the relation between short and long rates if that relation is complex; nor do they provide a basis for deciding whether short rates are influencing long rates or whether long rates are influencing short rates. In addition, we still are left with the puzzle that in spite of the fairly high correlation between current short and long rates their movements are disparate, especially at turning points.

For these reasons we have experimented with the following four higher order models containing two autoregressions of order 6 and two mixed models of order 12. These regression equations are

(1) $$Y_t = \sum_{s=1}^{6} b_s Y_{t-s}$$

(2) $$X_t = \sum_{s=1}^{6} a_s X_{t-s}$$

(3) $$Y_t = \sum_{s=1}^{6} b_s Y_{t-s} + \sum_{s=7}^{12} b_s X_{t+6-s}$$

(4) $$X_t = \sum_{s=1}^{6} a_s X_{t-s} + \sum_{s=7}^{12} a_s Y_{t+6-s}$$

Let us briefly summarize our main results:

Lagged X does not help in predicting Y .

Knowledge of past behavior of short term rates does not help to make better predictions of long term rates. The residual variance of Y in the fitted autoregression

$$Y_t = \sum_{s=1}^{6} b_s Y_{t-s}$$

is 0.00749.

This provides about the same reduction in the residual variance (slightly greater in fact) as does the regression

$$Y_t = \sum_{s=1}^{6} b_s Y_{t-s} + \sum_{s=7}^{12} b_s X_{t+6-s}$$

The R^2 for the two models are 0.977 and 0.978, respectively. The addition of six X predictors does not bring about any reduction in variance over that achieved in the autoregression of order six. This coupled with the evidence of the periodicities from the correlogram and periodogram would argue against the hypothesis that the direction of causation is from short term rates to long term rates.

Lagged Y does help some in predicting X.

When we compare the two models which attempt to predict short rates, we find some evidence that lagged Y may have some value in predicting X. Thus when we compare the autoregression of order 6 with the mixed model which also uses long rates to predict short term rates, we find that the residual variance is reduced from .06 to .05. The coefficient of determination in the autoregression of order 6 is .943 and it rises to .955 when we add the additional six Y predictors. It is also interesting to note that Y seems to affect X by means of successive first differences.

The high correlation between X and Y does not necessarily reflect a direct influence of X on Y. Instead the higher order models summarized above seem to point to a more complicated structure of the following type:

$$X_t = f(X_{t-1}, X_{t-2}, \ldots X_{t-6}) + a_1 \Delta Y_{t-1}, + a_2 \Delta Y_{t-3}$$

$$Y_t = g(Y_{t-1}, Y_{t-2}, \ldots Y_{t-6})$$

where X is determined partly by its own past history and partly by two successive first differences in long rates, and where Y may be subject to longer waves and changes much less frequently. In this kind of a model the high correlation between current rates results from a fairly complex structure and does not necessarily lend support for a policy of operating on short rates to influence long rates.

For both series the autoregression model seems to account for most of the variance. This would fit in with our notion that while interest rates are changing each period, they change their direction only infrequently. Normally both short and long rates move together and this has been especially true for the post-Accord period that we studied. This may explain why we get a fairly high correlation coefficient and why there appears to be a fairly close relation between short and long rates. From a policy point of view, what is crucial is not the closeness of the relation but rather his behavior at turning points. To the extent that divergent movements occur at turning points, we may still have a fairly good relation between short and long rates even though this relation is not stable and, indeed, may change over time. This would explain why during a recession we may observe a very substantial drop in short rates with very little repercussion on long rates. If this view is correct it would also imply that correlation coefficients would be higher if we divide the period into subperiods.[23]

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[23] Some evidence is presented in the next section.

C. Conclusion: Some Policy Implications

The statistical analysis presented thus far suggests that long rates are not particularly responsive to movements in short rates. A central bank which operates on the short rate in the hopes of influencing the long rate is not likely to succeed except in those cases where the rate structure would have changed in the desired manner without the policy action. This would suggest that it is not desirable for a central bank to limit its freedom by adherence to a bills-only policy.

Do we have any evidence that a central bank that is willing to act in all maturity sectors will have greater success in influencing the long rate? It is, of course, tempting to conclude that bolder action by the central bank can bring about desired changes in the long rate. At the same time it is clear that the evidence we have presented does not, in itself, necessarily justify such a conclusion. To find as we have that the central banks' purchase of short maturities did not lower the long rate substantially does not imply that a purchase of longs would have: first, because in recessions when the central bank wishes to lower the long rate it may be sticky; and second, because the effect of a given dollar purchase of bonds may not differ substantially from that of bills.

Indeed, it is precisely this latter conclusion that emerges from the excellent study that Okun prepared for the Commission on Money and Credit. Okun studied the period 1946-1959 and found that "the estimated effects of open-market actions are very similar, whether they are conducted by means of bills or of long bonds." [24]

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[24] Op. cit., p. 366.

Interestingly enough, although Okun's theoretical framework suggested that relative supplies should affect the rate structure, he nevertheless concludes that "if the results of this paper are anywhere near the mark, how much the Federal Reserve buys (or sells) is far and away more important than what issue it chooses to deal in" (Underlining in original).[25]

We should also emphasize, as Okun does, that it would be incorrect to interpret his results as a vindication of bills-only. Nevertheless, the results of this careful study provide considerable support for those who maintain that changes in the maturity structure of the Federal debt will not have substantial effect on the rate structure. A number of considerations suggest that this finding, based on his study of the years 1946-1959, may need modification when applied to later years. Thus, although he finds no evidence to support the alleged "thinness" in the market for longer maturities, he points out that the data he worked with may not support the view that the market is thin in part because the authorities limited their transactions because of this fear.[26] Just as this inhibition on the part of the authorities after the Accord may mask a possible "thinness" in the market, it, in conjunction with the systems' commitment to maintain a pegged interest rate structure prior to 1951, may also mask the effect of changes in relative supplies -- or the differential effect of open market transactions in bills or bonds.

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[25] Op. cit., p. 369.

[26] Okun gives the following analogy which he attributes to Henry Wallich. "The monetary authorities have consistently viewed the bond market as thin ice and they have therefore skated with great care. According to the data, they have never fallen through the ice. Yet, it cannot be justifiably concluded that the ice is solid and the caution gratuitous." Op. cit., p. 350.

Also for the period he studied he classified debt as short or long depending on whether its maturity was less than or greater than one year. This classification would not be appropriate in all circumstances and especially for a period such as 1961-1964 when significant policy-induced changes took place in the within-one-year category.

In addition our recent experience with "Operation Nudge" seems to allow a greater role for changes in relative supplies.[27] And while we cannot conclude that "Operation Nudge" is a great success, it seems to have succeeded to a greater extent than was thought possible, say, in 1961 when the policy was announced. Although the authorities appear to have succeeded in holding down long rates while short rates rose, there is clearly a problem in separating out the policy induced effects from cyclical and secular effects. In particular, how much of this was due to "Operation Nudge" (changes in the relative supplies of bills)? How much to the narrowing of the spread in the upward phase of the business cycle? And how much to the possibility that we are now in the midst of a long wave of lower interest rates? (A number of money market experts have expressed the view that the era of rising interest rates begun in 1946 may have ended sometime in 1959-1960 and that this secular movement has kept the long rate from rising.) Nevertheless, in spite of these uncertainties it would appear that "Operation Nudge" did succeed at least partially and this

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[27] "The data on changes in the maturity structure of publicly held Treasury debt seem to provide some support for the assumption that changes in the debt structure have influenced the term structure of Treasury yields." See "Changes in the Structure of the Federal Debt," Federal Reserve Bulletin, March 1963, pp. 308-309.

thus points to a role for changes in relative supplies.[28]

There is also additional evidence to support the view that the factors affecting short and long rates may have changed during this period. A study of scatter diagrams of long and short rates for the period 1953-1964 suggests that the relation between them may have changed in 1958 and possibly again in 1961. The table below shows the coefficients for the five regressions that we calculated for the various periods.

Our main findings are that an upward shift in the curve relating long and short rates took place in early 1958 (the intercept rises from 2.18 to 2.98); that there was a substantial change in slope in 1961 (the slope changes from .32 to .21); and that the change in 1958 occurred approximately in February (but we cannot readily date the 1961 change). These findings suggest that asset holders' preferences, or expectations, may have changed several times in the years since the Accord, as they began to adjust to a world of flexible interest rates. One interesting

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[28] Professor H. G. Johnson in a recent paper makes the following observations: "A related but subsidiary question about recent monetary policy relates to the effectiveness of the policy of twisting the rate structure. . . .When this policy was initiated, the results of contemporary research suggested that changes in the composition of the public debt would have relatively trivial effects on interest rates. . . . Meanwhile, the twist policy has apparently had more influence on the rate structure than was earlier predicted for it. A plausible line of explanation, but one difficult to explore, is that Federal Reserve policy pronouncements have a direct effect on the market's expectations." See H. G. Johnson: "Major Issues in Monetary and Fiscal Policies," Federal Reserve Bulletin, November 1964, pp. 1409-1410.

TABLE 4

REGRESSION COEFFICIENTS FOR THE RELATION $Y = a + bX$ [1, 2]

Period	a	b	r^2
1953-1960	2.23	.46	.65
1953-1958	2.18	.38	.91
1958-1960	2.98	.32	.86
1953-1964	2.24	.51	.63
1961-1964	3.38	.21	.61

1 Y is the rate on U. S. bonds and X is the rate on Treasury bills.

2 The standard errors for a range from .04 to .08 and those for b from .02 to .04.

hypothesis is that these expectational changes may reflect a learning process on the part of the public.[29] In early 1958 the expectations of the public may have become inelastic, reflecting their first experience (in over two decades) of having gone through a complete cycle under a flexible monetary policy.[30] Unlike the expectational change that we are postulating for 1958, the change in 1961 that we seem to find did not interfere with but may even have facilitated the effectiveness of the action taken by the authorities. It is as if the public expected "Operation Nudge" to succeed and acted on that basis. Put another way, what we find is consistent with the view that the public did not believe in bills-only but did believe that the term structure could be influenced by changes in relative supplies. And if, in retrospect, the public will conclude that "Operation Nudge" did not succeed to the extent that they expected, we may experience still another change in expectations and in the relation between interest rates.

The upshot of this discussion is that interest rate behavior may be changing and that we do not yet have enough evidence to quantify the effect of changes in relative supplies on rate structure. Further research in two particular areas would be highly desirable: An analysis

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[29] I am indebted to Professor Tobin who suggested this hypothesis.

[30] It may be worth recalling that Keynes' pessimism concerning monetary policy in a recession reflected precisely this possibility of inelastic expectations.

of "Operation Nudge;" and a testing of the learning hypothesis and the extent to which the expectational changes thwart or facilitate monetary action. In the absence of this evidence we cannot be certain that without the self imposed inhibitions the central bank would have been more successful in the bills-only period; but we do have some evidence that the rate structure may have undergone some evolutive changes and also that the rate structure is the resultant of a complex set of forces. These considerations suggest that the relation between the long rates and the short rates may be subject to substantial and abrupt shifts, especially at turning points when the stability of the relation is most relevant for the central bank. And for these reasons it is not desirable for a central bank to adhere to a bills-only policy or to limit its freedom in any way.

Given the complexity of the interest rate structure, the possibility that it is affected by a learning process, and the paucity of tested knowledge, it seems both advisable, and prudent, for the central bank to be more flexible in its actions and tactics, less dogmatic in its pronouncements, and more reluctant to take a policy posture that is at the extreme of the spectrum of opinion. Perhaps this may be the most important lesson to be learned from the bills-only episode; the evidence in recent years that the Federal Reserve is engaging in activities that may minimize a recurrence of extremism is therefore gratifying.

CHART 1: CORRELOGRAM OF INTEREST RATES

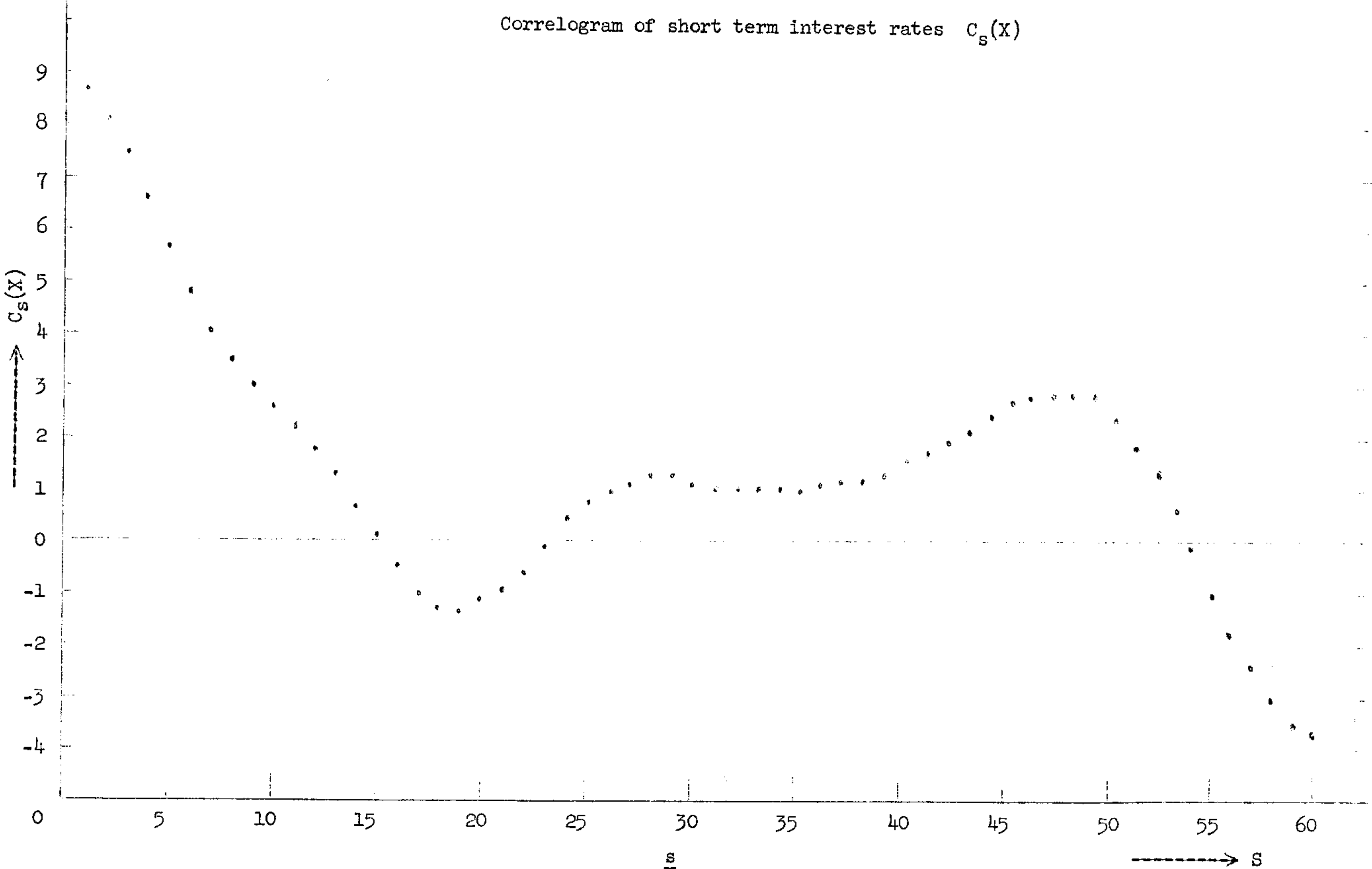
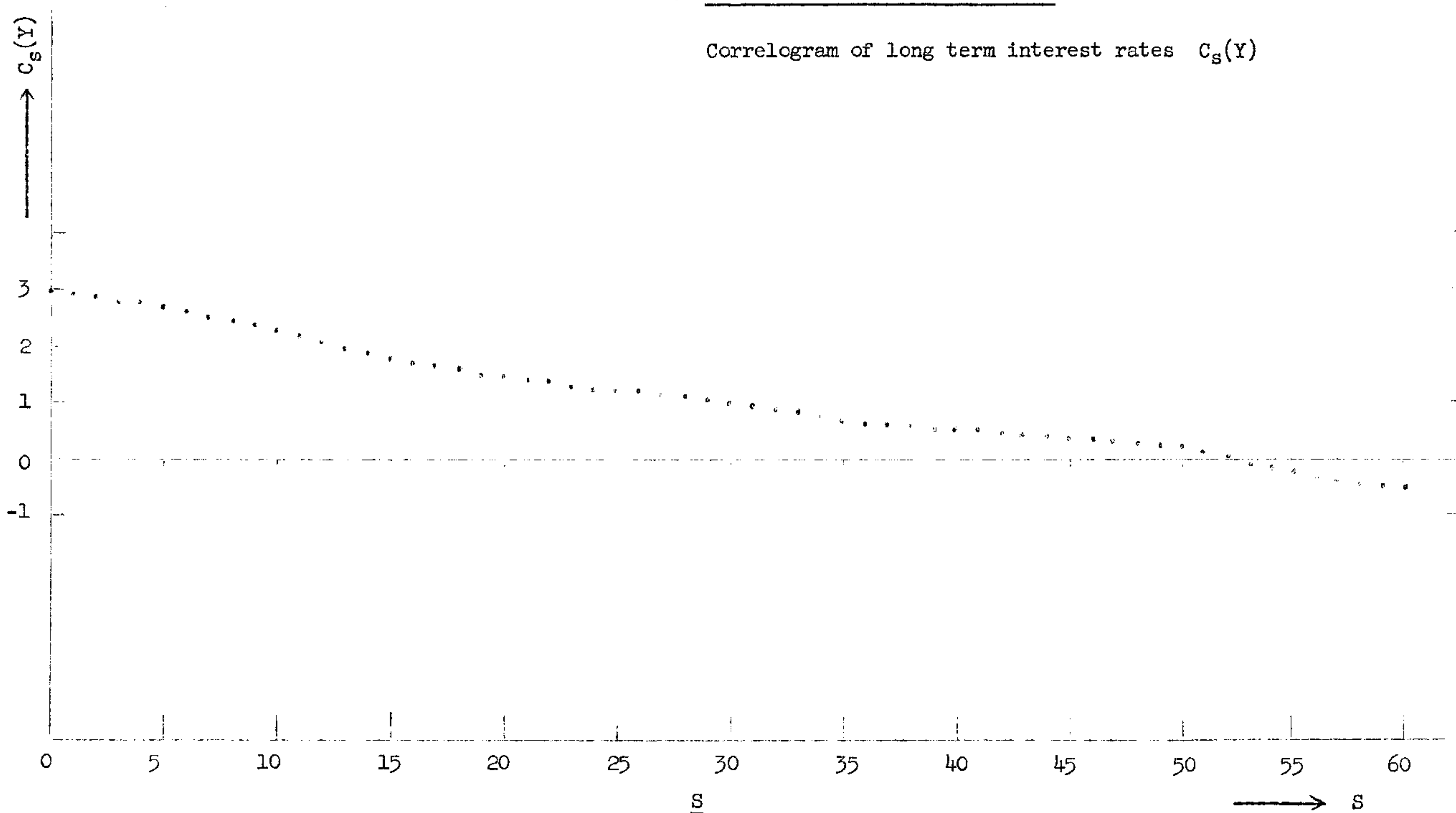
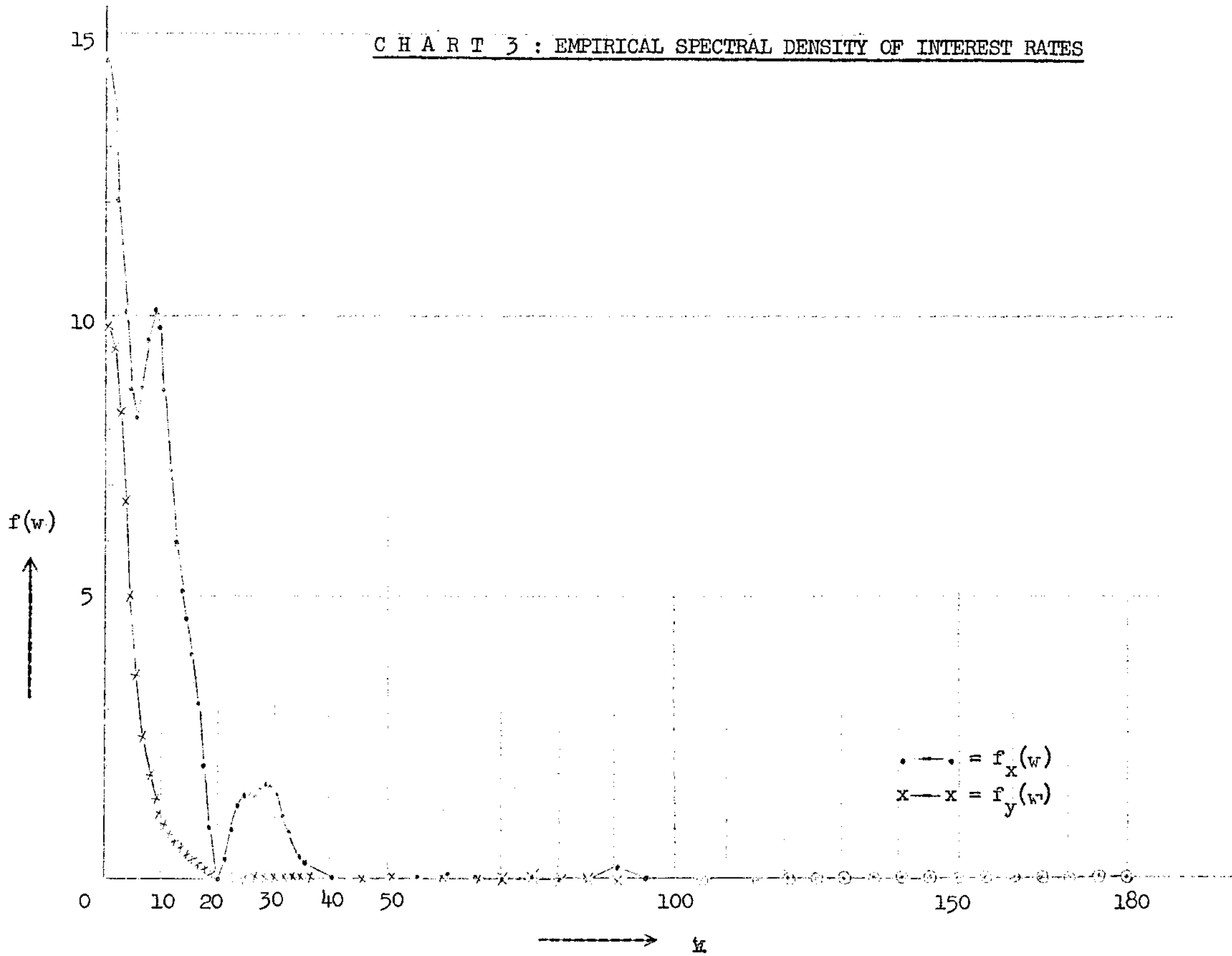


CHART 2 : CORRELOGRAM OF INTEREST RATES




C H A R T 3 : EMPIRICAL SPECTRAL DENSITY OF INTEREST RATES
f(w)
15
10
5
0
10
20
30
40
50
100
150
180
w
• — • = $f_x(w)$
x — x = $f_y(w)$

A P P E N D I X   I

Expectations, Relative Supplies of Debt and the Term Structure of Interest Rates

The expectations hypothesis as developed by Fisher, Keynes, Hicks and Lutz has been often taken to imply that the relative supplies of short and long term debt does not affect the term structure of interest rates. If the long rate is an average of expected short term rates, it can be controlled by the monetary authorities only to the extent that they can affect expected rates. And if expected rates are independent of current rates, a swapping of short term securities for long term securities will not affect the respective yields. Under these conditions the term structure of interest rates is independent of the relative supplies.[1] A number of assumptions are used to justify the view that the term structure is independent of relative supplies of debt. Among these are that short and long debt are perfect substitutes, that investment costs are insignificant, that neither lenders nor borrowers have risk aversion, and that expectations are inelastic. If we change some of these assumptions and assume, for example, elastic expectations, it is no longer true that the long rate cannot be influenced by the monetary

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1. For a thorough review of the development of neo-classical doctrine and for references to the literature, see J. W. Conard: An Introduction to the Theory of Interest, (Univ. of California, 1959), Part III. The expectational theory as developed by Hicks and Lutz was subjected to a critical review by J. M. Culbert in "The Term Structure of Interest Rates," Quarterly Journal of Economics, November 1957, pp. 485-517, and by D. G. Luckett in "Prof. Lutz and the Structure of Interest Rates," Quarterly Journal of Economics, February 1959, pp. 139-140, both of whom found the theory unsatisfactory. For some recent attempts to reformulate the Hicks-Lutz theory, see D. Meiselman: The Term Structure of Interest Rates, (Prentice Hall, 1962) who developed an operational test of this expectation hypothesis, B. G. Malkiel: "Expectations, Bond Prices, and the Term Structure of Interest Rates," Quarterly Journal of Economics, May 1962, pp. 197-218, and R. Kessel: The Cyclical Behavior of the Term Structure of Interest Rates, a forthcoming N.B.E.R. publication.

authorities or that the term structure is independent of the relative supplies of debt. Let us therefore review some different theories of term structure to focus explicitly on the effect of changes in relative supplies.

In a world of certainty, where short term and long term securities are perfect substitutes, forward rates and expected rates are identical. If any long rate were not the average of the relevant number of short rates, profits could be realized by arbitraging among maturities. In a world of perfect foresight, no investment costs and complete shiftability, the long rate is an average of the future short rates and the term structure of interest rates is independent of the relative supplies. Once we modify these assumptions and consider a world of uncertainty with imperfect foresight, we may distinguish three different theories of term structure.

1. <u>The No Risk Aversion Expectations Hypothesis</u>: This view postulates that default-free securities differing only with respect to term-to-maturity are relatively perfect substitutes, and that forward rates are unbiased estimates of expected rates. It also assumes that for the market as a whole there is no risk aversion, even though individual transactors may speculate or hedge on the basis of risk aversion. More specifically, it assumes that speculators who are indifferent to risk are sufficiently large as a class to determine market rates on the basis of mathematical expectation and that they dominate the market. Rates on long term maturities are averages of expected short term rates.

Meiselman formulated and tested an error-learning mechanism for the formation of expectations which postulates that expectations are revised whenever previously held expectations are in error. His main results are that the long rate is an average of expected short term rates, that neither risk aversion nor risk preference play important roles in the formation of forward rates, that forward rates are therefore unbiased estimates of expected rates. He interprets his results as contradicting the Hicks-Keynes theory of normal backwardation, the hedging-pressure theory of interest rate structure, and the liquidity-preference theory of interest.[2] Finally he concludes from his study that "changes in either Treasury debt policy with respect to the maturity composition of the public debt or Federal Reserve policy with respect to the maturity composition of its portfolio will have no long run effect on the yield curve, unless the initial disturbance alters expectations of 'the interest rate'" (p. 49) and that "The systematic covariation of segments of the yield curve contradicts the widely held view that the market is a 'segmented' one." (p. 60)

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2. These conclusions have been challenged by Wood and Kessel. See J. H. Wood: "Expectations, Errors and the Term Structure of Interest Rates", Journal of Political Economy, April 1963, pp. 160-171, "The Expectations Hypothesis, The Yield Curve, and Monetary Policy", Quarterly Journal of Economics, August 1964, pp. 457-470, and R. Kessel: op. cit.

2. The Risk-Aversion Expectations Hypothesis: This view postulates that short term and long term securities are not perfect substitutes and that for the market as a whole there is risk aversion. A lender engaging in a long term contract requires and expects to receive a "risk" or liquidity premium for taking this risk. The forward rate is therefore a biased estimate of the expected rate and exceeds it by the amount of the risk premium. This is the theory of "normal backwardation" developed by Keynes and Hicks. In the Hicksian formulation, the futures market is dominated by the lenders, and they, in turn, are conceived to be speculators with risk aversion who regard long loans as riskier than short loans. To the extent that the liquidity premium varies with the composition of the debt, the structure of interest rates is not independent of the relative supplies of short and long securities. This is therefore an intermediate position. The empirical work reported in Kessel's study support this view.

3. The Hedging Hypothesis: This view postulates that many transactors will hedge if the expected gains from speculation are not great enough to offset their distaste for uncertainty. Hedging behavior is conceived to be the response to risk aversion. Borrowers and lenders will have schedules of preference for short and long term funds which are related to the composition of their assets. The maturity composition of the existing assets and liabilities will affect the opportunity costs of hedging. The spread between short and long rates depends on the net hedging pressure. The equilibrium structure of rates requires that all excess demands are zero; this structure will, in

general, depend on the relative supply of shorts and longs.[3] This view which, in principle, allows relative supplies to have the greatest influence on rate structure is supported in the recent studies by Conard, Culbertson, and in the theoretical model used by Okun.[4]

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3. For the market as a whole, we may get this result without assuming risk aversion. Even if all investors are risk-neutral and act purely to maximize expected return and investment costs are zero, relative supplies would still affect the rate structure, so long as investors differed in their estimates of future long rates. See A. M. Okun: "Monetary Policy, Debt Management and Interest Rates: A Quantitative Appraisal", Stabilization Policies, (Prentice-Hall, 1963), pp. 338-340.

4. See also Neil Wallace: "The Term Structure of Interest Rates and the Maturity Composition of the Federal Debt", unpublished Ph. D. dissertation, University of Chicago, 1964.

APPENDIX II

The quantities C_s, r_s, $f(w)$ are shown below for x. The same calculations apply to Y.

1. The autocovariance coefficient of lag S is defined as follows:

$$C_s(X) = \frac{\sum_{t=1}^{N-S} X_t X_{t+s} - \frac{1}{N-S} \sum_{t=1}^{N-S} X_t \sum_{t=s+1}^{N} X_t}{N-S}$$

Note that $C_s(X) = C_{-s}(X)$

2. The autocorrelation coefficient of lag S is defined as

$$\pi_s(x) = \frac{C_s(x)}{C_o(x)}$$

3. The empirical spectral density function is:

$$f_x(w) = \sum_{s=-M}^{M} C_s(x) \left(1 - \frac{|s|}{M}\right) \cos ws$$

$$= C_o(X) + 2 \sum_{1}^{M} C_s(x) \left(1 - \frac{s}{M}\right) \cos ws$$

In our example N = the number of observations = 93 and M, the maximum lag calculated, = 60.

$f(w)$ is defined for $w = 0,1,2, \ldots 35$ by degrees
$w = 40, 45, \ldots 180$ by five degree intervals