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THE MONETARY MECHANISM: SOME PARTIAL RELATIONSHIPS

James L. Pierce

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. THE RESERVE ASSET PORTFOLIO</td>
<td>19</td>
</tr>
<tr>
<td>III. THE INVESTMENT AND LOAN ASSET PORTFOLIOS</td>
<td>37</td>
</tr>
<tr>
<td>IV. SOME COMPLICATING FACTORS</td>
<td>58</td>
</tr>
<tr>
<td>V. THE MODEL AND SOME EMPIRICAL EVIDENCE</td>
<td>97</td>
</tr>
<tr>
<td>VI. CONCLUSIONS AND POLICY IMPLICATIONS</td>
<td>142</td>
</tr>
<tr>
<td>APPENDIX A: INVESTOR EQUILIBRIUM</td>
<td>152</td>
</tr>
<tr>
<td>APPENDIX B: DESCRIPTION OF BALANCE SHEET DATA</td>
<td>159</td>
</tr>
<tr>
<td>APPENDIX C: THE COMPOSITION OF THE RESERVE ASSET PORTFOLIO</td>
<td>162</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>166</td>
</tr>
<tr>
<td>SELECTED BIBLIOGRAPHY</td>
<td>171</td>
</tr>
</tbody>
</table>
THE MONETARY MECHANISM: SOME PARTIAL RELATIONSHIPS

by

James L. Pierce

I. INTRODUCTION

The existence of an intimate connection between monetary policy and the commercial banking system is well known. The exact nature of this connection is far from perfectly understood, however. The purpose of this study is to attempt to specify both the nature and the degree of control that the monetary authorities can exert over commercial banks. It is generally agreed that the Federal Reserve System is capable of controlling the total reserves of its member banks. What is not agreed upon is the manner in which banks in turn respond to changes in reserve availability. It would appear that the reason that there is no clear consensus on this issue is that no mechanism of response has been formulated which allows one to predict bank reaction to policy.

In an interesting article, James Schlesinger outlines the course which monetary policy controversy has followed over the years. As Schlesinger demonstrates, the literature is replete with criticisms and defenses of monetary policy, with few if any of the combatants ever induced to change their positions. The paucity of compelling analysis by either side of the controversy cannot help but strike the reader.
It is reasonable to contend that the issues will not be resolved until the mechanism which lies between monetary policy and its ensuing effects on the income generating sectors of the economy has been specified. The total specification of this monetary mechanism no doubt lies far in the future. Some of the work has already begun, but most is yet to be done.

The links between monetary policy and the income generating sectors of the economy are complex and often subtle. The most fruitful approach to the total specification of these links appears to lie in determining parts of the total relationship in the expectation of finally putting the pieces of the puzzle together. It would of course be convenient if the total mechanism could be specified now. This unfortunately appears to be out of the question. The rather disheartening bickering which still characterizes much of the discussions of the effectiveness of monetary policy bears witness to the fact that the popular, highly simplified, and highly aggregated theories have not been successful. So much misunderstanding and educated guessing concerning monetary affairs arises from the fact that at the present there is no monetary theory which even approaches success in explaining the role of monetary policy in the economy. Theory as it exists today is inadequate for such a task. A usable monetary theory must be able to describe and predict how, and with what intensity, changes in policy variables are transmitted to the areas of the economy which have been chosen as targets for policy. Such a theory cannot abstract from the institutional and behavioral peculiarities which
characterize financial institutions. An important element in the
determination of the influence of financial factors on the economy lies
in the response patterns of financial institutions, which transmit changes
in financial variables to the income generating sectors of the economy.
It is in the spirit of providing a component of this total monetary
mechanism that the present study is undertaken. While the specification
of the response mechanism for commercial banks is admittedly a small part
of the total mechanism, it is an important one, and it represents a logical
first step in the specification of the monetary mechanism.

A primary concern of the study is the operation of monetary policy
in a cyclical context. An important issue in policy debates is the extent
to which static models lead to inferences which are not valid in the real
world of uncertainty and of change. Economic policy is exercised in a world
characterized by instability and by pervasive, cumulative movements in
relevant economic variables. In order to develop an analysis which allows
one to predict the effects of any particular policy act, it is necessary to
determine whether or not important behavioral relationships are altered in
the dynamic context. If they are altered, the analysis must be so modified
as to enable the prediction of the effects of dynamically determined
behavior patterns. The analysis of policy should not abstract from such
dynamic considerations. If the tests of hypotheses suggest that economic
behavior is not significantly influenced by the existence of cyclical
movements in the economy, so be it; one should not assume the lack of
significance, however, but test for it.
The behavior of commercial banks has been singled out for study because of the direct link that they enjoy with monetary policy. No small part of the controversy over such policy has been concerned with the role of commercial banks in the economy and with the degree of control which the Federal Reserve System can exert over them. Of particular interest here is the lending policies of banks. According to Warren L. Smith and others, the monetary authorities can exert little or no control over the volume of bank loans.² It is argued that banks respond freely to variations in loan demand by adjusting their grants of new loans to whatever level is dictated by the prevailing level of loan demand. Banks are conceived to adjust their other portfolio items in a manner which will bring their loan portfolios to the size dictated by the state of loan demand. Presumably, such discussions of bank behavior assume that loan demand fluctuates over a sufficiently narrow range to disallow one from ever observing banks with either no loans at all or with all their resources committed to loans. Over this range, the volume of reserves and the level of bond prices are seen to exert little or no influence on lending policies. If the contention is correct, the Federal Reserve does not control this important group of financial institutions. A compliment of this argument revolves around the extent to which banks use non-price means of rationing credit. Here the discussion involves the question as to whether or not banks restrict their loans by means other than price. Banks will find it necessary to ration loans only if they are not willing to freely substitute loans for other
assets. If, in fact, banks are not willing to carry out such free substitution, they will find it necessary to ration credit on some basis, non-price or otherwise.

The influence which the Federal Reserve exerts over commercial banks must take place through bank asset portfolios. At issue here is the extent to which the composition of bank asset portfolios is influenced by monetary policy. If this influence is to be determined, it is necessary to specify the factors which influence bank portfolio management. The role of policy in this process can be determined only when we know more about how banks manage their portfolios. As a result, no small part of the analysis is concerned with the factors which, in general, affect the size and composition of such portfolios. Banks are economic agents and as such must make economic choices. The commercial bank has limited resources which it must distribute in a profitable manner. The basic issue is the determination of the factors which influence this allocation of funds. If the determination is successful, it should prove possible to make more meaningful pronouncements on the role of policy than has heretofore been the case. Every effort is made to orient the analysis in a manner which will make it amenable to the study of policy implications. The primary goal is the determination of the role of policy, the portfolio analysis is a means to this end.

In order to reduce the scope of the study to more nearly manageable proportions, only the cyclical behavior of banks is analyzed. Longer-run implications and conditions are not treated. As long-run behavior is hardly
independent of cyclical movements, the analysis should provide insight
into the factors affecting bank behavior in the long-run.

Due to the complexity of the behavioral relationships involved,
a rigorous analysis of bank portfolio management in a world of uncertainty
and of change is far beyond the scope of the study. No profit maximization
conditions are established, and no programming techniques are used. The
development of a theoretically respectable model of bank behavior, which
can be used directly for the analysis of policy, is extremely difficult
to achieve. Some fairly rigorous models of bank behavior have been developed,
but they have a long way to go before they can be used as tools for the
analysis of the impact of policy. The inability to present the current
analysis in rigorous terms implies that, at best, a small step has been
taken toward obtaining satisfactory answers to some important questions.

The Framework of the Analysis

The basic framework used in the study is intended to both simplify
the analysis at hand and to serve as a basis for future research. It
should be stressed at the outset that the analysis is in a sense restricted
by its orientation. The analysis must be amenable to policy study, and its
framework must be constituted so as to allow statistical testing of the
hypotheses using available, or at least potentially available, data. While
the theoretical analysis may suffer by discussing only testable hypotheses,
it appears that in the area to be studied the fundamental problems are of
necessity empirical in nature. Most of the unanswered questions concerning monetary policy are concerned with the degree of response of economic agents to variations in policy variables; these questions can only be answered empirically.

The framework is basically a very simple one, which is designed to aggregate a typical commercial bank's asset portfolio sufficiently to simplify the analysis while retaining enough detail to allow a meaningful treatment of portfolio selection. The bank is conceived to have a total supply of available funds at its disposal. These funds are its deposit liabilities (less required reserves) and its capital account. The bank distributes these given funds among available assets.

The total asset portfolio is separated into three major components. The three basic groups are a transactions balance, a portfolio of relatively long-term, high yielding securities, and a group of non-financial loans. The analysis which follows will use this tripartite asset separation as its basic structure. A brief description of the components is given here, and a detailed analysis will follow. The transactions balance is called the portfolio of reserve assets; it provides the bank with a pool of highly liquid assets to be used for transactions purposes. The components of the portfolio are the bank's holdings of cash (excess reserves and balances due from other banks), short-term Treasury liabilities (bills, certificates, and notes and bonds maturing within 1 year), and other highly liquid assets.
The portfolio of relatively long-term securities is called the investment asset portfolio; it is held for income and diversification purposes, and it provides a potential source of speculation. This portfolio contains such securities as intermediate (1-5 year) and long-term Treasury bonds, municipal bonds, and special long-term issues. The non-financial loan portfolio is held for income purposes and is composed of all loans other than the extremely short-term loans made to brokers, dealers, and finance companies -- these are included in the reserve asset group. The following notation is used:

A: Total assets (less required reserves)
R: Reserve assets
I: Investment assets
L: Loan assets
F: Total available funds
D: Demand deposits (less required reserves)
T: Time and saving deposits (less required reserves)
C: Capital accounts

These terms are arranged in the following balance sheet identities:

\[
\begin{align*}
A &= R + I + L \\
F &= D + T + C \\
F &= A
\end{align*}
\]

* A complete list of the components of these identities is given in appendix B.
The commercial bank is given a total supply of funds, \( F \), which it allocates among the three asset groups. The portfolio decision to be considered here involves the determination of the desired shares of \( R \), \( I \), and \( L \) in the total asset portfolio.

The validity of this three asset approach rests on an important simplifying assumption. It is assumed that the bank makes two essentially independent decisions concerning the characteristics of its total asset portfolio. The first decision involves the determination of the desired distribution of funds among reserve, investment, and loan assets. The second decision involves the determination of the desired composition of each of the three asset groups. The bank is conceived to set general policy goals in terms of the relative sizes of \( R \), \( I \), and \( L \). Given these goals, it then attempts to optimally allocate funds within each of these asset groups. For example, the bank decides that it wants a particular proportion of its resources devoted to transactions purposes. Given the desired size of the reserve asset portfolio, it then decides what proportion of the transactions balance is to be in cash, what proportion in bills, etc. The bank goes through the same sort of decision process for the other two asset groups; it first determines their desired size and then their compositions. This sort of decision process involves the solution of two separate portfolio problems. The desired relative shares of \( R \), \( I \), and \( L \) in the total asset portfolio are determined. Then, the desired shares of the components of each of the three basic asset groups are determined.
It goes without saying that in reality the two decisions are not completely independent. The total size and the composition of the three asset groups are not wholly unrelated. If, for example, the bank decides to scrimp on its transactions balance, it can partially offset some of the possible costs involved by shortening the maturity of the reserve asset portfolio; i.e., it can substitute cash for bills. A complete treatment of the management of a bank's total asset portfolio would involve a simultaneous determination of the size and composition of the three asset groups. It would analyze the shares of all relevant assets in the total asset portfolio. Such an approach is beyond the scope of this study. It is assumed that the two decision processes outlined above exist and that they are sufficiently independent to allow the analysis to proceed.

Only the determination of the desired distribution of funds among the reserve, investment, and loan asset groups is treated here. In order to keep the size of this study manageable, it is necessary to relegate the analysis of the composition of the three asset groups to future research.* The success of the current analysis will give some indication of the possible fruitfulness of such research.

In order to determine how the bank allocates its given funds among reserve, investment and loan assets, it is necessary to establish the basis

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*A brief discussion of the determinants of the composition of the reserve asset portfolio is given in appendix C.
on which the asset groups compete for bank funds. Of primary interest is
the degree to which the bank's demand for R and I constrains its
acquisition of loans. By assuming that the bank determines the desired
relative shares of R, I, and L in total assets independently of
their compositions, it is possible to treat each asset group as if it were
composed of a homogeneous set of assets. That is to say, as if all assets
within each group were perfect substitutes. It is convenient to respectively
classify the reserve, investment, and loan asset groups by cash, long-
term bonds, and loans. Each asset has its own return and risk properties.
The portfolio problem involves an analysis of how a commercial bank will
allocate a dollar of F among the assets in a manner which will maximize
it "utility."

Before proceeding to the discussion of the determinants of the
composition of the bank's asset portfolio, it should prove useful to remind
the reader of some of the fundamental elements of general portfolio analysis.
As the name implies, such analysis is not interested in individual securities
per se, but rather in alternative portfolios which are typically composed
of more than one security. An investor is faced with the problem of choosing
an asset portfolio which in some sense best suits his needs. The future
returns on individual securities are not known with certainty. The investor
forms expectations of what these returns will be. His conception of future
security returns will condition the expected returns on the various port-
folios which are available to him. It is assumed that the investor is
not only interested in expected future returns but also in the risk that actual yields will differ from their expected values. Portfolio analysis commonly assumes that the investor considers the standard deviation of return to be the relevant measure of the dispersion of possible outcomes, i.e., of asset risk.

A basic simplification of portfolio analysis is the assumption that only two elements enter into the investor's utility function: expected average portfolio return and average portfolio risk (his estimate of the standard deviation of portfolio return). The essence of the theory lies in the manner in which the expected returns and risks of individual securities condition the average expected return and variance of the total portfolio. The return-risk properties of alternative portfolios, combined with the investor's utility function specify the optimal portfolio for the investor.

It is common to characterize an investor's preferences by an aversion to risk; he can be induced to accept more average portfolio risk only if sufficiently compensated by increased average return. Such preferences are conveniently characterized by a quadratic utility function of the form \( U = aR + bR^2 \) where \( U \) is utility, \( R \) is return, \( a \) is a positive constant, and where a value of \( b \) less than zero implies risk aversion. It may be recalled that expected utility is given by

\[
E(U) = aE(R) + bE(R^2) = aE(R) + b[E(R)]^2 + b\sigma^2,
\]

where \( \sigma^2 \) is the variance of return. Defining \( E_T \) and \( \sigma_T^2 \) to be the average portfolio
expected return and variance respectively, expected utility is given by
\[ E(U) = aE_T^2 + b\sigma_T^2. \]
Thus the expected utility of the investor depends upon the average expected return and variance of return of the portfolio.
If the return on each security is normally distributed, portfolio return will also be normally distributed and the distribution is completely specified by \( E_T \) and \( \sigma_T \).

It is now necessary to discuss the relationship between \( E_T \) and \( \sigma_T \) and the returns and risks of the securities which comprise the portfolio. To simplify this presentation, it is assumed that the investor considers only two assets, \( x_1 \) and \( x_2 \), to be relevant components in his portfolio. The returns on the two securities are taken to be random variables. The expected return and variance of the portfolio is discussed in terms of a dollar of invested funds. Expected return is given by
\[ E_T = w_1E_1 + w_2E_2, \]
where \( E_1 \) and \( E_2 \) are the expected returns on the two securities, and \( w_1 \) and \( w_2 \) are the shares of the two securities in the portfolio; and where \( w_1 + w_2 = 1 \) and \( w_1, w_2 \geq 0 \). The expected return on the portfolio per dollar invested is a weighted average of the expected returns on the two securities. The variance of the portfolio is given by
\[ \sigma_T^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{cov}(x_1, x_2), \]
where \( \sigma_1^2 \) and \( \sigma_2^2 \) are the variances of return of the two securities.
Recalling that the covariance of the two returns is defined as
\[ \text{cov}(x_1, x_2) = \rho_{12} \sigma_1 \sigma_2, \] where \( \rho_{12} \) is the correlation between the two returns, the variance of portfolio return is given by

\[ \sigma_T^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2. \] The average variance of return depends not only upon the variances of the two asset returns, but also upon the degree of interdependence between the returns. The necessary elements of portfolio analysis have now been defined and some of the implications are now discussed.

The risk or standard deviation of return on the portfolio is less than the weighted average of the individual risks, provided the two asset returns do not have a perfect positive correlation. Consider the case where \( \rho_{12} = 0 \), then

\[ \sigma_T^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 \quad \text{and} \quad \sigma_T = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2}. \]

Alternatively, consider the case in which \( \rho_{12} = 1 \), then

\[ \sigma_T^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 = (w_1 \sigma_1 + w_2 \sigma_2)^2, \quad \sigma_T = w_1 \sigma_1 + w_2 \sigma_2 \]
i.e., \( \sigma_T \) is a weighted average of the two risks. Finally, if \( \rho_{12} = -1 \), then

\[ \sigma_T = w_1 \sigma_1 - w_2 \sigma_2, \quad \text{and} \quad \sigma_T = 0 \quad \text{for} \quad w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}. \]

The relationship of portfolio risk to the variances and covariances of the individual securities immediately leads to a basic tenet of the theory of portfolio selection, namely that diversification tends to reduce portfolio risk for a given expected return. Assume \( E_1 < E_2 \) and \( \sigma_1 < \sigma_2 \), \( E_1, \sigma_1 > 0 \); and for simplicity assume that \( \rho_{12} = 0 \).
Under these conditions, and indeed for any $\rho_{12} < 1$, the minimum risk portfolio is not achieved when all funds are put into the security with lowest risk, i.e., when $w_1 = 1$. In the case of independence of return, $\sigma_T^2 = w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2$. Taking the first derivative of $\sigma_T^2$ with respect to $w_1$ and setting it equal to zero, the minimum $\sigma_T$ is obtained:

$$\frac{d\sigma_T^2}{dw_1} = 2w_1 \sigma_1^2 - 2w_1 \sigma_2^2 = 0.$$ 
The share of the low risk asset in a dollar of portfolio is given by $w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$, which equals unity only for $\sigma_1 = 0$.

Thus diversification of the portfolio will reduce $\sigma_T$ below what it would be if the investor chose $w_1 = 1$, even though $\sigma_1 < \sigma_2$, and it also leads to a greater $E_T$ than would exist if $w_1$ equaled unity. If asset returns are positively correlated, the ability to reduce risk through portfolio diversification is reduced, and it is eliminated when $\rho_{12} = 1$; in this case the minimum risk portfolio is achieved when $w_1 = 1$. If asset returns are negatively correlated, the risk reducing potential of diversification is increased, and in the case where $\rho_{12} = -1$ portfolio risk can be eliminated in the two asset case.

Portfolio analysis considers only those portfolios which minimize risk for a given expectation of return; these are often called the set of
efficient portfolios. Within the efficient set, the expected value of the portfolio return can only be increased at the expense of increased risk. The maximization of expected utility for a risk avverting investor will typically lead to a diversified portfolio. The portfolio chosen by this maximization will be a member of the efficient set.

In appendix A, the investor equilibrium in the two asset case is discussed in some detail. The conclusion of the appendix and of portfolio analysis in general is that for a risk avverting investor, the share of an asset in a portfolio is generally positively related to its expected return and negatively related to its risk. Other things being equal, an increase in the expected return of an asset, or a decrease in the investor's estimate of its standard deviation of return (risk), will typically result in an increased share of this asset in the total portfolio. This conclusion will be used frequently in the subsequent analysis.

Given that many of its liabilities are payable on demand, and given the small size of its capital account relative to its liabilities, the commercial bank can be conveniently assumed to be a risk avverting investor with a quadratic utility function. The assumption of the quadratic utility function is useful for the analysis of substitutions among assets. There is some question as to whether or not a corporation -- as opposed to an individual -- can be characterized by such a set of preferences. The concept of a utility function is a little difficult to interpret for a business. The discussion of the reserve asset portfolio revolves around
the transactions and precautionary demand for highly liquid assets. Some specification of a utility function is useful in accounting for the existence of the investment asset portfolio. By holding securities along with cash, the bank is behaving like a risk averting investor, even though it may not be one. If the bank did not act as if it averted risk, it could meet its transactions and precautionary demands by holding cash and simply put the remainder of its available funds into loans. In this case liquidity preference, rather than risk aversion, could be used as the basic behavioral assumption.

It is assumed that the three asset groups are substitutes for each other. Other things being equal, an increase in the expected return and/or a decrease of the risk of one asset will either result in a reduction of the shares of both of the other two in the total asset portfolio, or result in a decrease in the share of one asset and no change in the share of the other. It is not obvious that this is a good assumption. For example, an increase in the expected return on loans may induce the bank to reduce the relative share of I and to increase the shares of both R and L. It increases the share of L in total assets in response to the increased expected return, and it increases R in response to the increased average portfolio risk. In order to consistently predict the direction of portfolio shifts in response to variations in relative expected returns and risks, it is necessary to assume that such reactions do not occur. The empirical evidence will provide a test of this assumption.
The discussion now turns to the analysis of the bank's demand for reserve, investment, and loan assets. The entire discussion is carried out in terms of the portfolio choice of an individual commercial bank. The analysis which follows is essentially in two sections. The first section (chapters II and III) considers bank portfolio management under static conditions. The second section (chapters IV-VI) deals with bank behavior in a cyclical environment. Chapter IV is concerned with a modification of the analysis prompted by the introduction of dynamic considerations. In chapter V, the model is subjected to empirical test. In chapter VI, the discussion is summarized and some conclusions are attempted.
II. THE RESERVE ASSET PORTFOLIO

Like any other business, a commercial bank must have a fund of assets at its disposal which enables it to carry out its transactions in an orderly manner. The reserve asset portfolio represents such a group of assets. It permits the bank to compensate for the lack of synchronization between the inflow and the outflow of funds. Even if the timing and size of the flows of funds were known with certainty, the lack of perfect synchronization in their movements would necessitate the existence of such a transactions balance. This balance performs an important productive service for the bank. The reserve asset portfolio earns an implicit rate of return for its services; this return makes an important competitor for bank funds.

The isolation of the sources of flows of bank funds is most easily achieved by assuming that deposits created by the issue of new loans are withdrawn from the bank as soon as they are made, and that borrowers do not accumulate balances in the bank prior to loan repayment. Under these conditions, the volume of transactions which flows through the bank during any period of time, arises from the deposits and withdrawals of funds by its customers and from the pattern of grants and retirements of loans during the period. Only if the inflow of new deposits were exactly equal in size and timing to the withdrawal of funds, and only if the grants of new loans were conditioned by the date of repayment and
by the size of loans retired, could the bank operate without a transactions balance. The lack of synchronization of the flow of funds is a sufficient condition for the existence of the reserve asset portfolio.

Given the existence of the reserve asset portfolio, we are left with the more interesting problem of isolating the factors which determine its size. It is assumed for simplicity that \( R \) is composed only of cash. The only necessary assumption, however, is that reserve assets have lowest expected return, lowest risk, and highest liquidity of the three asset groups. It should be stressed that reserve assets need not, and typically are not, exclusively in cash form. They are assets which approach cash with respect to risk, return and liquidity. As mentioned in Chapter I, a brief discussion of the determinants of the composition of \( R \) is given in appendix C.

The representative bank has a given amount of funds at its disposal which it allocates among \( R \), \( I \), and \( L \). The proportion of total available funds allocated to the reserve asset portfolio is not technically fixed, but is a subject of economic choice. The volume and pattern of transactions does not automatically specify the size of \( R \). Just as the amount of cash in the transactions balance of interest theory is not uniquely determined by the level and pattern of transactions, the bank's total transactions balance is likewise not uniquely determined. Reserve assets must compete with other assets for a place in the bank's total portfolio. Funds allocated to the transactions balance earn an implicit
rate of return. This return arises from the costs saved the bank by holding cash rather than investment assets prior to disbursement of the funds. These costs primarily involve the unprofitable sale of investment assets to meet transactions needs, e.g., to meet a reduction in the level of demand deposits. The bank must weigh brokerage fees and other costs involved in shifts of funds back and forth from R to I against the interest income expected on investment assets.

The character of the bank's liabilities is such as to make the transactions balance of primary importance. Withdrawals of funds must be met immediately in the case of demand deposits and are normally so met in the case of saving deposits. The large number of such withdrawals coupled to a roughly equal number of deposits of funds tend to circumscribe the bank's ability to put idle funds into investment assets until they are needed for disbursement.

In a discussion of commercial banking, it cannot be realistically assumed that the future volume and pattern of transactions is known with certainty. The need to forecast the future volume and pattern of transactions, and the uncertainty which surrounds such forecasts alters and complicates the relationship between the volume of transactions and the desired reserve asset portfolio. In order to keep the discussion manageable, it is assumed that the level of loan demand is known and constant. As the bank knows the rate of loan repayment, the remaining source of variation in the volume of transactions is the level of deposits.
In a world of uncertainty, the bank must estimate the future number, pattern and volume of transactions. Such an estimate involves a forecast of the future number, pattern, and dollar amounts of both deposits and withdrawals of funds. Under these conditions, the reserve asset portfolio gains in importance. The portfolio not only provides a means of carrying out an estimated volume of transactions, it also serves as a buffer stock of liquid assets which insulates the other asset groups from unexpected variations in transactions. If the bank should experience an unexpected net loss of funds during any period of time, it is able to meet at least part of the loss from this stock. The use of the reserve asset portfolio in this capacity reduces (or eliminates, depending upon the size of $R$ relative to the loss of funds) the necessity of selling other assets under unfavorable terms. By a like token, if the bank should experience an unexpected net inflow of funds during any period,

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*It has no doubt occurred to the reader that the ability of the bank to borrow funds has not been mentioned as an alternative to selling earning assets to meet a deposit loss. Banks of course do borrow. The level of such borrowing is treated as a deduction from $R$. The impact on the bank of variations in the costs of borrowing are not treated in the main body of this study. Such costs are considered to be more important to the determination of the composition of $R$ than to its total size. It is assumed that variations in the costs of borrowing influence the allocation of reserve asset funds between, say, bills and cash but that they have little influence on the allocation of total funds between $R$ and the other two asset groups. The impact on the bank of variations in the costs of borrowing is treated explicitly in Appendix C.*
it will tend to view such an inflow as transitory in nature and simply hold it in cash form. It is generally not profitable to invest the funds in other assets if the inflow is strictly transitory.

If the movements of funds prove to be more than transitory in nature, the bank will revise its estimate of the future behavior of transactions and adjust R in accordance with this revised estimate. The use of the transactions balance as a buffer stock implies that the implicit return on R is not known with certainty. This return represents the saving to the bank of not making frequent and large movements into and out of other assets. When the future behavior of transactions is not known with certainty, the implicit return is likewise not known with certainty.

Before the analysis can turn to a discussion of the proportion of available funds devoted to reserve assets, it is necessary to develop a means of handling the expected volume of transactions. In the present context such a treatment involves an analysis of how the bank forms its conception of the future number, pattern, and dollar amounts of both deposits and withdrawals of funds during any period of time. In an effort to keep the analysis within the original framework, and in order to make it compatible with available data, it is necessary to relate the behavior of transactions to the level of deposits. This is most conveniently done by initially assuming that demand deposits are the bank's only source of funds, and hence its only liability. This assumption will be subsequently relaxed.
The problem is made manageable by making a series of related simplifying assumptions concerning the behavior of transactions. It should be pointed out that these are the same sort of assumptions made (either explicitly or implicitly) in analyses of the transactions demand for cash of interest theory. It is assumed that the number and total dollar amounts of deposits which the bank receives during any period of time are a positive function of the total level of deposits which prevail at the beginning of the period. It likewise assumed that the number and total dollar amounts of withdrawals during the same period are also a positive function of this level of deposits. Finally, it is assumed that the bank is only interested in the dollar volume of deposits and withdrawals made during the period. That is to say, in the sum of the values of individual deposits (number times average amount) and in the sum of the values of individual withdrawals during the period. This assumption implies that the bank is not interested in the pattern of deposits and withdrawals made during the period, only in the level of deposits which will prevail at the end of the period. If the period is considered to be the bank's reserve "week," this assumption is probably not too restrictive. By relating both the volume of deposits and of withdrawals to the level of deposits in this manner, a great simplification is accomplished. It should be stressed that these assumptions do not imply that total deposits and withdrawals need to be equal during the period; the actual level of deposits can change through time.
It is not sufficient to assume that the volume of deposits and of withdrawals during the period are simply positive functions of the level of deposits. Some further assumption must be made concerning the nature of this relationship. On the basis of somewhat inconclusive evidence, it is assumed that the relationship between the volume of transactions, as now defined, and the level of deposits is positive and more than proportional. A given rise in the level of deposits is associated with a more than proportional rise in the volume of deposits and of withdrawals. For example, the relationship might be of the form \( V = D^r \), where \( V \) is the volume of transactions (deposits and withdrawals), \( D \) is the level of deposits, and \( r \) is a constant where \( r > 1 \).

The basis for the assertion of such a relationship rests on the observed behavior of the rate of average turnover of demand deposits for various sets of commercial banks.\(^4\) The evidence suggests that bank debits rise faster than do bank deposits; the average rate of turnover of deposits has been increasing through time. Analyses of the transactions demand for money indicate that the transactions balances of businesses and of individuals should rise less than proportionally with the volume of transactions.\(^5\) In the current context, this implies that the rate of turnover of the bank's depositors' accounts should rise through time. The volume of transactions carried out by the customers increases through time but their average deposit balance need not rise in proportion. The observed behavior of the average rate of turnover of bank deposits provides
partial confirmation of this proposition. Given the assumption that the volume of bank transactions is positively and more than proportionally related to the level of its deposits, it is now necessary to discuss the bank's conception of the future behavior of deposits.

In determining the size of the reserve asset portfolio which it desires to hold at the end of any period, the bank must estimate the level of deposits which it expects to prevail at the end of that period. If it were to allocate funds to the reserve portfolio solely on the basis of the expected level of deposits, it would run the risk of having insufficient assets in the transactions account to meet unexpected net losses of funds. It would also have its portfolio balance disturbed if it received an unexpected influx of funds. Under these conditions, the bank would either find itself in the position of having to sell earning assets in order to meet an unexpected deposit loss or of having to make frequent adjustments in its portfolio in order to invest an unexpected increase in deposits. In either case the reserve account would fail to fulfill its function as a buffer stock. The bank must allow a safety margin for error in its forecasts, if R is to properly insulate the rest of the asset portfolio from unexpected variations in the level of deposits. The behavior of this safety margin is as crucial as the deposit forecast to the determination of the desired reserve asset portfolio.

It is assumed that the bank's experiences have prompted it to treat its deposit forecast as the midpoint (mean) of a range of possible deposit
levels. The bank is thus assumed to treat the future level of deposits as a random variable (the nature of its distribution is discussed below). The range of possible deposit levels includes those levels which the bank considers most likely to exist at the end of the period. For example, the bank may believe with, say, 95% confidence that the actual level of deposits will lie within some stated range. The deposit forecast is simply the mean of this range. The risk, as the bank sees it, that the actual level of deposits will differ from its expected value is approximated by the width of the confidence interval for any given degree of confidence. The wider the interval, the less confidence the bank places in the forecast. The reserve asset portfolio which the bank desires is assumed to vary positively with the risk of experiencing an unexpected change in the level of deposits. In a world of uncertainty, the desired R depends upon both the deposit forecast and upon an estimate that the forecast will be wrong.

The problem of attempting to relate the bank's conception of the most likely level of deposits and of the dispersion of possible outcomes around this level to some measurable variable is approached by assuming certain characteristics of the bank's deposit level. The desired reserve asset portfolio is expressed in terms of the expected value and the standard deviation of the future level of deposits. An estimate of the standard deviation of the level of deposits is used by the bank as a measure of the risk that the actual deposit level will differ from its
forecasted or expected value. It is convenient to specify a probability
distribution for the total level of deposits -- a random variable -- which
possesses certain desirable characteristics. The distribution must dis-
allow negative deposit levels which, for example, rules out the normal
distribution. It is also desirable that the distribution have the property
that the standard deviation of deposits is positively related to mean
deposit size. This second criterion is based on the assertion that when
the mean level of deposits is large, the bank views the likelihood of a
large deviation of the actual level of deposits from this mean to be greater
than when average deposit size is small. On the basis of the two criteria
discussed, the lognormal distribution is particularly well suited as a tool
for the illustration of the principles involved.

It is convenient to begin the discussion by assuming that the bank
has \( N \) depositors, each with a given size deposit at the beginning of any
period. It is further assumed that the total level of deposits at the end
of the same period is a random, lognormally distributed variable with
expected value \( \alpha \) and variance \( \beta^2 \). The basic properties of the
distribution are easily presented.\(^6\) Consider a random variable, \( x \),
developed over the range \( 0 < x < \infty \). If \( y = \log(x) \) is normally
distributed with mean \( \mu \) and variance \( \sigma^2 \), then \( x \) is lognormally
distributed. The mean of \( x \), \( \alpha \), is given by \( \alpha = e^{\mu+\frac{1}{2}\sigma^2} \), and the
variance, \( \beta^2 \), by \( \beta^2 = e^{2\mu+\sigma^2}(e^{\sigma^2}-1) = \alpha^2 \eta^2 \) where \( \eta^2 = (e^{\sigma^2}-1) \)
and $\eta$ is the coefficient of variation of the distribution. The distribution of $x$ is completely specified by the parameters $\mu$ and $\sigma^2$. The general shapes of the frequency functions $f(x)$ and $f[\log(x)]$ are given graphically as follows:

Values of $x$ are restricted to those strictly greater than zero, and $f(x)$ is skewed to the right. Values of $y = \log(x)$ are not bounded, and the distribution is normal.

Abstracting from seasonal and other systematic influences, it is assumed that the bank uses the current level of deposits as a measure of the deposit level which it expects to prevail at the end of the period; i.e., $E(x_{t+1}) = \alpha_{t+1} = x_t$ and therefore, $E(y_{t+1}) = \mu_t$, where $x$ now
denotes the total level of deposits. It is further assumed that with \( N \) fixed the bank considers the parameter \( \sigma^2 \) to be a constant for all \( t \).

This assumption implies that given percentage deviations of \( x \) from its expected value are viewed by the bank as having the same likelihood of occurrence no matter what the current level of \( x \) (or of \( \mu \)). It can be seen from the expression above that \( \eta \) depends only on \( \sigma^2 \) and, hence, is a constant under this assumption. As the standard deviation of \( x \) is given by \( \beta = \alpha \eta \), \( \beta_{t+1} = \alpha_{t+1} \eta = x_t \eta \). The bank considers the standard deviation of \( x \) in period \( t+1 \) to be proportional to the current level of deposits, \( x_t \). When the standard deviation of the logarithm of the level of deposits is a constant, the standard deviation of actual deposit size is linearly related to the deposit level.

Up to this point, it has been assumed that \( \sigma^2 \) is a constant. Such an assumption can be justified only when \( N \), the number of deposits, is fixed. Given \( N \), any variation in mean deposit level must arise from changes in the size of existing accounts. A doubling of the level of deposits, given \( N \), is equivalent to replacing each original individual account with two perfectly correlated accounts, each with a mean size equal to that of the original account. In this situation, there are no economies of risk; the standard deviation of deposits doubles.

When the assumption of a fixed number of deposits is relaxed, the strict proportionality between mean deposit level and the standard deviation
of this level disappears. In general, an increase in the bank's average deposit level in part represents an increase in the number of deposits. If these new deposits are not perfectly correlated with the old, or with each other, the standard deviation of deposit level will be less than proportionally related to mean level of deposits; $\sigma^2$ will decline. The contribution which these new deposits make to $\alpha$ and $\beta$ depends upon the average size and the standard deviation of each and upon the extent of correlation which they experience, either with each other or with the original deposits. Only if the new deposits possess perfect correlation with the old will changes in the bank's total deposit level which arise from changes in the number of deposits result in a proportional increase in deposit standard deviation. In this limiting case, variations in the level of deposits which stem from changes in $N$ are equivalent to equal changes in the size of existing accounts.

Changes in the bank's deposit level arising from changes in $N$ produce a less than proportional increase in aggregate deposit "risk". The standard deviation will vary proportionally with the average level of deposit only if variations in the total deposit level arise solely from changes in the size of existing accounts. Such a phenomenon is indeed unlikely to occur. The bank will view the relationship between the expected deposit level and its estimate of the standard deviation of this level to be less than proportional.

The reserve asset balance which the bank desires to hold at the end of any period depends upon the expected level of deposits at the end
of the period and upon the risk that the actual level of deposits will differ from the expected, i.e., upon the estimate of the standard deviation of deposits. The expected level of deposits determines the expected volume of transactions, under the assumption that the level of deposits and the volume of transactions are positively related. Analyses of the transactions demand for money indicate that the transactions balance should vary positively, but less than proportionally with the volume of transactions.\textsuperscript{7} Just as these analyses imply that there are economies in the use of the transactions balance, the discussion here implies that the commercial bank is also able to effect transactions economies. It has been asserted, however, that the volume of transactions which the bank carries out per period is more than proportional to the level of its deposits. The nature of the relationship between the desired reserve asset portfolio and the expected level of deposits depends upon both the relationship between the level of deposits and the volume of bank transactions and the relationship between the volume of transactions and the transactions balance. Unfortunately, it is impossible to accurately specify the functional form involved. For lack of anything better, it is assumed that the desired reserve asset portfolio is proportional to the expected level of deposits.

Reserve assets are also held as a buffer stock. In the face of a deposit forecast which is not wholly reliable, the bank is induced to hold additional reserve assets as a safety margin. This margin acts to insulate the loan and investment portfolios from unexpected deposit variation. It
has been shown under fairly general conditions that the bank's conception of the unexpected movements in the level of deposits will vary positively, but less than proportionally, with the size of deposits. Large deposit size afford economies of risk. The portion of reserve assets which represents a safety allowance will vary positively with deposit size. It is again not obvious what form the relationship will take. While the standard deviation of the deposit level is less than proportionally to the level of deposits, it is not clear that the standard deviation of transactions is less than proportionally related to the level of deposits. Here again, the inability to specify functional forms restricts the analysis. It greatly simplifies the empirical work if it is assumed that the portion of \( R \) which represents a buffer stock also varies proportionally with deposit size. Under this condition, the total desired reserve asset portfolio varies linearly with the level of deposits. While the bank itself experiences economies of transactions and risk, its customers experience the same sort of economies. Both the bank and its customers are able to expand their transactions balances less than proportionally with the volume of transactions. The economies of transactions of depositors enable them to expand their volume of transactions more rapidly than they need increase their transactions balances, i.e., their deposit levels. It is these economies of the customers which prevent the bank from achieving its own economies from large deposit liabilities.

The composition of the bank's demand deposits also influences the size of the desired reserve asset portfolio. Given the size of the bank's
demand deposits, the larger the proportion of highly active and/or erratic
deposits in the deposit total, the larger the desired reserve asset portfolio.
Such deposits tend to increase both the expected volume of transactions and
the variability of the volume. Conversely, the larger the share of highly
stable deposits the smaller the desired \( R \). Not only the size but also
the composition of deposits are instrumental to the determination of the
size of the bank's desired reserve asset portfolio.

It has been assumed that demand deposits comprise the only source
of funds available to the bank. Given that time deposits represent roughly
40% of the bank's total deposit liability, this assumption must be relaxed.*
With their generally greater predictability and their lower rate of turn-
over, these deposits exert a different influence on \( R \) than do an equal
quantity of demand deposits. As a given level of time deposits is associated
with a smaller volume of transactions than the same level of demand deposits,
the need to hold funds in highly liquid form is reduced; more funds are
available for use in the investment and loan portfolios.

* The capital account, the remaining element in \( F \), is omitted from the
analysis of this chapter on the empirical observation that it is not suffi-
ciently variable to have a significant influence on \( R \).
Given the size and composition of the total supply of funds, the desired reserve asset portfolio will vary negatively with the expected rates of return on investment and loan assets; and it will vary positively with their risks, i.e., risk of default and/or of capital loss.* The higher the return or the lower the risk on either of these two asset groups, the more willing will the bank be to economize on its transactions balance. The desired reserve asset portfolio will also tend to vary positively with the transactions costs involved in the purchase and sale of competing assets. Such costs involve the expenses of making frequent short-term adjustments in the investment asset portfolio. Frequent movements into and out of I can be expensive in terms of avoidable capital losses and brokerage fees. Such costs are discussed in more detail in the next chapter. The reader is reminded that the costs of borrowing are considered to influence the composition of R but not its total size.

In summary, the size of the desired reserve asset portfolio depends upon the expected size and standard deviation of the future volume of transactions, upon alternative returns and risks, and upon the costs of making frequent adjustments in the portfolios of earning assets. The highly

*It may be recalled that these assertions rest on the assumptions that both I and L are substitutes for R and that the bank's utility function is quadratic in return.
simplified analysis which has been carried out implies that such factors can be represented by the size and composition of the total supply of funds, by the expected return and standard deviation of return on investment and loan assets, and by the size of brokerage fees and other costs of transactions incurred by shifting funds among assets. The characteristics of the investment and loan asset portfolios must be introduced before the determinants of \( R \) can be more fully isolated. The introduction of these two portfolios is the subject of the next chapter.
III. THE INVESTMENT AND LOAN ASSET PORTFOLIOS

The investment asset portfolio is intermediate with respect to the other two portfolio groups in terms of expected return, * liquidity, and risk. Investment assets are assumed to be homogeneous assets whose expected return and risk are greater than those of reserve assets. Securities in the investment portfolio are marketable, but they lack the high liquidity of reserve assets. The existence of brokerage fees and of short-term variations in price render investment assets ill-suited for transactions purposes.

The existence of an organized market for investment assets suggests that the bank could conceivably use them for transactions purposes. It could invest any short-term inflow of funds in I and sell securities to meet deposit outflows. During the period that the funds are in I, they would earn the prevailing market rate of return on such assets. If this yield were to exceed the costs involved in frequent shifts into and out of securities, I would dominate the reserve asset portfolio as a transactions balance. In order to justify the exclusion of the investment asset portfolio

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*Some conception of the relative magnitudes of the expected rates of return on I and L is given in an interesting study of bank revenues and costs conducted by the Federal Reserve Bank of Boston. While the study deals only with relatively small banks, it does provide some indications of the orders of magnitudes involved. For the banks in the study, the actual net yield on loans for 1962 was 2.75% and the net yield on investments was 1.08% for that year. These yield figures are net of all cost including interest payments on time and savings deposits, and the costs of servicing demand deposits.
from conscious use by the bank as a transactions balance, it is necessary to demonstrate the conditions under which I could be used for this purpose.

There are definite costs associated with buying and selling investment assets. Only if revenues are expected to exceed these costs during any period of time will the bank use investment assets for purposes of conducting its transactions. Brokerage fees are a primary factor in the determination of the costs involved in frequent purchases and sales of investment assets. The existence of such fees makes the expected return on investment assets, in part, a function of the length of time they are held. The longer the securities are held, the less significant brokerage fees become in the determination of the net yield.

The issue here is the minimum length of time that investment assets must be held before they become a profitable use of bank funds. Only if this period is extremely short, e.g., a week or less, will it be profitable for the bank to use I as a transactions balance. This minimum length of time is called the holding period for investment assets. If the bank were to sell any investment assets before their holding period has elapsed, their rate of return would be less than the return on reserve assets. The concept of the holding period can be defined precisely.

The definition is given in terms of the holding period for an individual security in the investment asset portfolio. To facilitate the definition the following notation is used:
\( \bar{P} \): par value of the bond

\( P_o \): bid price of the bond at time of purchase

\( P_t \): bid price at time of sale

\( i \): annual coupon rate on the bond

\( s_t \): spread between the bid and ask prices

\( t \): time in years

\( r_I \): annual net yield on the bond

In the determination of the holding period, the bank must forecast the future yield on the bond. The expected yield on the security at time \( t \) is given by the following expression:

\[
E(r_I)_t = \frac{E(P)_t - E(s)_t + (iP)_t - P_o}{(P_o)_t}
\]

Rearranging terms yields,

\[
E(r_I)_t = \frac{iP}{P_o} + \frac{E(P)_t - P_o}{(P_o)_t} - \frac{E(s)_t}{(P_o)_t}
\]

The expected rate of return on an investment asset purchased at time \( 0 \) and held until time \( t \) is given by the sum of the coupon yield on the security, the expected capital gain or loss on the bond during the period, and the expected brokerage charge spread over the period. The holding period for an investment asset is defined to be the minimum length of time which the security must be held in order to equate its expected yield to
that of a reserve asset. Under the assumption that reserve assets are in cash form, the definition implies that the security need be held only until its expected yield equals zero.*

The shorter the length of the holding period, the stronger the incentive for the bank to hold investment assets for short periods of time and to sell them whenever resources are needed to meet an outflow of funds. In other words, the stronger the incentive to use I as a transactions balance. If, for example, it were necessary to hold investment assets for only one day before they began to earn a positive expected rate of return, there would be little or no need to hold a cash balance. Investment assets could be used profitably for even the most short-term transactions purposes.

It is not obvious that the holding period would ever be sufficiently short for the bank to consistently and consciously use I as a transactions balance. Aside from the influence of brokerage fees, the existence of uncertainty with respect to future security prices will also be a determining factor in the calculation of the length of the holding period. If the bank calculates the length of the holding period solely in terms of $E(P)_t$, it runs the risk that the actual price at time of sale will differ from the expected. The expected return upon which the holding period is based is not known with certainty. The belief that the actual return at

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*This statement is modified in the discussion which follows.
time \( t \) could differ from that expected will induce the bank to purchase investment assets only if their expected return exceeds zero sufficiently to compensate it for the risk of experiencing an actual return which is negative.

The possibility of incorrectly forecasting the future price of investment assets tends to extend the length of the holding period beyond the point at which \( E(r_I)_t = 0 \). Under conditions of uncertainty, reserve assets clearly dominate investment assets whenever the expected return on the latter is no greater than zero.* The holding period must be long enough to provide an expected return on investment assets which exceeds zero sufficiently to compensate the bank for the risk that the actual return at the end of the period will be negative. In attempting to evaluate the length of the holding period for investment assets, it should be recalled that the portfolio only contains securities whose maturity exceeds one year. Due to their longer maturity, investment assets are subject to relatively large short-term price fluctuations. Such variations in price make the estimation of the expected return on investment assets difficult even for short time horizons. Bid-ask differentials also tend to be larger and more variable for the longer-term securities.

Some conception of the length of the holding period is obtained by considering an example. Assume that the bank is considering the purchase

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*The risk aversion which induces the bank to insist on a positive rate of return in the determination of the minimum holding period explains the positive implicit yield on \( R \).
of a $1000, five year bond with a coupon rate of 4%. The bond will pay $1000 at maturity and the bid-ask spread is known and constant at $.25 per $100. Assume that the bank does not expect the price of the bond to change: \( E(P_t) = P_o = P \). Finally, assume that the bank is not concerned with the possibility that \( E(P_t) \neq P_t \). Under these conditions, the bank needs to hold the bond sufficiently long to allow the accumulated interest income to equal the brokerage cost incurred at sale, i.e., until \( E(r_I)_t = 0 \). If the bank sells the security before the end of the period, it expects to earn a negative return. Substituting the given information into the expression for \( E(r_I)_t \), we obtain \( t = .06 \) years for \( E(r_I)_t = 0 \). The holding period for the security is 25 days.

On the basis of this example, it certainly appears that the holding period for investment assets is too long for them to be used for transactions purposes. The existence of uncertainty with respect to the actual price which will prevail at the end of the period induces the bank to require an expected return greater than zero in the computation of the holding period. Even in the absence of such risk aversion, a holding period of 25 days is certainly too long for investment assets to serve as a transactions balance. It is asserted that the holding period for investment assets is too long to make them suitable for short-term transactions purposes. Positive brokerage

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*The value of the bid-ask differential upon which this example rests is representative of such differentials for bonds of 5-6 year maturity.*
fees and unexpected price movements tend to make the holding period for investment assets far too long for the bank to profitably replace R with I. The frequent and often large transactions which the bank must carry out imply that funds cannot be invested in I long enough for such assets to be used as part of the transactions balance. Frequent and large movements into and out of I would tend to reduce the net yield on this asset group below the yield on reserve assets.

It should be stressed, however, that unlike loans, investment assets are marketable. This gives I an important advantage over L. Investment assets do compete with reserve assets for funds. Investment assets certainly do not dominate reserve assets as a use of bank funds. Both have a place in the bank's total assets portfolio. The bank is not insensitive to variations in their relative rates of return. The shorter the holding period for investment assets, the greater the inducement for the bank to substitute I for R. The shorter the period of time necessary for investment assets to earn a positive expected rate of return, the smaller the risk that the bank will have to sell them prior to the end of the holding period. The bank is able to put more confidence in short-term deposit forecasts, and hence, is better able to judge the relative attractiveness of I if the holding period is short.

Up to this point, the analysis has been interested only in the determination of a planning horizon for the bank which is of minimum length. Such horizons have significance for the discussion of the possible use of I for purposes of transactions. The proposition that the bank will
consciously use investment assets as a balance to carry out the expected volume of transactions has been rejected. It is now necessary to discuss the bank's demand for investment assets in terms of longer horizons.

The expected return on \( I \) is partially dependent upon the length of the bank's planning horizon. If the only relevant investment period for the bank is the time to maturity of investment assets, their expected return is known with certainty. With such an horizon, the expected and actual yield is given by \( \frac{i P}{P_0} \). It is rather unlikely that the bank is only interested in current yield to maturity as implied by this horizon. Given that investment assets can be sold prior to maturity, the bank is interested in the expected return on \( I \) over shorter periods of time. The actual length of the bank's planning horizon depends upon its conception of the possible pattern of changes in the actual yields on the three asset groups from the present to the maturity of investment assets. If the bank considers it possible that it will be induced to shift from \( I \) to either of the other two portfolio groups, the behavior of the expected return and risk (estimated standard deviation of return) on investment assets becomes important.*

It is assumed that in the static environment being considered, the existing price of investment assets equals the expected price for all \( t \). The relevant planning horizon is assumed to be sufficiently long to render

*Concern over the size of its capital account through time will also induce the bank to use shorter horizons.
the influence of the bid-ask spread negligible. Under these conditions, the current yield on $I$ equals the expected yield. Even though $P_o = E(P)_t$, the bank is aware that the actual price at time $t$ can differ significantly from its expected value. The variability of actual security prices imparts a risk to the holding of $I$. The bank cannot know with certainty that the actual and expected yields on a security held from time $o$ to time $t$ will be equal. Perfect certainty exists only if the security is held from time $o$ to its maturity. If, for example, it is possible that loan demand will increase in period $t$, it is possible that $P_t$ will be less than its expected value and that the actual return on $I$, $(r_{I})_t$, will be small or even negative. Under these circumstances the bank would have been better off by simply holding reserve assets until loan demand rose. The possibility of conditions such as these induces the bank to hold investment assets only if their expected return is sufficiently large to compensate it for the risk that the expected and actual yields will differ.

In deciding upon the allocation of funds between investment and reserve assets which it desires to make at any point in time, the bank must compare expected transactions needs against the expected return and risk associated with holding $I$. Given these considerations, the desired allocation depends upon the bank's utility function. Under the assumption that the function is quadratic in return, and given expected transactions needs, the desired proportion of investment assets in a dollar of available funds tends to vary positively with the expected return on $I$ and negatively
with the estimated risk on these assets. A complete determination of the shares of R and I in available funds can only be specified after the loan portfolio has been discussed.

The Loan Portfolio:

The relative shares of the reserve and investment asset groups in the total asset portfolio are conditioned by the bank's decision to hold loan assets. Of the three asset groups, loans have the highest expected rate of return, the greatest risk, and the least liquidity. The market in which these assets are purchased differs drastically from the markets for reserve and investment assets. These features alone are sufficient cause for the isolation of the loan portfolio. The expected return on loans is, of course, conditioned by the interest rate charged by the bank. Such return tends to exceed the return on the other two asset groups. Of the three asset portfolios, only loans possess significant risk of default. Due to the relatively small capital account of a typical commercial bank, such risk is of crucial importance.  

The market for loans is highly imperfect. No organized, wide scale market exists for bank loans. Most loans involve negotiation between the bank and the borrower, and many loans are highly specialized in character. While some competition for new loans exists among banks, it is often imperfect indeed. Needless to say, the ability of the bank to dispose of any of its loans prior to their maturity is all but nonexistent except for a very small
portion of the portfolio.* The bank cannot acquire a seemingly endless quantity of loans at the existing market price. It is fully aware of its ability to influence the terms on which it grants loans. It is assumed, on the other hand, that the bank does not consider itself capable of influencing the prices of $R$ and $I$.

The essential features of the loan account are its relatively high rate of expected return, its comparatively large risk, and its very low liquidity. Under the assumption of the quadratic utility function, the share of funds allocated to the loan portfolio will vary positively with the expected net return on loans** and negatively with the bank's estimate of loan risk. Given the nature of the market for loans, the bank does not consider expected return and risk to be beyond its control. It is able to exert an important influence on both. Customer demand for bank loans is a decreasing function of the interest charge on loans. The expected return on new loans is thus conditioned by the number of new loans granted. The bank is also able to influence crucial characteristics of the loans it grants; characteristics which bear directly on the risk of the loans. Such factors as the size of each individual loan, average loan maturity, and the relative strength of loan guarantees all represent important elements in the bank's

*The reader is reminded that bank borrowing is excluded from the analysis. The only remaining means of disposing of existing loans is to sell participations to other banks. The ability to make such sales is limited.

**The effects of variations in the costs of granting and servicing loans on the size of $L$ are not explicitly treated in this study. Some partial evidence on such effects is available.6
conception of loan risk. These factors are as crucial to the bank's portfolio decision as the interest charge on loans. Loans are a matter of negotiation between borrower and lender in which both return and risk are determined. The interest rate set and the number of loans granted at this rate will depend upon both the bank's and its customers' views concerning loan size, maturity, and guarantees. The discussion which follows is essentially an adaptation of Donald Hester's analysis of commercial bank lending behavior to the framework of this study. 7

Other things being equal, the bank prefers high interest rates on its loans, relatively small average loan size, short maturities, and strong guarantees. Obviously, high interest charges correspond to high expected return on L. Relatively small loan size, per loan, tends to increase the bank's ability to diversify, and it reduces the strain on the borrower of repayment. The bank prefers relatively short maturities on its loans in the belief that the longer the maturity the greater the uncertainties involved. These uncertainties are associated with both default risk and with risks connected with the holding of an illiquid asset. The bank, of course, favors strong guarantees as these tend to directly reduce default risk. Borrowers on the other hand tend to prefer low interest charges, larger average loan size, longer maturity, and weak guarantees.

Hester's analysis suggests that it is useful to consider such factors as the interest rate on loans, average loan size, average maturity, and the strength of guarantees as comprising what he calls a set of loan terms. 8
If stringent loan terms are associated with high interest charge, small average loan size, short maturity, and strong guarantees, then it is possible to say that the bank bargains for stringent loan terms and the borrower for less stringent terms. The supply of loans is an increasing function of this stringency and the demand a decreasing function of loan term stringency. It should be stressed that the set of loan terms is essentially a vector. The lack of a common dimension prevents the elements in the set from being represented by a single number. Mester's empirical work indicates that to a certain degree both the bank and the borrower are willing to trade one element in the set for another in their negotiations.

In general it is asserted, however, that the more stringent the terms of lending, the greater the bank's desired loan portfolio, and the smaller the desire of individuals to borrow. Given the size and composition of its total supply of funds, and the return and risk on investment assets, the bank must decide on the size of the loan portfolio which it wants to hold. The bank's decision to hold loans is conditioned by its estimate of the credit-worthiness of loan applicants. The bank uses such characteristics as the size and composition of borrowers' assets, the size of their inventories, their sales, profits, etc. for this purpose. Given borrower characteristics and competing uses of funds, the bank will wish to hold more loans the greater the stringency of loan terms. Borrowers, on the other hand, are motivated by such factors, or characteristics, as expected future levels of sales and profits, costs of alternative sources of funds, etc. Other things being equal, borrowers will demand more loans when loan terms are less stringent.
That loans are the riskiest assets which banks hold, and that there are alternative uses of funds are sufficient reasons to make the supply of loans a positive function of the stringency of loan terms. Given the supply of available funds, the bank is willing to hold a larger loan portfolio only if the terms on these assets are made more stringent. The bank is willing to expand \( L \) only if the interest charge on loans is higher and/or the risk is lower. Given \( F \), the bank can expand \( L \) only at the expense of \( R \) and \( I \). Such expansion increases the risk associated with unexpected deposit losses and it increases default risk. Such increases in total portfolio risk will be accepted only if loan terms are sufficiently increased to compensate the bank for the added risk. Part of the increase in loan terms will probably represent a reduction in loan size, maturity, etc. Increases in the stringency of loan terms of this sort represent attempts to reduce the risk associated with the holding of \( L \).

It is likely that the supply of loans function is nonlinear. Given the bank's basic risk aversion, it will require ever increasing stringency of loan terms to induce it to add a constant increment to its loan portfolio. The bank's relatively small capital account coupled to ever present transactions needs should be sufficient cause for the supply function to be nonlinear, given \( F \). As the share of \( L \) in total assets rises, the bank becomes increasingly vulnerable to deposit loss and to loan default. Further increases in the share of loans in total assets should become increasingly difficult to induce. It is of course impossible to increase loans beyond the point at which all available funds are devoted to loans.
Given the characteristics of borrowers and of lenders discussed above, the actual dollar amount of loans and the terms on which this amount is issued are established by negotiation between the bank and its customers. The total volume of loans and the terms on which they are issued are simultaneously determined. An equilibrium exists when the demand and supply of loans are equal. Such an equilibrium implies a set of loan terms which equates the amount of loans the bank wishes to hold to the amount of loans borrowers wish to issue. It should be stressed that both the supply and the demand are for a stock of loans. The analysis is interested in the stock of loans which the commercial bank wishes to hold as assets and the stock of loans which borrowers are willing to owe to the bank during any period of time. The existence of equilibrium implies that there is no tendency for L to change. The issue of new loans equals the retirement of loan outstanding.

The discussion given so far implies that the shares of R, I, and of L in total assets are conditioned by the size of F, by its composition, by the expected return and risk on I, and by the stringency of loan terms. The actual size of the three portfolios is importantly influenced by the demand for loans. It is this demand which interacts with the supply function to determine the set of loan terms. The equilibrium values of the reserve, investment, and loan asset portfolios cannot be determined independently of the demand for loans.

Unfortunately, an adequate specification of the demand for loans appears to be impossible. Very little is known about the general features
of the demand for bank loans. Such a demand function is of necessity very complex. An adequate specification of the function would involve a description of the behavior of most, if not all, of the economy. The demand for bank loans emanates from many important sectors of the economy. The specification of the demand function not only requires the estimate of such factors as the existing level of aggregate sales and profits in the relevant sectors, but more importantly, it requires estimates of the future levels of these variables. The demand for loans is not uniquely determined by the future levels of these factors, however. Information must be obtained on both the costs of alternative sources of funds and borrowers' attitudes concerning these sources. The relationship between the aggregate level of future profits, and allied variables, and the demand for bank loans will be conditioned by these alternative sources of funds and their relative costs.

Rather than attempt to specify the demand for bank loans function, the bank's supply function and the equilibrium value of L can be estimated by means of an approximate identification of the supply function. The argument which follows is essentially the same as the analysis of approximate identification presented by E. J. Working.\textsuperscript{10} The supply of loans function has been specified in terms of F, its composition, the expected return and estimated risk on I, and the set of loan terms, T. The only factors to which loan demand has been specifically linked are those in T. Obviously, loan demand depends upon many factors other than the stringency of loan terms. Such factors as expected future profits, sales, etc., are
often much more important to borrowers than the terms on which they can obtain loans from the bank. The loan demand function is far from being completely specified. An observed relationship between the set of loan terms and the size of the bank's loan portfolio represents the locus of points of equilibrium between the demand and supply functions. This locus will approximate the bank's supply function if the variance of the stochastic term of the supply function is small compared to that of the demand function. Under this condition, the "shifting" demand function will trace out the shape of the "stable" supply function in the space defined by the factors specifying the supply function. This condition is certainly met in the case at hand. The unexplained variance of the supply function is surely much smaller than the unexplained variance of the demand function. The latter has been specified only in terms of \( \tau \). This being the case, a more complete specification of the demand schedule is unnecessary for approximate identification of the supply function.

The unobserved level of loan demand interacts with the supply function to produce an observed \( \bar{L} \) and an observed set of loan terms, \( \tau \). The "stability" of the supply function relative to demand enables us to discuss the supply function without actually observing the level of loan demand. Given the other factors which influence the supply of loans, stringent loan terms are associated with a high level of loan demand and a relatively large observed \( \bar{L} \). Conversely, less stringent terms imply a lower level of demand and a smaller observed \( \bar{L} \). The ability to observe the actual loan portfolio and the set of loan terms is sufficient to "identify" the supply function.
The discussion of the determinants of the desired size of the reserve, investment, and loan asset portfolios in the static case is now essentially complete. The specification of the supply function in terms of observed $T$ and $L$ immediately implies that the bank's desired reserve and investment asset portfolios are also specified. The interdependence of the three portfolio groups implies that the stringency of loan terms also exerts its influence on $R$ and $I$, via their competition with $L$. Other things being equal, an increase in the stringency of loan terms (increase in the level of loan demand) will induce the bank to shift funds from $R$ and $I$ into $L$.

The relationships which have been developed for the static case are easily presented in summary form. To facilitate the presentation, the following notation is used:

- $D$: Total demand deposits*
- $D_v$: Particularly active and/or erratic demand deposits
- $D^*$: Demand deposits other than $D_v$
- $T$: Total time deposits
- $C$: Capital account
- $E(r_I)$: Expected yield on investment assets
- $\sigma_I$: Estimated risk of return on $I$
- $\mathcal{T}$: The set of loan terms.

*All deposit items are, as before, considered net of required reserves.
Recalling the identities

\[ F = D' + D_v + T + C \]
\[ = R + I + L, \]

the desired level of each of the portfolio groups is given by the following expressions:

\[ R = f_1(D', D_v, T, C, \text{E}(r_1), \sigma_1, \gamma) \]
\[ I = f_2(D', D_v, T, C, \text{E}(r_1), \sigma_1, \gamma) \]
\[ L = f_3(D', D_v, T, C, \text{E}(r_1), \sigma_1, \gamma) . \]

Brokerage fees are omitted in the interest of simplicity.

Under the assumption that the three asset groups are substitutes, the desired levels of the three portfolios are determined by the same set of factors. It is obvious that given \( F \), only two of the three groups need be specified; the third equals the difference between \( F \) and the sum of the other two. Given the interdependency of the three groups, the portfolio eliminated is arbitrary. No attempt is made at this point to specify the actual form of these functions. Functional forms will be specified after bank behavior in the dynamic context is discussed. Such a discussion is the topic of the next chapter. A testable model is presented in Chapter V. As the next chapter will indicate, it is necessary to make some fundamental changes in the functions to render them testable.

Before turning to the dynamic case, some mention must be made of several special characteristics of the loan portfolio. Because no organized
market exists for the purchase and sale of loans, the management of the loan portfolio is more difficult and complicated than the manipulation of the reserve and investment asset portfolios. When the market price of investment assets rises, all the securities in this portfolio immediately experience an increase in value. It has been asserted that the bank will seek to increase the size of I if its expected net yield rises. The existence of a market for these securities allows the bank to experience an immediate gain on the existing assets in the portfolio, and it need only add a relatively small increment to the portfolio to regain equilibrium. Such is not the case with loans. If the expected return on loans rises and/or the estimate of loan risk falls, the bank can experience the gain only on new loans. Only those loans which are retired during the relevant period provide a means of experiencing an immediate gain from a loan portfolio which is fixed in dollar amount. Aside from the rate of loan repayment, the bank can realize the gain on loans only by reducing R and I. This can lead to a rather complicated sequence of events. If the bank suddenly finds itself in the position of being able to grant loans on more favorable terms, it will relend funds obtained from loan repayments and it may also reduce R and I below their final equilibrium values. The dollar size of L will swell as the bank attempts to obtain more of the possible gain. As loans continue to be repaid, the bank will use part of the funds to replenish its depleted R and I portfolios. It is able to achieve the higher average return on L with a smaller dollar investment in the portfolio. Thus one may observe a large reduction in R and I and a comparable rise in L followed by a
rather gradual increase in the dollar size of \( R \) and \( I \) and fall in \( L \) until their new equilibrium levels are achieved. These levels may well be quite different from their initial disequilibrium values.

A second complication also arises from the lack of an organized market for loans. There is an upper limit to the rate of reduction of \( L \). If the bank finds itself in the position of having to grant loans on less favorable terms, it will seek to reduce \( L \). It cannot reduce its loan portfolio at a rate greater than the rate of repayment of loans. In order for the empirical analysis to be carried out in Chapter V, it is necessary to make some assumptions concerning the features of \( L \). In particular, it is assumed that the average maturity of the loan portfolio is sufficiently short to allow a substantial proportion of total \( L \) to be retired each period. Obviously, the shorter the time period, the more tenuous this assumption becomes. The greater the rate of loan repayment per period, the less important these adjustment problems become. The analysis implicitly has assumed that variations in the attractiveness of loans, relative to \( R \) and to \( I \), are never great enough to prevent the bank from using the funds liberated by loan repayment as the adjustment factor. Available evidence does suggest that the average maturity of bank loans is quite short.*

This evidence implies that possible rigidities in the management of the bank's loan portfolio are not as great a problem for the analysis as one might at first expect. As the next two chapters will indicate, rather simple assumptions are made concerning the existence of lags in the bank's portfolio adjustments.

*The Federal Reserve System's 1957 survey of business loans indicates that over 60% of the total dollar value of member bank loans to business have a maturity of one year or less. The dollar value of such loans maturing during short periods of time is rather large.
IV. SOME COMPLICATING FACTORS

The functional relationships summarized in Chapter III must be modified and their forms made explicit before it is possible to develop a model of bank behavior which is amenable to empirical test. The purpose of this chapter is to introduce and discuss some complicating factors which importantly influence the specification of the final model. These complications arise from an inability to identify the supply and demand functions for bank loans and from the existence of cyclical patterns of movement in the levels of loan demand and of bond prices. The discussion which follows first treats the modifications of the static model which arise from the inability to identify the underlying relationships, and it then turns to the dynamic problems.

In Chapter III, the supply function for bank loans was "identified" by relating the size of the loan portfolio to the set of loan terms \( T \). The approximate identification of the supply function through \( T \) was considered to be necessary given the inability to observe the demand for loans function for either an individual bank or for any number of banks. Unfortunately, it is not possible to observe the set of loan terms either. The empirical work of the next chapter utilizes time series data. Information on the components of the set of loan terms is not available in the form of time series.* The only component which is at all accessible is the interest

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*Hester was able to obtain cross sectional data on some of the components of \( T \) by interviewing bankers and by utilizing existing survey data.\(^1\)
rate charged on loans by various commercial banks.\textsuperscript{2} The work of both Hester and Guttentag strongly implies that the interest rate charged on loans is a poor proxy variable for the total set of loan terms.\textsuperscript{3} The short-term inflexibility of the interest rate on loans implies that banks use other means to adjust their loan portfolios.

The discussion up to this point has been conducted as if the set of loan terms could be observed. This was done to illustrate the theoretical issues involved. Banks do attempt to ration the volume of credit they extend. The inability to observe the rationing devices does not argue against their existence. Stress has been put on $\mathcal{T}$ and upon the ability of this set to provide approximate identification of the supply of loans function. It appears that a meaningful solution to the problem of identifying the supply function lies in the development of a usable series for $\mathcal{T}$. No such estimates so far exist.

It is shown below that without information on the set of loan terms, identification of the supply of loans function is impossible. The inability to identify the function exists even if it is possible to accurately specify a demand for loans function. This writer has attempted elsewhere to identify the supply function indirectly through the use of a proxy variable for $\mathcal{T}$.\textsuperscript{4} I am now convinced that the use of the proxy creates more problems than it solves, and it is not used here. By rejecting the use of the proxy variable, it must be concluded that identification of the bank's supply of loans function is impossible without information on $\mathcal{T}$. 
The actual size of the bank's loan portfolio at any moment of time cannot be determined independently of the demand for bank loans. This, of course, implies that the size of the other two portfolio groups likewise is not determined. It is necessary to attempt to specify a demand for loans function. While the function to be introduced is a highly simplified one, the stringency of loan terms must be included. It is the dependence of both the demand and the supply of loans on \( T \) which prevents the identification of either function.

The demand for bank loans is conditioned by two sets of factors. The first is comprised of borrowers' conceptions of the future values of their incomes, profits, sales, etc. There is an incentive to borrow only if the funds so obtained can be put to productive use in terms of financing the purchases of such items as inventories and capital equipment. The demand for such investment is conditioned by the expected future levels of sales and profits. The second set of factors represents the relative costs of obtaining funds from various sources. The individual bank often represents only one of several possible sources of funds to the borrower. The demand for bank loans depends upon the attractiveness of such loans relative to other sources. Such attractiveness is represented by the stringency of the terms of bank loans relative to the terms of borrowing from other sources.

In order to present testable relationships, it is necessary to make some simplifying assumptions concerning the determinants of the demand for bank loans. In particular, it is assumed that the current level of economic activity determines the profit and sales expectations of borrowers. The
higher the level of aggregate income, the greater actual and expected sales and profits; and the greater the demand for bank loans. The second assumption deals with the influence of alternative costs of funds. It is assumed that the relative costs of borrowing do not change. When the stringency of bank loan terms rises, the terms of borrowing from other sources rise in proportion. This assumption permits the exclusion of alternative costs of funds from the analysis.

In the interest of simplicity of exposition, it is assumed that the stringency of loan terms can be represented by a single element in the set. The actual element selected is arbitrary. It is convenient to use the rate of interest on bank loans as the representative of the stringency of loan terms. No explicit use is made of the interest charge, or of any other loan term, in the model to be developed. The contention made previously that the interest charge is a poor proxy for \( R \) still holds. The interest rate is considered to be unobservable, and it is eliminated from the expressions below. The argument is greatly simplified if an abbreviated version of the supply of loans function is used. The use of this form does not alter the conclusions to be drawn.

The supply and demand equations which follow are taken to be linear in their arguments. Linear relationships are used solely for purposes of simplicity of exposition. As the analysis will soon indicate, the supply of loans function is not linear in loan terms. In the present context, this implies that the bank expands the quantity of loans supplied less than proportionally to a given change in the interest rate charged on its loans.
The specification of the simplified model uses the following variables:

- \( L_s \) : the dollar value of loans supplied per unit of time
- \( L_d \) : the dollar value of loans demanded per unit of time
- \( L \) : the observed dollar value of the bank's loan portfolio
- \( F \) : the total supply of funds
- \( Y \) : an aggregate income or sales variable
- \( r \) : the interest charge on new loans
- \( i \) : the market yield on investment assets
- \( u \) and \( v \) : stochastic terms

These variables are combined to yield two structural behavioral equations and an identity:

\[
L_d = a_1 - a_2 r + a_3 Y + u
\]

\[
L_s = b_1 + b_2 r + b_3 F - b_4 i + v
\]

\[
L_s = L_d = L .
\]

The interpretation of these expressions is fairly straightforward. The dollar value of loans demanded varies negatively with the interest cost of borrowing and positively with the level of economic activity. The supply of loans equation is an abbreviated and slightly modified version of the function described in Chapter III. Here, the dollar value of loans supplied varies positively with the interest charge on loans, positively with the supply of funds available to the bank, and negatively with the yield on investment assets. Under the assumption that the bank expects no change in the prices
of investment assets, the market yield equals the expected return on \( I \). The third expression is an identity which states that the volume of loans demanded equals the volume supplied.

It should be noted that the aggregate income variable appears only in the demand equation. It is not obvious that the supply of loans function is unaffected by \( Y \). It is quite conceivable that the bank's conception of loan default risk is influenced by the general level of economic activity. Under this condition, the supply of loans varies positively with \( Y \). The bank is willing to supply a greater volume of loans at any rate of interest the higher \( Y \). As the discussion below will indicate, it is not possible to disentangle the influence of the supply of loans from that of the demand in the determination of the observed loan portfolio, \( L \). The inability to identify the supply function implies that the exclusion of \( Y \) from \( L \) is not essential to the present discussion. Such an exclusion simplifies the analysis and is retained.

The interest charge on new loans, \( r \), is used here as a proxy for \( \Gamma \); it is considered to be unobserved. It is possible to eliminate \( r \) from the system of equations and to obtain a "reduced form" equation which relates the size of the loan portfolio to the observed exogenous variables \( F \), \( Y \), and \( i \). An expression for \( r \) is derived from the demand equation. Substituting this expression into the supply equation, and combining it with the identity yields an expression for the observed size of the loan portfolio of the general form:

\[
L = A_1 + A_2 Y + A_3 F - A_4 i + Z,
\]
namely,

\[
L = \left[ a_1 + b_1 + \frac{a_2 b_1}{a_2} + \frac{a_2 b_1}{b_2} \right] + \left[ a_3 + \frac{a_2 b_2}{a_2} \right] Y + \left[ b_3 + \frac{b_2 a_2}{b_2} \right] F
\]

\[
- \left[ b_4 + \frac{b_4 a_2}{b_2} \right] i + \left[ \frac{b_2}{a_2} \right] u + \left[ 1 + \frac{a_2}{b_2} \right] v .
\]

The observed size of the loan portfolio varies positively with the level of aggregate income, positively with the size of the bank's total resources, and negatively with the yield on investment assets. The size of the portfolio is determined both by the supply and by the demand for bank loans. The separate influences of these two functions on the observed \( L \) could be determined only if it were possible to observe \( \nabla \), which for simplicity has been represented by \( r \). The set of loan terms cannot be observed; this rules out the possibility of identifying the structural equations. The structural parameters \( a_2 \) and \( b_2 \), which enter into the determination of all the coefficients of the reduced form, cannot be identified.

The coefficients of the reduced form equation represent a mixture of the influences of the supply and demand functions. The parameter \( a_2 \), the interest rate parameter of the demand for loans equation, enters into the determination of all of the reduced form coefficients. The role of this parameter in the reduced form demonstrates the importance of the set of loan terms in the determination of the loan portfolio. Any observed relationship between \( L \) and the exogenous variables \( Y \), \( F \), and \( i \) will mirror the influence of \( \nabla \) on the unobserved demand and supply functions.
Up to this point, the relationships in the structural equations have been assumed to be linear. This assumption was made to simplify the introduction of the general reduced form expression. Very little, if any, information can be offered concerning the form of the demand for loans function. Given the extreme simplicity and the rather artificial nature of this function, it appears that little further damage can be done by retaining the assumption that the function is linear. Rather more can be offered concerning the form of the supply of loans function. On the basis of the analysis of the last two chapters, some observations can be made concerning the actual form of the function. The discussion of the relationship between the volume of the bank's transactions and the size of the reserve asset portfolio given in Chapter II implies that the relationship between \( L_s \) and \( F \) is proportional. Given the relatively low risk associated with acquiring investment assets, it is asserted that the relationship between the supply of loans and the yield on investment assets can be accurately considered to be linear.

No such assertions of linearity can be made for the relationship between \( L_s \) and \( r \). In Chapter III, it was argued that the bank will expand the volume of loans it supplies less than proportionally to a given increase in the stringency of loan terms. In the present context, this implies that the relationship between the dollar value of loans supplied and the interest charge on loans is less than proportional. Given the size of its resources, \( F \), an increase in \( L_s \) represents a rise in the share of loans in total assets. Such a rise reduces bank liquidity and increases portfolio risk. The relatively small size of its capital account induces the bank to
have a strong aversion to such risk. The higher the loan rate of interest (stringency of loan terms) the larger is \( L \), and, given \( F \), the smaller is the bank's incentive to further increase the volume of loans for a given rise in \( r \).

The general shape of the bank's supply of loans function is shown in figure 1 for conditions of given \( F \) and \( i \). As \( r \) rises, the quantity of loans supplied approaches \( L = F \) asymptotically.

![Figure 1](image-url)

An increase in the total supply of funds or a decrease in the yield on investment assets will induce the bank to supply a greater volume of loans at every rate of interest; the curve shifts to the right. Obviously the curve shifts to the left if \( F \) decreases or \( i \) increases.*

*The response of \( L_s \) to changes in \( F \) and \( i \) is subject to the restriction that \( L \) is defined over the range \( 0 \leq L \leq F \).
The shape of the supply function implies a less than proportional relationship between the observed size of the loan portfolio and the level of aggregate demand in the reduced form equation. Such non-proportionality is illustrated by observing the locus of equilibria of the demand and supply functions in figure 2.

Each demand schedule is drawn under the condition of a given level of aggregate income, \( Y \). The curve labeled \((L_d)_1\) represents the relationship between the dollar value of loans demanded and the loan rate of interest, given the level of income, \( Y_1 \). Similarly, the demand schedules \((L_d)_2, \ldots, (L_d)_5\) represent the same relationship at the higher levels of income \( Y_2, \ldots, Y_5 \). The higher the level of income, the greater the quantity of loans demanded at any rate of interest.
The shape of the supply function implies that the observed loan portfolio will respond less than proportionally to a given change in the level of aggregate income (a given shift in the level of loan demand). The larger $Y$, and hence the larger $r$, the smaller the response of the loan portfolio to a given change in the level of income. The asserted relationship between $L_s$ and $r$ is sufficient to obtain this response of $L$ to variations in $Y$. The information in figure 2 is represented in a different form in figure 3. Given $F$ and 1, the locus of equilibria of the fixed supply schedule and the family of demand schedules is as follows.

*This is the case under the assumption that the supply of loans is not influenced by the level of aggregate income. If the dollar value of the loans supplied were considered to depend upon the level of $Y$, it would be necessary to assume that a given change in $Y$ exerts a greater influence of $L_d$ than on $L_s$. This assumption would be sufficient to provide the conclusion made in the text. It is certainly true that as the share of total assets devoted to loans approaches unity, any influence of $Y$ on $L_s$ would be small.
The relationship between $Y$ and $L$ represents the 'reduced form' of the demand and supply equilibria. As $Y$ expands, the observed size of the loan portfolio rises with it; and the share of loans in total assets approaches unity asymptotically. The shape of the curve in figure 3 follows directly from the points of equilibrium of the demand and supply schedules in figure 2.

Given that the reserve and investment asset portfolios are assumed to be substitutes for $L$, the relationship between $L$ and $Y$ immediately implies relationships between $R$ and $I$ and the level of income. The higher $Y$, the greater the expected return on the loan portfolio relative to the returns expected on $R$ and $I$. An increase in $Y$ induces the bank to shift funds from $R$ and $I$ into loans. The implied relationships between the level of income and the dollar sizes of the reserve and investment asset portfolios appear in figures 3a and 3b below.
The discussion of the static characteristics of the model is now essentially complete. It has been shown that the inability to observe the set of loan terms presents new problems for the analysis. In particular, it necessitates the introduction of a demand for loans function into the discussion -- a hazardous task at best. A highly simplified and rather artificial demand function has been specified so that the analysis could continue. In the absence of a series for $T^r$, this demand function only provides a means of obtaining a reduced form expression for the observed dollar size of the loan portfolio.* Identification of the coefficients of the supply and demand functions is impossible. Finally, it has been argued that the relationship between $L$ and $Y$ tends to be less than proportional.

Before turning to a statement of the model to be tested, it is necessary to introduce some dynamic factors which exert an important influence on the relationship between $L$ and $Y$. The dynamic complications arise from the interaction of the cyclical movements in the level of income and the prices of investment assets.

Some Dynamic Factors:

Up to this point, the analysis has been essentially static. The existence of uncertainty has been introduced by framing the discussion in

*Obviously, the dollar size of $R$ and $I$ also depend upon $L_d$. This requires the use of reduced form expressions for these portfolio groups as well.
terms of the expected value and the estimated standard deviation of both asset returns and of future deposit levels. Subjective probability beliefs, however, have been assumed to be given and constant. The discussion has been interested in specifying the determinants of the relative shares of R, I, and L in total assets. To facilitate the analysis, it was assumed that these determinants were constant. Under these conditions, it was obviously not possible to discuss bank portfolio adjustments through time. While the static analysis is useful for a discussion of the effects of infrequent, discontinuous changes in the determinants of R, I, and L, it is of little value in the analysis of bank reaction to frequent and continuous changes in these factors. Cyclical variations in the level of income and in the prices of investment assets are of particular interest in this context.

It is convenient to analyze the influence of fluctuations in the level of income and in the prices of investment assets in terms of the structural equations for $L_s$ and $L_d$. There is obviously a resultant response of the reduced form relationship to fluctuations in these factors. As specified, variations in the level of income can influence only the demand for loans function, and movements in the prices of investment assets can influence only the supply function. In the discussion which follows, the bank is described as responding to changes in the "level of loan demand." Fluctuations in this level are to be interpreted as variations in the quantity of loans demanded, per unit of time, in response to movements in the level of aggregate economic activity, $Y$. 
In the static case, changes in such factors as the level of loan demand, the total supply of funds, and the expected return on investment assets are considered to be permanent. The bank has no reason to believe that a movement in any of these factors will be followed by further movements in the same direction. While deposits, bond prices, and loan demand are subject to random movements, the bank has been conceived to consider their current levels to the levels expected to prevail at the end of the period. The expected change in these factors in this case is zero. The bank is aware, of course, that their actual levels can differ from the expected. If, for example, the level of income should decline during any period, fewer loans will be demanded at the existing terms of lending. The bank must decide how it can best live with a permanently less attractive loan portfolio. It will increase \( R \) and \( I \) in accordance with the expectation that loan demand will not change again. Once the adjustments are complete, the actual reserve and investment portfolios will be increased to their desired levels and \( L \) will be reduced to its new, permanently lower level. Barring any further change in the level of income, and resultant change in \( L_d \), the bank has no incentive to alter the composition of the total asset portfolio. In the absence of any previous pattern in the movements of \( L_d \), the bank has no reason to believe that any given change in the level of loan demand will be followed by further change. In this sort of environment, the bank does not look more than one period to the future; it has no reason to believe that loan demand will change again.
This is not the sort of environment in which the bank finds itself. The levels of income and of bond prices (bond yields) have demonstrated an historical tendency to move continuously through alternating periods of expansion and contraction. Put more simply, they have been subject to cyclical fluctuations. While these cycles have differed markedly in period and amplitude, they share the properties of being both cumulative and self-reversing. The levels of income and of bond prices have demonstrated a recurring tendency to alternate between expansion and contraction.*

In the light of this experience, it is highly unlikely that the bank will not take the possibility of further cyclical movements in loan demand and bond prices into account in the formation of its expectations. Just as the bank is aware of seasonal movements in the levels of loan demand, deposits, etc., so too is it aware of possible cyclical patterns in these variables. In a dynamic world, the bank's planning horizon is necessarily lengthened. The past becomes a useful guide to the future.

If the level of loan demand should rise during any period, and if it has previously expanded for several periods, the bank will expect further increases in this variable. Movements in the level of income and in bond prices tend to be cumulative; a rise in their levels is most likely to be

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*It must be stressed that these cyclical fluctuations are not conceived to have the predictable features of, say, a sine wave. The levels of income and of bond prices have not demonstrated any historical tendency to yield cycles of constant period and amplitude. Given the variability of the cyclical patterns in income and bond yields, the actual course which future movements in these variables will take is not known with certainty.
followed by further increases. The same argument obviously holds in reverse
during cyclical contractions. Declines in loan demand and in bond prices are
likely to be followed by further declines. While these movements tend to be
cumulative, they are also self-reversing. This second feature of the movements
in loan demand and in bond prices should lead the bank to believe that neither
the level of loan demand nor the level of bond prices will permanently main-
tain their current direction of change. It believes that the longer loan
demand and bond prices have been moving in the same direction, the greater
the likelihood that they will reverse their direction of movement in the near
future.* The assumption of this sort of expectations is most applicable to
a situation in which cycles have been of fairly short duration. Such has
been the case with post-war cycles in the levels of income and bond prices in
the United States.7 These are the cyclical variations which are relevant to
this analysis.

In the context described, it is useful to consider the bank's planning
horizon to encompass a full cycle in the level of loan demand and/or bond prices.

*This second assumption implies that the bank views the length of time from
the beginning of the current period to the turning points in the levels of
loan demand and of bond prices as random variables which it treats in terms
of a set of subjective probability beliefs. In particular, it implies that
the longer the period of time during which these factors have been expanding
(contracting), the greater the likelihood that a turning point will be reached
at the end of the period. The assumption of such a set of probability beliefs
follows from the assertion that the bank expects cyclical movements in loan
demand and bond prices to continue in the future. Bank behavior in terms of
these probability beliefs is not formally treated in the analysis. While such
beliefs are implicit in the discussion which follows, they are not introduced
with sufficient precision to allow one to empirically test for their existence.
It realizes that the income from its loan portfolio tends to be low in recession and high in prosperity. It also realizes that fluctuations in the prices of investment assets exert an important influence on the return it will earn on its investment asset portfolio. The bank must live with the cyclical fluctuations in these exogenous variables. A reasonable way to live with them is to take them into account in the formation of its decisions. Observed short-term adjustments in the composition of the bank's total asset portfolio represent attempts to manage the portfolio in accordance with the longer horizon.

The desired composition of the bank's total asset portfolio at any moment of time is importantly influenced by its expectations concerning the future cyclical movements in loan demand (income) and bond prices. These arguments must be made more precise. This is accomplished, in part, by modifying the static model. It is difficult to explicitly introduce the dynamic elements into the analysis with any degree of precision. The alterations which are made to the static model are qualitative in nature. Intuitively, it does not appear impossible to put the expectational factors discussed into rigorous form. While the complexities involved have prevented such a treatment here, such an analysis appears to be well worth attempting.

The basic premise on which the discussion of this section rests is that the bank is able to determine the current phase of the cycles in the level of loan demand and in bond prices. It knows when the level of loan demand is in a phase of general expansion (contraction), and it knows when the level of bond prices is experiencing a period of general increase (decrease).
Given the bank's ability to determine the current phase in the cycles in these two variables, it will expect the level of loan demand and of bond prices to continue to decline if they are in a contraction phase, and it will expect them to continue to rise if they are in an expansion phase. Such knowledge of the phase of the cycles prevents the bank from being fooled by transitory (random) reverses in the direction of movement of loan demand or in bond prices.

The analysis begins by considering bank reaction to cyclical fluctuations in the level of loan demand. In this discussion, the level of bond prices (and, hence, bond yields) is held constant. Once this reaction has been traced out, bond prices are allowed to vary along with loan demand. At the beginning of any period, the bank must decide upon the allocation of funds among the three asset groups which it desires at the end of the period. Such a decision necessitates forecasts of changes in relevant factors which will occur during the period. Given the movements in the level of loan demand, the size of $F$, etc., expected to occur during the period, the bank will determine a set of loan terms which will allow it to achieve the desired portfolio mix. In order to facilitate the discussion, it is assumed that all factors except the level of loan demand are constant. This assumption implies that the bank's subjective probability beliefs concerning the size

*It is assumed in this chapter that the bank is able to adjust the dollar size of the components of the total asset portfolio with sufficient speed to achieve the desired portfolio mix at the end of the period. The shorter the lapse of time between the beginning and the end of the period, the more tenuous this assumption becomes. The possibility of rigidities and frictions in the adjustment of $R$, $I$, and $L$ to their desired levels is discussed in Chapter V.
and composition of \( F \) and the return on \( I \) do not change. Their current levels represent the levels expected to prevail at the end of the period. Under these conditions, only variations in the level of loan demand resulting from changes in \( Y \) can induce the bank to alter the current composition of its total asset portfolio.

Consider a situation in which the level of income has just begun its expansion phase, i.e., it has recently passed its lower turning point. This situation corresponds to the level of loan demand \( (L_d)_1 \) in figure 2. The bank is aware of the turning point and expects the level of loan demand to continue to rise for some time. During the previous contraction in \( Y \), the level of loan demand had declined and the bank reduced the relative share of loans in total assets, i.e., there was a movement down the supply function of figure 2. This decline induced a corresponding increase in the shares of \( R \) and \( I \) in total assets. The large dollar size of \( R \) and \( I \) relative to total assets implies that the marginal expected returns from these two asset groups is relatively low. Even though loan terms are not stringent (a small value of \( r \) in the diagram), the reserve and investment asset portfolios provide relatively weak competition for funds. The implicit return on \( R \) is low; the portfolio is sufficiently large to provide a very high degree of liquidity. The diversification potential of the investment asset group has been exploited along with its potential as a possible source of liquidity.

Under the conditions described, a shift of funds into loans is attractive to the bank even without a substantial increase in the stringency of loan terms.
During the early periods of expansion of the level of loan demand, the bank is willing to increase the volume of loans it supplies rather freely. A given increase in the level of loan demand will produce a relatively large increase in the size of $L$. As the expansion in loan demand continues, the bank becomes increasingly unwilling to further increase the volume of loans it supplies without the compensation of increasingly favorable loan terms. Given $F$, the share of loans in total assets rises along with the level of loan demand. As $R$ and $I$ are drawn down, they become increasingly attractive to the bank. In response to both the competition for funds exerted by $R$ and $I$ and the increasing portfolio risk, the slope of the supply of loans schedule declines. A given change in the stringency of loan terms produces a smaller change in the dollar value of loans supplied. An ever increasing rate of expansion of the level of income is required to increase the level of loan demand sufficiently to achieve a constant rate of increase in the size of $L$.

The discussion the effects of a cyclical expansion in the levels of income and of loan demand has simply put the information supplied by figures 2 and 3 into verbal form. In terms of figure 2, the expansion of income is associated with upward shifts in the demand curves. The higher the level of a demand curve, the later the stage in the expansion of $Y$ in this cyclical context. The response of the observed dollar size of the loan portfolio to the expansion in the level of income is summarized in figure 3. Given the total supply of funds available to the bank, $L$ will expand by a smaller amount for a given increase in $Y$, the larger the existing share of loans in total assets.
Presumably, the same elementary argument holds in reverse for periods of declining loan demand. When the levels of income and of loan demand reach their upper turning point, the loan portfolio is large relative to total assets and loan terms are quite stringent. When the level of income begins to contract, loan demand falls with it. The bank expects the level of loan demand to continue to decline. In the early stages of the contraction, a given decline in the level of loan demand will produce a relatively large reduction in the stringency of loan terms. A reduction of loan terms from their former high does not greatly increase risk per loan or significantly reduce expected return per loan. The initial reduction in the stringency of loan terms represents the partial satisfaction of the demands of previously unsatisfied borrowers. Most, if not all, of the reduction in loan terms stringency takes the form of reduction of "risk" factors (increase in average loan size, maturity, etc.) rather than the actual interest charge.* In the early stages of the decline in the level of loan demand, loans are still quite attractive; the loan portfolio still offers a relatively high expected rate of return at comparatively low risk. The early stages of the decline in income and loan demand are associated with a supply of loans function whose slope is quite small. The decline in \( L \) in response to a given decline in \( Y \) is relatively small.

As the cyclical contraction continues, the slope of the supply of loans function increases significantly. The expected return on the loan portfolio

*The possibility of such a reaction illustrates why \( r \) is a poor proxy for \( ?! \)
declines and the reserve and investment asset portfolios become increasingly attractive uses of bank funds. The existence of attractive alternative uses of its funds implies that the bank becomes increasingly unwilling to make further concessions in the terms of lending. Given the expected rates on R and I, the declining expected return on new loans becomes less worth the risk. The lower the level of income, the greater the reduction of the size of the loan portfolio for a given decline in Y.

So far, the introduction of cyclical variations in the levels of income and loan demand has not produced any startling results. The relationship between the level of income and the dollar size of the loan portfolio presented in figure 3 appears to hold in the dynamic case. When the cyclical movements in the prices of investment assets are allowed to interact with the cycle in aggregate income and loan demand, the argument is significantly altered, however. The locus of equilibria represented in figure 3 changes its shape during cyclical contractions.

a. The bond market:

Bond yields as well as loan demand experience cyclical variations. The cycles in the level of income and in bond yields tend to be roughly co-incident. Interest rates rise (bond prices fall) with the level of income during cyclical expansions and fall with it during contractions. The
rough coincidence of the two cycles is not surprising.* Interest rates rise during the expansion in loan demand as a result of the increasing level of aggregate income, with which the level of demand for bank loans is also associated. There is an excess demand for funds and interest rates generally rise. During contractions in aggregate demand, there is an excess supply of funds and interest rates tend to fall. Thus, at the upper turning point in loan demand, bond prices tend to be low and at the lower turning point they tend to be high.

Under these circumstances, the bank’s initial response to a decline in the level of loan demand should be different from that discussed above. When the levels of income and of loan demand begin to fall, the bank has incentive not to reduce the stringency of loan terms greatly, but rather to allow the loan portfolio to fall rapidly. The bank puts the liberated funds into the bond market in the expectation that the price of investment assets will rise. At the very time that the relative attractiveness of loans is beginning to decline, the expected return on I rises sharply. This increase in the expected return on investment assets will induce a relatively large shift of funds into investment assets in the early stages of the decline in the level of income.

* In one sense the coincidence is surprising. The existence of cyclical movements in the level of bond prices implies that there are profits to be made by moving into and out of bonds according to the phase of the cycle. It is rather surprising that attempts of speculators to buy cheap and sell dear have not removed the cycle from bond prices and market yields. A partial explanation of why speculators have been unable to remove the cycle is attempted at the end of this chapter.
When the level of loan demand reaches its upper turning point and begins to decline, the bank will expect it to continue to fall for some time. The relatively high current yield on investment assets coupled with the expected decline in the future level of loan demand and the expected rise in security prices tend to drive the expected return on I above the return expected on loans. During the early phase of the contraction in Y, the return which the bank expects to earn on its loan account is less than the return expected on its investment assets. The existence of the differential treatment of capital gains vis-à-vis other sources of profit implies that the after tax return expected on I compares quite favorably with the return expected on loans net of tax.\(^9\)

Despite the dominant position of the investment asset portfolio which develops as the levels of aggregate income and of loan demand begin to decline, the bank is restricted in its ability to shift funds into the investment portfolio. It has to rely on loan repayments to provide the primary source of funds for the purchase of securities. During the previous expansion in the level of loan demand, both the reserve and the investment asset portfolios were drawn down. The relative attractiveness of loans induced the bank to reduce the shares of both reserve and investment assets in its total asset portfolio. The reduction of the relative share of R in total assets implies that the implicit return on reserve assets is high at the upper turning point in loan demand. Given the relatively small size of the portfolio, any further attempt to reduce the reserve asset portfolio will tend to drive its implicit return toward that of investment assets. It is unlikely that there is a substantial shift of funds from R to I. Most,
if not all, of the funds must come from the repayment of existing loans.

Given the smaller risk associated with holding \( I \) rather than \( L \), the bank will grant new loans only if their expected return is greater than the return expected on investment assets. The increase in the expected return on investment assets which occurs at the upper turning point in \( Y \) implies that the supply of loans function shifts upward at that time. The bank wishes to hold fewer loans at the existing terms of lending. It will grant new loans only at rates commensurate with the increased expected return on investment assets.

The actual response of the supply of loans function at the upper turning point in \( Y \) depends upon the state of bank expectations concerning future movements in bond prices. These expectations have an important effect on the relationship between \( L \) and \( Y \) during the decline in the level of loan demand. The nature of the dependence of the loan-income locus on these expectations is illustrated by means of two different assumptions concerning the determinants of the expectations.

The first mode of behavior is one in which the future is forecasted on the basis of the most recent past. In particular, one in which the bank uses the prices of securities in previous periods to condition the price expected to prevail at the end of the current period. Let \( E(P_t) \) be the price of investment assets which is expected to exist at the end of period \( t \). The behavior described assumes that

\[
E(P_t) = w_1 P_{t-1} + w_2 P_{t-2} + \ldots + w_n P_{t-n},
\]

where

\[
1 < w_1 > w_2 > \ldots > w_n > 0 \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1.
\]

In this situation, the expected
price is a weighted average of previous prices. If the bank forms its price expectations in this manner, it is likely that in the early period after the actual turning point in bond prices, the expected price will still be declining. In subsequent periods, however, the expected price will begin to rise, but it will remain below the current price so long as prices are rising.

In this case, the supply of loans function shifts continuously upward as the expected price rises. The expected return on I is rising and in each period the bank is willing to supply a smaller quantity of loans at the existing terms of lending. At each point in time, however, the declining current yield on I reduces the expected return on investment assets below the level suggested by rising expected price. This implies that the shifts in the supply schedule diminish in importance as the yield declines. In this sort of world, the response of L to the declining level of loan demand is relatively large. The locus has the following general shape:

Figure 4

\[ L \]

\[ Y \]
The second behavioral assumption appears in some respects to be more realistic than the first. It is assumed that at the upper turning points in income and bond yields, the bank specifies an expected maximum price of investment assets which it expects to prevail at the end of the cyclical rise in security prices. The bank compares the current price to this expected maximum. It has the greatest incentive to shift into investment assets at, or just before, the upper turning points in income and security yields. Here, the difference between current and expected price is at a maximum and the current yield on I is also at its highest level; the expected return is at a maximum.

If the bank is able to perceive the existence of upper turning points in Y and i, it will shift funds quickly and sharply into I to obtain the greatest expected return. The supply of loans schedule shifts sharply upward. The bank will restrict rather severely the value of loans it is willing to grant at existing loan terms.

In this kind of world, the expected return on the investment asset portfolio achieves its maximum value at the cyclical turning point and then declines throughout the period of falling Y and i. This decline is attributable to both the narrowing gap between expected and actual price, as security prices rise, and to the declining market yield on securities. The supply of loans schedule shifts upward at the turning point and then proceeds to shift downward throughout the period of decline in the level of income and in the market yield on investment assets. The relationship between Y and L implied by this sort of behavior has the following general shape:
The locus shifts downward at the upper turning point in $Y$ but retains the general shape of the locus in the static case. Its slope at any point is lower, however, because of the downward shifts in the supply of loan schedule.

Actual bank response doubtlessly lies somewhere between the two extremes. It appears unlikely that the bank is able to accurately forecast cyclical turning points. Bank expectations probably respond in a lagged manner during periods of transition from cyclical expansion to contraction.* Several periods of consecutive decline in $Y$ and $I$ are required to convince it that a turning point has occurred. It appears unreasonable to assume,

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* The expected gain associated with the early purchase of investment assets is so great that the bank may attempt to forecast the lower turning point in the level of security prices. It is conceivable that it will attempt to shift funds into $I$ just before security prices are expected to reach their lower turning point. The implications of such behavior are discussed at the end of this chapter.
however, that the bank never awakens to the fact that once the decline has begun, it will continue for some time. It should be both willing and able to estimate some range of possible maximum values of the future price of investment assets. While the actual estimate may change as the downward movement continues, it remains true that there is a high premium associated with early and substantial shifts of funds into investment assets at or shortly after the turning point.

Within the context of the short-term cyclical contractions in aggregate demand and interest rates which have characterized the postwar period, the expectations described concerning maximum security prices should be realized fairly closely. If such expectations have been realized in the past, the bank will have ample reason to believe that they will be realized in the future.

It appears that the relationship between the level of income and the bank's loan portfolio is most likely to have the following shape during cyclical contractions:

Figure 6
At high levels of $Y$ and $L$, i.e., for values of $Y$ just following the peak, the response of $L$ to declines in $Y$ is relatively large. This represents the influence of the upward shifts in the supply of loans schedule. At intermediate levels of $Y$, the slope of the locus declines. The smaller slope is the product of the downward shifts in the supply schedule associated with declines in the expected return on $I$. At low levels of $Y$, i.e., during periods of late contraction, the slope of the locus again increases. This increase in slope is attributable to two sets of factors; both restrict further declines in the supply schedule.

First, at low levels of $Y$, the expected return on loans becomes increasingly less worth the risk of default. Second, liquidity becomes increasingly attractive in the face of declining expected returns on $I$ and $L$. This liquidity is associated not only with possible deposit movements, but also with the increasing likelihood that the level of loan demand will rise in the near future. In the late stages of the decline in the levels of loan demand and interest rates, the reserve asset portfolio should be a relatively large recipient of bank funds.

The assertion of a stable relationship between $Y$ and $L$ during periods of decline in the level of income stems from an implicit assumption concerning the rate of loan repayment. It is assumed that this rate is

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* The existence of lagged reactions at the upper turning point tends to remove the discontinuity of figure 5.
sufficiently great to allow the bank to reduce $L$ at the desired rate. This assumption implies that the excess of the value of loans repaid over the value of new loans issued equals the desired decline in $L$. If the bank wishes to reduce $L$ at a rate greater than the rate at which loans are repaid, it will not be successful, however. If the bank issues no new loans, the rate of loan repayment sets an upper limit to the rate of decline in $L$. It is assumed that the level of loan demand does not decline sufficiently fast for this upper limit to become operational.*

The situation which exists at the lower turning point in the level of loan demand helps to explain why the bank is so willing to increase the volume of loans it supplies when the level of loan demand rises. Its holdings of reserve and investment assets are large relative to total assets and their expected returns are low.

One reason that the bank was willing to increase the shares of $R$ and $I$ in total assets during the cyclical contraction was that it expected the level of loan demand to rise in the future. Previous cyclical movements in the level of loan demand led the bank to expect the contraction to be followed by an expansion. It expected a future rise in the expected return on loans. The rise in the level of loan demand indicates that its expectations have been born out; the expected return on loan is, in fact,何况

* Some implications associated with relaxing this assumption are discussed at the end of the chapter.
rising. Under these conditions, a given increase in the level of loan demand leads to a relatively large increase in the size of the loan portfolio. The costs of a transfer of funds into loans are low. The bank can gain increased portfolio expected return at the cost of very little increase in asset risk and decreased liquidity. The low expected returns on R and I make loans particularly attractive.

The expectation factors concerning the expected return on investment assets are at work at the lower turning points in income and bond yields as well as at their upper turning points. There is probably a lagged response involved as the bank becomes convinced that the fall in security prices has begun. Once it is convinced that prices are going to continue to fall, it attempts to estimate the minimum price. At this time, the expected return on investment assets is at a minimum. The spread between actual and expected price at the future cyclical peak is large and negative, and the market yield is at a low. The incentive to purchase additional investment assets is slight.

As the expansion continues, the expected return on I rises. This increase is attributable to the rising market yield on securities and to the smaller excess of actual over the future minimum price. The rise in the expected return on I leads to upward shifts in the supply of loans schedule. Such shifts produce a smaller increase in L, for a given rise in Y, than would be the case if the supply schedule were to remain stationary. In the early stages of the rise in Y, the upward shifts in the supply schedule should be relatively insignificant. As the process
continues, i.e., as the price of investment assets approaches the minimum and as the yield rises, the shifts will assume a greater importance. The increasing possibility of experiencing upper turning points in $Y$ and $i$ should give investment assets an added appeal. The increasing likelihood of a cyclical rise in security prices make investment assets particularly attractive. The locus has the following shape during periods of cyclical expansion.

Figure 7

The primary difference in bank behavior between the upper and the lower turning points in income and market yields lies in its ability to capitalize on the expected movements in bond prices. At the upper turning point, the bank has a great incentive to move into the bond market in order to capitalize on the expected rise in security prices. At the lower turning point, there is no such incentive to quickly shift funds into loans. Due to the lack of an organized market for loans, there is no means by which expected
future increases in the level of loan demand can increase their current value to the bank. The expected return on loans rises only as the actual level of loan demand rises. The bank does not attempt to shift funds into loans at the lower turning point in the same manner in which it shifts resources into investment assets at the upper turning point. It must wait for the increases in loan demand to materialize before increasing $L$.

The argument has now gone full circle. The interaction of the cycles in the levels of aggregate income and in interest rates leads to the conclusion that there are essentially two loci representing the relationship between $Y$ and $L$: one for cyclical expansions, figure 7, and one for contractions, figure 6. It should be stressed, however, that the discussion of expectational factors has served to stress the ambiguity of the analysis. The relationship between $Y$ and $L$ at any point of time is importantly influenced by the current state of bank expectations. The expectations involve bank beliefs concerning both future security prices, relative to their current levels, and future movements in the level of loan demand. The relationship between $Y$ and $L$ appears to be quite sensitive to the actual behavioral assumptions made concerning the manner in which expectations are formed.

There is no one set of expectational assumptions which is obviously superior to all others. Due to the inability to accurately specify the determinants of the expectational factors, it is not possible to make an accurate statement concerning bank response to variations in security prices and in the level of loan demand. The empirical evidence of the next chapter provides some information concerning such response.
The next chapter provides a statement of the modified model and it subjects this model to empirical test. Before turning to this chapter, it appears worth while to venture some conclusions concerning the macro-implications of bank reaction to cyclical movements in the levels of aggregate income and bond prices. The analysis in this study has been concerned with the portfolio management of an individual commercial bank. The analysis does provide some insight into the economic impact of changes in the composition of the asset portfolios of the banking system as a whole. The abrupt changes in relative expected rates of return associated with cyclical turning points in the levels of loan demand and of bond prices are of particular interest here. While the level of aggregate demand and of bond prices can be assumed to be exogenously determined for an individual bank, it is not so reasonable to extend this assumption to cover the entire banking system. Both the level of loan demand and the level of bond prices are influenced by bank portfolio policy.

It has been argued that at the upper turning point in the levels of loan demand and security yields, the individual bank has to rely on loan repayments to provide it with funds for the purchase of investment assets. This argument tends to generalize to a statement concerning the behavior of the banking system. It appears likely that most banks will find themselves in the position of having relatively small reserve asset portfolios at the upper turning point. These banks are restricted, in total, in their ability to purchase securities. Such restrictions appear to provide a partial explanation of the observed cyclical behavior of bond prices. Even if all
banks agreed that bond prices will rise, they are not able to capitalize fully on this expectation, either individually or collectively. The banking system is not able to obtain a sufficient quantity of funds to produce an immediate reduction in the price of investment assets. Banks which want to purchase investment assets have no means of obtaining the necessary funds except through loan repayments. Funds flow into these banks only as loans are repaid. The volume of funds which the banks are able to put into the securities market per unit of time is effectively conditioned by the dollar value of loans repaid during the period.

During the early stages of the decline in the level of loan demand, the banking system is not able to provide an effective demand for securities which is sufficiently large to drive the price of investment assets to its expected high. This can be accomplished only through time as funds are liberated from the loan accounts of the banks. The slower the rate of loan repayment, the longer this process takes. As security prices move upward, interest rates are falling, obviously. The fall in interest rates and the partial realization of the rise in security prices reduce the expected return on investment assets. The reserve asset portfolios of these banks, which are still small, becomes relatively more attractive. Some portion of the liberated funds flow into the transactions account. As a result of the shift of these funds into \( R \), the excess demand for securities is reduced. The rigidities of the loan portfolio coupled with the depleted transactions balance produce a reaction of the banking system which provides a relatively gradual increase in the price of investment assets through time.
It has been assumed that the representative bank uses the upper turning point in the level of its loan demand as the signal to move into investment assets. Given the high premium associated with the early purchase of securities in a market which is expected to rise, it is conceivable that the bank will start to shift funds into investment assets before the level of loan demand actually begins to fall. While the expected return on loans is high just prior to this turning point, the relatively large share of loans in total assets implies that portfolio default risk is high also. If the bank is convinced that the turning point will materialize in the near future, it has an incentive to purchase $I$ before the decline in the level of loan demand actually materializes. Given the small size of $R$, this shift of funds can be accomplished only if the supply of loans is restricted or if the bank is willing to further reduce its reserve assets portfolio. Part of the purchase of $I$ will be financed by a reduction in $L$; the bank will supply fewer loans at the existing terms of lending.

To the extent that the banking system reacts in this manner, such behavior tends to promote the realization of the expected turning point in the level of loan demand. The restriction of the supply of funds leads to an increase in the stringency of loan terms and to a decrease in total volume of loans outstanding. In this context, the upper turning point in the level of aggregate demand, $Y$, is no longer completely exogenous. Bank expectations concerning the timing of the turning point leads to aggregate portfolio manipulations which help to produce it.

Bank reactions at the lower turning point in the levels of income and bond prices help to explain the ensuing expansion. In the early stages of
the expansion in the level of loan demand, banks are willing to increase the
size of their loan portfolios without greatly increasing the stringency of
their loan terms. The ability of the banks' customers to borrow on favorable
terms tends to promote the early expansion in aggregate income. In the early
stages of the expansion, there is no great incentive to immediately sell all
the previously acquired securities. Banks can continue to receive their
interest income from investment assets until the funds are needed for loans.
While security prices are expected to fall, the extent of the fall expected
in the early stages of the increase in Y is not sufficient to induce banks
to sell their securities prior to the realization of the expected increase in
the level of loan demand. As the level of loan demand rises, the banks sell
their holdings of investment assets and security prices fall. The stronger
the initial increases in the level of loan demand, the greater will be the
early sales of investment assets. As the level of loan demand continues to
rise, the banks increase the stringency of their loan terms by a greater amount
for any given increase in the level of Y. Such behavior tends to inhibit
further increases in loan demand; the cycle continues.

It should be stressed that the intent of this short discussion has been
not to demonstrate that the banking system "causes" the cyclical movements in
aggregate income and in bond prices. Rather, the intent has been only to
demonstrate that the conclusions reached concerning the behavior of an individual
bank produce some interesting implications concerning the behavior of the
banking system. In particular, the reactions of the banking system to expected
cyclical movements in the levels of loan demand and bond prices tend to create
forces which promote the realization of these expectations.
V. THE MODEL AND SOME EMPIRICAL EVIDENCE

The analysis of the last chapter utilized a simplified version of the supply of loans function to derive a reduced form equation which specified the size of the loan portfolio in terms of a set of exogenous variables. It is now possible to combine the more detailed supply relationships summarized in chapter III with the demand for loans function to yield the complete reduced form expressions. In order to obtain the final model, it is necessary to introduce the cyclical factors in a manner which allows them to be subjected to empirical test. It is also necessary to allow for the possible existence of lags in the adjustment of \( R, I, \) and \( L \) to their "equilibrium" levels.

It has been argued that the reaction of the observed dollar size of the bank's loan portfolio to variations in the level of income depends upon the phase of the cycle in \( Y \). The non-linear relationships between \( L \) and \( Y \) described in figures 6 and 7 of chapter IV are approximated by four linear segments. These four segments are introduced into the reduced form relationships by means of four dummy variables. The relationship between \( L \) and \( Y \) takes the form:

\[
L_t = c_0 + c_1(X_{El}^{t-1}) + c_2(X_{E2}^{t-1}) + c_3(Y_{Cl}^{t-1}) + [c_4(X_{El}^{t-1}) + c_5(X_{E2}^{t-1}) + c_6(Y_{Cl}^{t-1}) + c_7(Y_{C2}^{t-1})] Y_{t-1} ;
\]

\[
L_t = c_0 + [c_1(X_{El}^{t-1}) + c_4(Y_{El}^{t-1})] Y_{t-1} + [c_2(X_{E2}^{t-1}) + c_5(X_{E2}^{t-1}) Y_{t-1}] + [c_3(Y_{Cl}^{t-1}) + c_6(Y_{Cl}^{t-1})] Y_{t-1} + c_7(Y_{C2}^{t-1}) Y_{t-1} .
\]

* The justification for the use of a lagged income variable is given below.
The dummy variables $X_{E1}$ and $X_{E2}$ represent the first and second halves of the expansion in the level of income respectively; the variables $X_{C1}$ and $X_{C2}$ are dummies representing the first and second halves of the contraction in $Y$ respectively. Each variable assumes a value of unity during the relevant time period and a value of zero at all other times.

This model allows for the existence of four different relationships between $L$ and $Y$. The use of four intercept terms allows the model to display possible discontinuities as well as changes in slope. The analysis of chapter IV requires that $c_4 > c_5$, $c_6 > c_7$. The discussion of that chapter also implies something about the behavior of cyclical intercept terms, namely that $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$. The justification of these hypotheses concerning the sizes of the intercepts merits some discussion.

At the lower turning point in $Y$, there is an increase in the expected return on loans and a decrease in the expected return on investment assets; the supply of loans function tends to shift downward. At the same time, because of increased profit expectations associated with the turning point, borrowers are apt to demand more loans at the existing terms of lending. Both of these factors produce an increase in the dollar value of loans at a given level of income. As the expansion in income enters its second phase, the supply of loans schedule is shifting upward as a result of the increasing expected return on investment assets. This should result in a downward shift in the locus. At the upper turning point in $Y$, investment assets assume a greatly increased expected return and business expectations worsen. Both of
these factors produce a downward shift in the income-loan locus. As the contraction in the level of income enters its second stage, investment assets have lost much of their attraction and the locus shifts upward.

It is now possible to state the reduced form equations in terms of the full functional relationships of chapter III. A list of variables used in these relationships appears as follows:

- $D$: Total demand deposits
- $D_V$: Particularly active and/or erratic demand deposits
- $D'$: Demand deposits other than $D_V$
- $T$: Total time and savings deposits
- $C$: Capital account
- $i$: Market yield on investment assets
- $Y$: Aggregate income variable
- $X_{EL}$, $X_{E2}$: Cyclic dummy variables representing the first and second halves of the expansion in $Y$
- $X_{CL}$, $X_{C2}$: Cyclic dummy variables representing the first and second halves of the contraction in $Y$.
- $u_i$: Stochastic terms

The expressions for the three asset groups involve the same set of exogenous variables:

(I) $R_t = a_{01} + a_{11}(D')_{t-1} + a_{21}(D_V)_{t-1} + a_{31}(T)_{t-1} + a_{41}(C)_{t-1} + a_{51}(i)_{t-1}$

(II) $I_t = a_{61}(X_{EL})_{t-1} + a_{71}(X_{E2})_{t-1} + a_{81}(X_{CL})_{t-1} + [a_{91}(X_{EL})_{t-1}$

(III) $L_t = a_{101}(X_{E2})_{t-1} + a_{111}(X_{CL})_{t-1} + a_{121}(X_{C2})_{t-1} + Y_{t-1} + (u_i)_t$
where $a_{11}$ represents the slope coefficient for $(D')_{t-1}$ in each of the three equations, $a_{21}$ represents the slope coefficient for $(D_r')_{t-1}$ in each of the three equations, etc. $(u_{11})_t$ represents the stochastic term in each equation.

It is assumed that the relative proportions of $R$, $I$, and of $L$ in the bank's total assets at the end of any period are determined by the set of exogenous variables which exist at the beginning of that period. The levels of $D'$, $D_r$, $T$, $C$, and $I$ which prevailed at the end of the previous period are the levels the bank expects to exist at the end of the current period. It is also assumed that the current level of loan demand is determined by the level of income of the previous period.

The use of deposit variables which are lagged a period reduces, and hopefully eliminates, the problems associated with bank creation of demand deposits in the acquisition of earning assets. When the bank purchases earning assets from its cash holdings, there is a simultaneous increase in the level of its deposits. These deposits are likely to be withdrawn quickly from the bank. While they remain with it, however, any observed relationships between $D_t$ and $I_t$ or $L_t$ would represent, in part, the influence of $L_t$ and $I_t$ on $D_t$. The use of the current level of deposits as a determinant of the levels of $I_t$ and $L_t$ would impart an upward bias to the estimated slope coefficients. It is assumed that all deposits derived from the acquisition of earning assets during the period have left the bank by the end of that period. If this assumption is valid, the estimated slope
coefficients for $D_{t-1}$ are not biased. The use of a deposit variable which is lagged a period introduces serious complications into the estimation of $R_t$. These complications are discussed below.

Given a change in an exogenous variable, it is by no means clear that the three asset groups will fully adjust to their "equilibrium" levels by the end of a single period of time.* The time necessary for $R$, $I$, and $L$ to fully adjust to variations in the exogenous variables in the system depends upon the speed of adjustment of both the bank and its loan customers to changes in these variables. If, for example, the bank should receive an increase in the level of its demand deposits in period $t-1$, the total reaction to this increase may not be completed by the end of period $t$. The increase in $D$ tends to induce the bank to increase the value of loans it is willing to supply at the existing terms of lending. If the increase in $D$ represents a deposit of funds by a new customer, the bank may wish to put the funds into reserve assets until it can evaluate the permanence of the increase. Until the bank has decided that the funds will remain with it for some time, the shift in the supply of loans schedule will be small. It appears likely that the longer the new account remains with it, the more certain the bank will become that the funds will not leave in the near future. In this case, the reaction of the supply of loans schedule to the increase in $D$ will tend to be distributed through time.¹

* The extent of adjustment obviously depends upon the length of period.
Even in the absence of expectational factors, some time is required to overcome the inertia in the system. This inertia is associated with the speed with which the bank makes its portfolio decisions and with the minimum time necessary to find borrowers and to process new loans. The most important form of inertia arises from the speed of reaction of borrowers to the increased availability of bank loans. When the bank decides that it wishes to increase $L$, it will find it necessary to reduce the stringency of loan terms to attract new borrowers. The reaction of borrowers to the more favorable terms of lending takes time. Possible expectational factors coupled to the inertia of the bank and its customers to changes in the exogenous variables tend to produce a reaction to the increase in $D$ which is distributed through time.

The paths of reaction of the three asset groups to variations in the set of exogenous variables are of considerable interest. Consider a situation in which all exogenous variables have been constant for a sufficiently long period of time to allow all reactions to have completely worked themselves out, i.e., $R$, $I$, and $L$ are constant. Assume that in period $t=0$, the level of demand deposits rises and permanently remains at its higher level. The probable composite response of the bank and of its borrowers to this increase in the level of deposits is illustrated in the three figures below. Time is represented on the horizontal axes.

---

* The reaction, per unit of time, obviously depends upon the units in which time is measured.
In the initial period, all of the increase in $D$ is mirrored in an equal increase in $R$. This is true by hypothesis; $I$ and $L$ are assumed to be insensitive to variations in the current level of deposits. In period $t=1$, the bank responds to the increase in its available funds by purchasing securities and by lowering the stringency of its loan terms. Some new borrowers are attracted into the bank and both $I$ and $L$ rise; $R$ falls by the amount by which $I$ and $L$ have risen. This process is repeated in each successive period until a new point of equilibrium is reached. At this point, the marginal expected rates of return on the three asset groups are again in a position in which it is no longer in the bank's interest to further reduce the stringency of its loan terms or to purchase more investment assets.

If it is valid to assume that one time period must pass before $L$ and $I$ respond to a given change in $D$, it is invalid to assume that $R$ responds
in this manner. If the rise in $D$ in period $t=0$ produces no change in $L$ and $I$ in that period, $R$ must rise by the full amount of the increase in $D$, by definition. The role of the reserve asset portfolio as a buffer stock implies that unexpected variations in $D_t$ are mirrored in equal movements in $R_t$.

The expressions for $R_t$, $I_t$, and $L_t$ which are implicit in the above diagrams are of the form:

$$R_t = a_0 + a_{11} D_t - a_{12} D_{t-1} - a_{13} D_{t-2} - \ldots - a_{1n} D_{t-n+1}$$

$$I_t = b_0 + b_{11} D_t + b_{12} D_{t-1} + b_{13} D_{t-2} + \ldots + b_{1n} D_{t-n+1}$$

$$L_t = c_0 + c_{11} D_t + c_{12} D_{t-1} + c_{13} D_{t-2} + \ldots + c_{1n} D_{t-n+1}.$$

By hypothesis, $a_{11} = 1$, $b_{11} = c_{11} = 0$; and by definition

$$\sum_{i=2}^{n} (a_{1i} + b_{1i} + c_{1i}) = 0.$$

The actual paths depicted in the diagrams involve the following assumptions:

$$a_{12} < a_{13} \ldots < a_{1n} < 0, \quad \sum_{i=1}^{n} a_{1i} < 1; \quad 1 > b_{12} > b_{13} > \ldots > b_{1n},$$

$$\sum_{i=1}^{n} b_{1i} < 1; \quad 1 > c_{12} > c_{13} > \ldots > c_{1n}, \quad \sum_{i=1}^{n} c_{1i} < 1.$$

The special properties of the reserve asset portfolio only apply for variations in exogenous variables which comprise part of the bank's total supply of funds. There is no large initial movement in $R_t$ associated with variations in $i_t$ or $Y_t$. 
The income variable is treated differently in the distributed lag model. The model will be tested only for periods of cyclical expansion. This allows us to use a transformation of the income variable which possesses the properties associated with the income-loan relationship of figure 7 in Chapter IV. The transformed variable, \( Y^* \), measures the deviation of the current level of income from the level of the base year, \( Y_0 \), relative to the current level of income, i.e., \( Y^*_t = \frac{Y_t - Y_0}{Y_t} = 1 - \frac{100}{Y_t} \), where \( Y_0 = 100 \).

This variable has the following desired characteristics: \( \frac{dY^*_t}{dt} > 0 \), \( \frac{d^2Y^*_t}{dt^2} < 0 \), and \( Y^*_t \) approaches unity as \( Y_t \) becomes large.

The discussion of lagged reactions yields the following expressions for the three asset groups:

\[
\begin{align*}
\text{(IV) } R_t &= a_{b1} + a_{dl}^1(D')_{t-1} + \ldots + a_{dl}^n(D')_{t-n} + s_{v1}^1(D_v)_{t-1} + \ldots + s_{v1}^n(D_v)_{t-n} \\
\text{(V) } I_t &= a_{b1}^l + a_{dl}^l(D')_{t-1} + \ldots + a_{dl}^n(D')_{t-n} + s_{v1}^l(D_v)_{t-1} + \ldots + s_{v1}^n(D_v)_{t-n} \\
\text{(VI) } L_t &= a_{b1}^l(i)_{t-1} + \ldots + a_{dl}^n(i)_{t-n} + s_{v1}^l(i^*)_{t-1} + \ldots + s_{v1}^n(i^*)_{t-n} + (v^*_t)_{t-1}.
\end{align*}
\]

It is quite conceivable that the structure of the lagged responses in these expressions is much more complex than the one assumed in figures la, lb, and lc. The sum of the coefficients for the lagged values of each exogenous
variable equals the total response of the portfolio group to a given variation in that variable. This is not strictly true for the response of \( R_t \) to variations in the variables which comprise the bank's supply of funds. The dependence of \( R_t \) on the current levels of these variations in \( D' \), \( D_v \), \( T \), and \( C \) is the sum of the coefficients plus unity.

Serious estimation problems are introduced by eliminating the current levels of the bank's supply of funds from the expressions for \( R_t \). This can be seen by considering three examples concerning the properties of the movements in \( D' \). If \( D' \) is subject to trend, \( R_t \) will appear to be positively related to \( D'_{t-1} \). High levels of \( D'_{t-1} \) will be associated with high levels of \( R_t \). If \( D' \) is subject to relatively infrequent movements, a negative coefficient will be observed. The bank will have an opportunity to begin its portfolio adjustments. Finally, if the movements in \( D' \) from period to period are large and random with no evidence of trend, there may be no observed relationship between \( R_t \) and \( D'_{t-1} \). In this case \( R_t \) will simply insulate \( I_t \) and \( L_t \) from the variations in \( D' \). As will become clear below, these problems are crucial to the evaluation of the empirical analysis.

Equations I - III represent the initial response of \( R \), \( I \), and \( L \) to variations in the set of exogenous variables. Equations IV - VI, on the other hand, represent the total response of the three portfolio groups to changes in these variables. Both sets of relationships are tested in the empirical section which follows.
Some Empirical Evidence:

Now that the basic models have been introduced, all that remains is to describe the data used in the statistical work and to present the evidence. It will soon become evident that the lack of sufficient data severely restricts the usefulness of the empirical analysis. It is not possible to test adequately the simple hypotheses which appear in equations (I) - (VI).

The asset and liability data used in the statistical analysis are derived from the weekly balance sheets of some 85 large commercial banks.* The weekly data cover the period from January, 1960 through December, 1962. The specific banks included in the sample are those which are particularly active in the money market. Such activity is represented by frequent and relatively large purchases and sales of Federal funds and by other behavior which suggests that the banks are willing and able to manage their portfolios closely. The banks in the sample are large and they have access to all the short-term assets which comprise the money market. All Federal Reserve Districts are represented in the sample.

The sample of banks selected represents in many ways the easiest test of the hypotheses which have been presented. If any banks concern themselves with relative marginal rates of returns on their assets and if any make the fine adjustments in their portfolios suggested by the analysis, they are the ones in the sample.

* The banks comprise a subset of the "Weekly Reporting Member Bank" series which is regularly published in the Federal Reserve Bulletin.
The analysis of this study requires individual bank data of the sort which comprises the elements of the sample. Unfortunately, disclosure rules of the Federal Reserve System prevent the use of individual bank data in the empirical work which follows. While the analysis of this study has been conducted in terms of the behavior of an individual commercial bank, it is necessary to test the hypotheses using time series data which are aggregated over all banks in the sample.* The use of such data obviously introduces aggregation problems into the analysis. It appears, however, that because the banks in the sample were chosen on the basis of their roughly similar sizes and motives, these problems are at least reduced; although they are certainly not eliminated. The aggregate data obviously restrict the empirical analysis.

The most undesirable feature of the sample is the extremely short period of time which it covers. The time period is not only short, it is also not characterized by substantial variation in either the balance sheet items or in the level of income. It is the length and character of the time period which appears to place the most severe constraints on the analysis.

The coefficients of the six reduced form expressions are estimated using standard least squares regression analysis. The data represent raw, weekly observations on all variables. The income variable used is the

* All necessary adjustments such as deduction of required reserves from the deposit items were made to the individual bank data.
Federal Reserve Board's index of weekly department store sales. This measure is obviously an inadequate measure of total economic activity, but it is probably the best weekly series available.

None of the data are adjusted for seasonal variation. A set of seasonal dummy variables is used which estimates the seasonal variations in \( R, I, \) and \( L \) which are not accounted for by the seasonal movements in the exogenous variables of equations (I) - (III). Only monthly seasonal variation is considered. This requires the use of the same seasonal dummy variable for each week of a given month. The seasonal dummy variables assume a value of unity for all weeks of the relevant month and a value of zero at all other times. The equations are estimated using 11 seasonals and the total constant term.

The actual variables used in the empirical analysis appear as follows:

\[
D : \text{Total demand deposits} \\
D_g : \text{Government demand deposits} \\
D_b : \text{Demand deposits due to other banks} \\
D' : \text{Total demand deposits other than } D_g \text{ and } D_b \\
T : \text{Total time and savings deposits} \\
C : \text{Capital account} \\
i : \text{The market yield on 3-5 year Government securities} \\
Y : \text{Weekly index of department store sales} \\
X_{E1}, X_{E2} : \text{Cyclical dummy variables, expansions} \\
X_{C1}, X_{C2} : \text{Cyclical dummy variables, contractions} \\
S_i : \text{Seasonal dummy variables, } i=2, ..., 12.
\]

* Seasonal dummy variables are not needed in the estimation of equations (IV) - (VI). The reason for the exclusion is made clear later.
The series for \( D_g \) and for \( D_b \) are the only components of total demand deposits which could be isolated from the data. It is not clear that these two deposit groups represent a good measure of particularly active and/or erratic deposits. While Government deposits tend to fluctuate over a very wide range, notice is given to the banks before funds are deposited or withdrawn from the account.\(^5\) Such notice obviously increases the predictability of the account. Deposits due to other banks represent part of the transactions balances of the depositor banks and, therefore, should be quite variable and erratic. However, a substantial portion of \( D_b \) represents compensating balances left with the banks in payment for services associated with the correspondent relationship.\(^6\) The use of \( D_b \) in this capacity reduces the variability and lack of predictability of the series. The isolation of \( D_g \) and \( D_b \) from total demand deposits does provide a potential source of useful information, however. If the banks in the sample are observed to react differently to variations in these two deposit groups than they do to \( D' \), the existence of regularly reported data on these two accounts is of value.

The time series covers the period from January 1, 1960 through December 26, 1962. The four cyclical dummy variables equal unity over the following sub-periods.

\[
\begin{align*}
X_{e1} & : \text{June 21, 1961 - December 26, 1962} \\
X_{e2} & : \text{January 6, 1960 - June 29, 1960} \\
X_{c1} & : \text{July 6, 1960 - December 21, 1960} \\
X_{c2} & : \text{December 28, 1960 - June 14, 1961}
\end{align*}
\]
The dates of the turning points in the level of Y were estimated visually using the monthly, seasonally adjusted series for department store sales. It should be noted that the dates of these turning points do not correspond to the ones which are commonly considered to represent the 1960 - 1961 recession.* As the department store sales index is used in the empirical work, its turning points must be used to date the general expansion and contraction phases.

Given the imprecise character of the theoretical discussion and the nature of the data, the evidence which follows should be considered to be the results of a pilot study. There is an insufficient number of observations to allow one to draw any convincing conclusions from the results. No small part of the discussion and evaluation of the empirical evidence is concerned with the interpretation of the estimated coefficients. For many coefficients, there are no specific hypotheses to be tested.

Some experimentation has been carried out in the choice of functional forms; some of these forms are mentioned below. Data for the period from July, 1959 through December, 1959 were available but not included in the sample. This exclusion of observations is justified on the grounds that any distortion of behavior produced by the steel strike of 1959 and by its aftermath would produce a disproportionate distortion of the estimates of the relationships. Given the small total number of observations available,

---

* For example, the upper and lower turning points in the Federal Reserve Board's Index of Industrial Production occurred in June, 1960 and February 1961, respectively.
the estimates of the coefficients of the reduced form equations are particularly sensitive to any such distortions. In total, the results reported below are suggestive at best. The model must be tested against new, and hopefully better, data before any truly meaningful conclusions can be drawn concerning the estimates of the coefficients.

The estimates of the coefficients of equations (I), (II), and (III) appear below in table 1. Although it is only necessary to estimate two of the three equations, all three are estimated in order to test the consistency of the results. Each equation is estimated independently of the other two. The standard errors appear in parentheses below the coefficient estimates, and d is the value of the Durbin-Watson statistic.
Table 1

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<td>.033 (.214)</td>
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<td>( s_{12} )</td>
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<td>.238 (.411)</td>
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Table 1 (Cont.)

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<td>$I_t$</td>
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<tr>
<td>$L_t$</td>
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* Evidence of significant serial correlation of residuals at 95% level.
Due to the reduced form character of the three equations and to the existence of lags in adjustment, both the interpretation and evaluation of these results are exceedingly difficult. The role of each exogenous variable is discussed in terms of its impact on each of the three asset groups.

A good example of the problems introduced into the analysis by the existence of lags is the estimated coefficient for $D'_{t-1}$ in equation (I). During the period studied, the short-term movements in this deposit item were both large and erratic. There is no evidence of a trend in $D'$ over the period.* The week to week movements in the variable were large, but its average variation over longer periods was slight. Under these conditions, the dependence of $R_t$ on $D'_{t-1}$ tended to produce an inconsistent estimate for the slope coefficient in equation (I).

By hypothesis, changes in the current level of $D'$ are mirrored dollar for dollar in changes in the same direction in the reserve asset portfolio. Such response represents the role of $R$ as a buffer stock. If $D'_t$ should move in the same direction for several periods, any observed relationship between $R_t$ and $D'_{t-1}$ will involve a large, positive slope coefficient, i.e., one which approaches unity. When $D'_t$ changes its

* In gaining access to the data used in this study, the author was discouraged from reproducing any part of it, including the aggregate series. It is not possible, therefore, to present the series which are discussed in the text.
direction of movement, this relationship between $R_t$ and $D_{t-1}'$ is destroyed. At the turning point in $D'$, $D'_t < D'_{t-1} > D'_{t-2}$; $R_t$ falls even though $D'_{t-1} > D'_{t-2}$. The more frequent these changes in direction of $D'_t$, the smaller the estimated coefficient for $D'_{t-1}$. Given the behavior of $D'$ during the period, it appears that the estimated slope coefficient in equation (I) represents the influence of both of the factors described. $D'_t$ moved in the same direction in a sufficient number of consecutive weeks to provide a positive slope coefficient, but it also changed directions sufficiently often to significantly reduce the size of the coefficient.

The coefficients for $D'_{t-1}$ in equations (II) and (III) suggest that the sort of lagged adjustment depicted in figures 1b and 1c were at work. The small size of the estimated coefficients implies that at the end of a week, very little progress has been made toward the adjustments of $I_t$ and $L_t$ to their final equilibrium levels. A $1.00$ increase in $D'_{t-1}$ produces an approximate increase in $I_t$ of $.09$ and an increase in $L_t$ of $.05$. Both $I_t$ and $L_t$ are insulated from the disturbances associated with the unexpected changes in $D'_t$ discussed above. The slope coefficients in equations (II) and (III) should be unaffected by such disturbances. The coefficients in these two equations suggest that in the absence of these disturbances, the coefficient for $D'_{t-1}$ in equation (I) would have been of the order of magnitude of $-.14$, rather than the observed coefficient of $.31$. 
The estimated coefficients for $D_g$ and $D_b$ in the three equations are discussed together. Here again, the interpretation of the results is derived from the observed behavior of these two deposit components during the time period studied. During the period, both $D_g$ and $D_b$ were subject to relatively large, infrequent movements. During the weeks which lay between these large movements, neither variable displayed the large week to week variations which characterized $D'$. This sort of behavior of the two deposit components allows one to observe the lags at work in the equations for all three portfolio groups.

During the week in which the banks experienced a relatively large change in either of these two deposit levels, there is an equal change in $R_t$ in the same direction. In the next period, the banks begin the adjustments of their asset portfolios to their new levels. The general absence of further change in the deposit item implies that the banks can, in fact, begin to make the desired adjustments in $R$. These adjustments are always in the opposite direction from that of the initial period. For example, if $D_g$ should increase during week $t$ and then remain approximately at this level for several weeks, the reaction of $R$ over this period will be similar to that described in figure 1.a above. The reserve asset portfolio will experience a sharp increase in week $t$ and then decline at a decreasing rate over weeks $t+1$ through $t+n$.

The negative coefficients for $(D_g)_{t-1}$ and $(D_b)_{t-1}$ in equation (I) imply that this sort of reaction path existed during the period of time
studied. The positive coefficients for the two deposit items in equations (II) and (III) represent the other side of the adjustment in $R$. The role of $R_t$ as a buffer stock implies that the estimated coefficients in equation (I) tend to be less trustworthy than the estimates in the other two equations. The approximate responses of the three portfolio groups to variations in $(D_g)_{t-1}$ and in $(D_b)_{t-1}$ are evident, however. A $1.00$ increase in $(D_g)_{t-1}$ is roughly associated with a $0.30$ decrease in $R_t$, a $0.15$ increase in $I_t$, and a $0.15$ increase in $L_t$. A $1.00$ increase in $(D_b)_{t-1}$ is roughly associated with a $0.40$ decrease in $R_t$, a $0.17$ increase in $I_t$, and a $0.25$ increase in $L_t$. The existence of serially correlated residuals in the three equations prevents one from drawing any inferences concerning the differences in the sizes of the coefficients in equations (II) and (III).

It is of some interest to note that the estimated coefficients for $(D')_{t-1}$ in equations (II) and (III) are markedly smaller than the coefficients for $(D_g)_{t-1}$ and $(D_b)_{t-1}$ in the same equations. This difference in the sizes of the coefficients is compatible with the observation that the large random variations in $D'$ made deposit forecasts so difficult during the period that variations in $D'$ were absorbed almost exclusively in $R$. The banks were not willing to allow short-term variations in $D'$ to disturb their investment and loan asset positions. The banks responded more slowly, in terms of $I$ and $L$, to variations in $D'$ than they did to movements in $D_g$ and $D_b$. 
At first glance, the estimated coefficients for $T$ in the three equations would appear to be unreasonable. In terms of the previous discussion of lags in bank portfolio adjustments, the coefficients in equations (II) and (III) appear to be far too large. These coefficients suggest that the lags in response to variations in $T_{t-1}$ are completed in approximately a week. The evidence discussed so far strongly implies that the lags in adjustment to variations in the levels of the demand deposit items are significantly longer than a week. There appears to be little justification for arguing that the lags in portfolio adjustments associated with movements in the level of time and savings deposits is so short. Given their greater predictability and cost, a relatively large proportion of any increment in $T$ should find its way into loans. It is the loan account which represents the greatest source of inertia.

During the period studied, the week to week variations in $T_t$ were small relative to its mean and the series was subject to a definite upward trend. Under these conditions, the future level of $T$ was highly predictable. The banks should have been able to forecast future levels of $T$ with sufficient accuracy to allow them to make their portfolio decisions on the basis of expected future levels of $T$. This implies that the lags worked themselves out before the increase in $T$ actually materialized. The ability to make such adjustments is particularly important for the loan account. The banks were apparently able to carry out their loan negotiations before they actually obtained the funds they were to lend. On the basis of this argument, the lags in adjustment exist, but they are associated with expected rather than actual $T$. 
The estimated coefficients in equations (I), (II), and (III) approximate the "equilibrium" values of these coefficients.

In the light of this interpretation, the estimated coefficients appear to be reasonable. A $1.00 increase in $T_t$ is associated with an approximate increase in $I_t$ of $.30 and an approximate increase in $L_t$ of $.70. Such a reaction strongly suggests that the level of time and saving deposits was an extremely important element in the determination of $L$ during the period.

The interpretation of the coefficients for $C_{t-1}$ is similar to that given for the estimated coefficients for $(D_g)_{t-1}$ and for $(D_b)_{t-1}$. During the period studied, the capital account displayed the same sort of properties observed for $D_g$ and for $D_b$. The level of $C$ was subject to relatively large, intermittent movements. The week to week variations in the level of the capital account were small; it was subject to a slight upward trend. All movements in $C$ were small relative to the variations in the other components of the banks' supply of funds.

The relative sizes of the estimated coefficients in the three equations are of some interest.* The negative slope coefficient in equation (I) implies that $C_t$ was sufficiently stable to allow one to observe the beginning of the process of portfolio adjustment. The relatively large coefficient for $C_{t-1}$ in equation (II) is of particular interest. This slope coefficient implies that the banks are willing to make relatively large shifts of funds into the

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* Due to the residual nature of the capital account, the coefficient estimates should be viewed with some skepticism.
investment asset portfolio. A $1.00 increase in the level of $C_{t-1}$ is associated with a $.35 increase in the level of $I_t$. The small coefficient for $C_{t-1}$ in equation (III) implies that there is a much longer lag associated with the placement of the funds into loans. The coefficient estimates in the three equations appear to be consistent in the sense that their sum is approximately equal to zero.

Some of the most interesting results of the empirical analysis stem from the estimated coefficients for $i_{t-1}$ in the three equations. The loan equation is discussed first, as the interpretation of the results is the most straightforward for this equation. The coefficient for $i_{t-1}$ in equation (III) strongly implies that the asset groups do compete for bank funds. Banks do adjust to variations in marginal yields. The estimated coefficient implies that a rise in the market yield on 3-5 year Government securities of 1 percentage point (e.g., from 5 to 6%) results in an approximate decline of $1.0 billion in the aggregate loan portfolio. The elasticity of $I_t$ with respect to $i_{t-1}$ calculated at the means of the variables is -.1. A 1% rise in $i_{t-1}$ (e.g., from 6.00 to 6.03%) is associated with an approximate decline in $I_t$ of .1%. It should be pointed out that the possible existence of lags in adjustment suggests that the total response of $L$ to variations in $I$ is larger than the slope estimate suggests.

The coefficients for $i_{t-1}$ estimated for equations (I) and (II) require some explanation. According to the model developed, the reserve
asset portfolio is composed exclusively of cash and earns only an implicit rate of return. In fact, only a small and variable proportion of \( R \) is in cash form. Most of the transactions balance is held in the form of highly liquid nonmoney assets. These assets earn the going market yield on such assets. The estimated coefficients for \( i_{t-1} \) in equations (I) and (II) mirror the influence of this short-term yield on bank portfolio decisions.

The positive coefficient for \( i_{t-1} \) in equation (I) is indicative of the difficulties associated with using the same set of exogenous variables to estimate all three equations. The relevant variable to use in equation (I) is not \( i \), but rather some index of the level of \( i \) relative to the short-term rate. The greater the short-term rate in period \( t-1 \), relative to \( i_{t-1} \), the greater the desired level of \( R_t \). On the other hand, such an index has no particular relevance for the determination of \( L_t \). It is obvious that both \( i_{t-1} \) and the index cannot be used in the same regression because the two variables would be highly correlated.

The positive coefficient for \( i_{t-1} \) in equation (I) stems from two properties of the relationship of the short-term rate to \( i \). First, the two rates tend to move together. Generally speaking, the short-term rate is high when \( i \) is high, and it is low when \( i \) is low. Second, the short-term rate varies over a wider range than \( i \). These two factors imply that a given movement in \( i \) tends to be coupled with a movement in the short-term rate which is not only in the same direction, but which is often larger in magnitude. Such a reaction suggests that when \( i \) rises, the short-term
rate tends to rise by a larger amount and \( R \) becomes more attractive relative to \( I \). Obviously, the opposite reaction occurs for a given decline in \( i \).

The effects of the relationship between the short-term rate and the yield on 3-5 year securities which have been described are consistent with the estimated coefficients for \( i_{t-1} \) in equations (I), (II), and (III). A rise in \( i_{t-1} \) tends to make \( R_t \) more attractive relative to \( I_t \), and it also makes the loan portfolio less attractive relative to both of the other two asset groups. There is a relatively large shift of funds out of loans and a relatively small shift of funds out of investment assets. The reserve asset portfolio receives funds from both portfolios. While the use of the single interest rate variable tends to cover up the competition between \( R \) and \( I \) for bank funds, it certainly does not hide the competition of \( L \) with both of these asset groups.

The discussion now turns to the coefficients estimated for the income variable. Here, the interpretative difficulties are even greater than before. The added difficulties arise from the use of a highly simplified and unrealistic demand for loans function and from the use of an unsatisfactory variable to measure the level of aggregate demand in the economy. These factors imply that it is hazardous to attempt to draw any conclusions from the coefficient estimates concerning the role of the demand for bank loans in the determination of the relative sizes of the three portfolio groups.

One final complication must be mentioned before the estimates are discussed. The period studied was not characterized by large movements in
any index of the level of aggregate economic activity. This was particularly true of the index of department store sales. On a seasonally adjusted basis, this index varied over a very narrow range. This sort of behavior of $Y$ is not conducive to adequate tests of the hypotheses made concerning the relationship of $Y$ to the three portfolio groups. Given the strong growth of time and savings deposits which occurred during the period, it is unlikely that the level of loan demand ever reached a sufficiently high level, relative to the banks' resources to produce the nonlinearities which were discussed in Chapter IV.

The results obtained from the use of the four cyclical dummy variables to analyze the influence of the level of loan demand on $R$, $I$, and $L$ are first summarized and then discussed in more detail. As the analysis is concerned with the relative sizes of the coefficients, lags in adjustment are not discussed. Presumably, the lags are the same in all phases of the cycle.

The hypotheses made at the beginning of this chapter concerning the relative sizes of the coefficients for the four dummy variables in the loan equation must be rejected. The income-loan locus has a small, positive slope during the first half of the expansion in $Y$, and a larger positive slope during the second half of the expansion. It was argued that the slope would be smaller, not larger. The locus has a negative, but relatively small slope during the first half of the contraction in $Y$ and no slope at all during the second half of the contraction.

The investment asset portfolio displayed some of the expected properties. The portfolio does appear to become more attractive during periods of declining income, as witnessed by the relatively large negative
slope in the first half of the contraction. The slope is still negative but much lower during the second half of the contraction. The results for the reserve asset portfolio are surprising. The slope coefficients are positive for all four cycle phases.

Given the rather surprising nature of the evidence for equations (I) and (III), an effort must be made to account for the observed results. Each cycle phase is studied in some detail.

The relatively large and negative intercept coefficient for $X_{t-1}$ in equation (III) makes no sense in terms of the analysis of this study. It implies that during the first half of the expansion in $Y$, there is a downward shift in the income-loan locus below the average level which exists during the second half of the contraction in $Y$. The positive intercept term in equation (I) implies that there is an upward shift in the relationship between $Y_{t-1}$ and $R_t$ during the same period. The negative coefficient for the investment asset portfolio is consistent with the analysis.

The estimated slope coefficients for the first half of the expansion in the level of income, are also inconsistent with the hypotheses. According to the analysis of Chapter IV, the slope coefficient in equation (III) should be positive and large during the period. This does not appear to be the case. The estimated coefficient in the loan equation is small and probably equal to zero. This implies that during the early stages of the expansion in $Y$, a given increase in the level of income is associated with a small or zero increase in the level of $L$. The slope coefficient in equation (I) is
both positive and relatively large. A given increase in \( Y_{t-1} \) is associated with a relatively large increase in the level of \( R_t \). The observed positive slope coefficients in the three equations imply that the estimates are inconsistent. The sum of the coefficients should be zero.

About the only rationalization which one can give to this evidence is that the levels of loan demand and of \( Y \) were not related during the first half of the expansion in \( Y \). In particular, the level of demand did not rise with \( Y \) during the early stages of expansion.* This interpretation is at least consistent with the observed upward shift in the reserve asset relationship and with the large positive slope in equation (I). If the level of loan demand did not rise, there would be a tendency for \( R \) to be relatively attractive as the banks waited for the increase in the level of demand to materialize. It must be stressed, however, that this argument rests on pure supposition; there is no means of observing the demand for bank loans.

The results for the second half of the expansion in \( Y \) are somewhat more satisfying.** The slope and intercept terms for equation (III) are both

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* This possible lack of response cannot be the product of the date of the lower turning point in the index of department store sales. As mentioned earlier, the lower turning point in aggregate activity appears to have occurred before the turning point in the index.

** Experimentation with the cutoff points between the first and second halves of the expansion in \( Y \) did not produce any marked changes in the relative sizes of the coefficients for \( (X_{E1})Y \) and \( (X_{E2})Y \) in equation (III).
consistent with the hypotheses. The intercept term suggests a very small and probably zero upward shift in the relationship. The slope coefficient is both positive and relatively large. The coefficient implies that a rise in $Y_{t-1}$ from 100 to 200 results in a $1.4$ billion increase in $L_t$. The intercept term in equation (I) implies that there is a decline in the relation between $R_t$ and $Y_{t-1}$. The slope coefficient in the reserve asset equation is still positive, but it is much smaller than the coefficient estimated for the early stages of increase in $Y$. All three slope coefficients are again positive which implies another inconsistency.

The results for the second half of the expansion in the level of income are quite compatible with the results which were expected to prevail during the first half. If the level of loan demand did not rise during the early stages of the expansion in $Y$, the period covered by $X_{E2}$ includes both halves of the expansion in the level of loan demand. If this supposition is correct, the hypothesis concerning the nonlinearity of the income-loan locus has not been tested. In the light of this possibility, the relationship between $Y$ and the three portfolio groups was tested for possible curvature by using a quadratic form for $Y_{t-1}$. This form yielded no evidence of nonlinearities for the period.

Of the four sub-periods studied, the estimates associated with the first half of the contraction in $Y$ are perhaps the most informative. The three intercept terms suggest that both $I$ and $L$ become more attractive relative to $R$ during the period. Neither the positive intercept term in
equation (II) nor the negative intercept in equation (I) contradict the hypotheses. The positive intercept in equation (III) does not support the hypothesis made concerning this term. It was argued that loans lose much of their attraction in the early stages of the contraction because of the increase in the expected return on investment assets. This was not the case for the period studied. However, the positive intercept in equation (II) does add substance to the proposition that investment asset become relatively more attractive. It appears that the substitution was out of $R$ rather than $L$, in this sample.

The slope coefficient in equation (I) is not consistent with the analysis of Chapter IV. The positive slope coefficient for the reserve asset equation implies that the substitution of $I$ for $R$ results in a reserve asset portfolio which declines with the level of income. The negative slope for the investment asset equation implies that the banks purchase securities as the level of income falls. The intercept and the slope coefficients for equation (II) give some support to the contention that there is a shift of funds into the investment asset portfolio during the early stages of the decline in $Y$. The shift appears to come at the expense of $R$ rather than $L$, however.

It was asserted that during the first half of the decline in $Y$, the slope coefficient in equation (III) would be positive and relatively large. The coefficient estimated is negative and small. There are two essentially opposing interpretations which can be used to rationalize the existence of a negative slope coefficient. First, it is conceivable that
the level of loan demand continued to rise during the first half of the contraction in $Y$. This would imply that during the period, the loan portfolio continued to increase as the level of income fell. This interpretation is compatible with the positive intercept term in equation (III) estimated for the same period. Second, it is possible that the level of loan demand did fall along with $Y$, but that the rate of loan repayment could not keep pace with the declining demand. Such a phenomenon would result in an intercept term which is observed to be positive and in a slope coefficient which is small and meaningless.

It is not obvious that either interpretation is correct. The fact does remain that the hypothesis concerning the relationship between the loan portfolio and the level of income during the first half of the contraction is not supported by the evidence.

The intercept terms estimated for the second half of the contraction in $Y$ indicate downward shifts in equations (II) and (III) and an upward shift in equation (I). The intercepts for (I) and (II) support the original hypotheses made for this segment of the cycle. The hypotheses concerning the slope coefficients do not fare quite so well. The positive slope in equation (I) implies that $R$ continues to decline as $Y$ falls. There does appear to be a decline in the size of the slope, viz., from .029 in the first half of the contraction to .016 in the second half. The fact remains, however, that the analysis requires the slope to be positive. The slope coefficient in equation (II) is consistent with the analysis. It is still negative and it is substantially smaller than in the first half of the contraction. The
slope coefficient in the loan equation is positive, but no doubt equal to zero.*

On the basis of the available evidence, the analysis of the relationships between the level of income and the sizes of the three portfolio groups must be considered to be a failure. The four linear segments displayed relatively few of the asserted properties. The four positive slopes in equation (I) and the behavior of the slopes in equation (III) are most troublesome for the analysis. The loan portfolio appears to respond to variations in \( Y \) only during the second half of its expansion. The most encouraging results were obtained in equation (II). Here, there is strong evidence that the banks actively shift funds into investment assets during the early stages of the decline in \( Y \).

As mentioned before, no great faith can be put in the estimates for the income variable. The index of department store sales is not a good proxy variable for the aggregate level of activity in the economy. Even if the index were a good measure of this activity, it is by no means clear that the relationship between the level of aggregate demand and the level of loan demand is a simple, linear one. These difficulties are clearly displayed in the inconsistent estimates for \( (X_{E1})Y \) and \( (X_{E2})Y \) for the three equations.

* The zero slope coefficient for \( (X_{C2})Y \) in equation (III) coupled to the small negative coefficient for \( (X_{C1})Y \) in the same equation supports the argument that \( L_t \) could not keep pace with the decline in the level of loan demand.
The behavior of the income variable over the period studied makes it extremely difficult to draw any conclusions concerning the usefulness of the approach. The fluctuations in Y relative to the movements in the banks' supply of funds were very small during the period. This makes it difficult to test the hypotheses. If the demand for bank loans is related to Y in a stable manner, this demand also fluctuated over a narrow range.

It is quite conceivable that if Y had been subject to much greater variation, relative to F, the negative relationship between R_t and Y_{t-1} would have been observed. It is also conceivable that under these circumstances, the nonlinear relationship between L_t and Y_{t-1} would have been observed, also. More data are required if the hypotheses are to be subjected to a valid test.

Distributed Lags:

One of the principle complications which arose in the interpretation of the estimates of equations (I), (II), and (III) lay in the existence of lags in adjustment. This implies that if the structure of the lagged relationships depicted in equations (IV), (V), and (VI) can be successfully estimated, some useful information will be obtained concerning the determination of the sizes of the three portfolio groups.

It is extremely difficult, if not impossible, to estimate the structure of the distributed lag system. The three equations cannot be estimated in the form in which they are stated. The high correlation between the levels of any of the exogenous variables in two successive weeks prevents direct
estimation of the expressions. Koyck and Friedman have developed techniques which provide indirect estimates of the time path of reaction of the dependent variable to variations in the exogenous variables. Both of these techniques use the level of the dependent variable in the preceding period as one of the exogenous variables in the expression for the dependent variable in the current period. More complex versions also include values of the exogenous variables in preceding periods in the expression.

These methods cannot be used here. The use of weekly data renders the correlation between the lagged dependent variable and the other exogenous variables too high to allow one to obtain consistent estimates of the coefficients. The same argument applies to the use of the lagged values of any of the exogenous variables. The techniques for obtaining estimates which are approximately consistent are too costly to be used here.

The method used in this study is a very simple one; but one which is extremely hazardous in terms of obtaining "good" results. The expressions in equations (IV), (V), and (VI) are tested in the form of first differences. While the values of a variable at two adjacent points in time may be highly correlated, it is not obvious that two adjacent changes in the variable are likewise correlated. To the extent to which this statement is valid, it is possible to obtain unbiased estimates of the coefficients in the three equations.

In order to reduce the number of variables to a more manageable number, the three demand deposit groups are added together. Due to the hypothesis concerning differences in bank behavior in cyclical expansions as opposed
to contractions, only the periods of expansions are considered. There is an insufficient number of observations to allow one to make separate estimates for periods of cyclical contraction. Each variable is lagged eight weeks. The following estimates are obtained for equations (IV), (V), and (VI) subject to the first difference transformation.
Table 2

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<td>$\Delta t-3$</td>
<td>-.057 (.373)</td>
<td>-.137 (.234)</td>
<td>.027 (.098)</td>
</tr>
<tr>
<td>$\Delta t-4$</td>
<td>-.392 (.367)</td>
<td>.125 (.231)</td>
<td>-.018 (.097)</td>
</tr>
<tr>
<td>$\Delta t-5$</td>
<td>-.101 (.350)</td>
<td>.092 (.220)</td>
<td>.040 (.092)</td>
</tr>
<tr>
<td>$\Delta t-6$</td>
<td>-.253 (.327)</td>
<td>.162 (.206)</td>
<td>-.019 (.086)</td>
</tr>
<tr>
<td>$\Delta t-7$</td>
<td>-.305 (.263)</td>
<td>.126 (.165)</td>
<td>-.078 (.069)</td>
</tr>
<tr>
<td>$\Delta t-1$</td>
<td>1.818 (1.315)</td>
<td>.139 (.826)</td>
<td>.584 (.346)</td>
</tr>
<tr>
<td>$\Delta t-2$</td>
<td>.301 (1.102)</td>
<td>-.307 (.692)</td>
<td>-.195 (.290)</td>
</tr>
<tr>
<td>$\Delta t-3$</td>
<td>.092 (1.173)</td>
<td>.393 (.757)</td>
<td>-.076 (.309)</td>
</tr>
<tr>
<td>$\Delta t-4$</td>
<td>.989 (1.149)</td>
<td>-.611 (.722)</td>
<td>.391 (.303)</td>
</tr>
<tr>
<td>$\Delta t-5$</td>
<td>-.062 (1.148)</td>
<td>.336 (.721)</td>
<td>.065 (.302)</td>
</tr>
<tr>
<td>$\Delta L_t$</td>
<td>$\Delta I_t$</td>
<td>$\Delta R_t$</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>$\Delta L_{t-6}$</td>
<td>$-1.424$</td>
<td>$0.53$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.105)$</td>
<td>$(.694)$</td>
<td>$(.291)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta L_{t-7}$</td>
<td>$1.712$</td>
<td>$0.509$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.178)$</td>
<td>$(.740)$</td>
<td>$(.310)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-1}$</td>
<td>$0.827$</td>
<td>$0.406$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.129)$</td>
<td>$(.709)$</td>
<td>$(.297)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-2}$</td>
<td>$2.138$</td>
<td>$0.696$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.104)$</td>
<td>$(.693)$</td>
<td>$(.291)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-3}$</td>
<td>$2.605$</td>
<td>$0.585$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.139)$</td>
<td>$(.715)$</td>
<td>$(.300)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-4}$</td>
<td>$0.561$</td>
<td>$0.400$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.193)$</td>
<td>$(.750)$</td>
<td>$(.314)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-5}$</td>
<td>$1.069$</td>
<td>$0.136$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.255)$</td>
<td>$(.788)$</td>
<td>$(.331)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-6}$</td>
<td>$2.756$</td>
<td>$0.336$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.217)$</td>
<td>$(.764)$</td>
<td>$(.320)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-7}$</td>
<td>$1.174$</td>
<td>$0.729$</td>
<td></td>
</tr>
<tr>
<td>&amp; $(1.172)$</td>
<td>$(.736)$</td>
<td>$(.309)$</td>
<td></td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>$0.577$</td>
<td>$0.562$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$2.159$</td>
<td>$2.310$</td>
<td>$1.915$</td>
</tr>
</tbody>
</table>
The values of the Durbin-Watson statistic imply that the use of first differences proved to be a useful means of removing the serial correlation from the residuals. In evaluating these results, it must be recalled that the first difference transformation almost invariably reduces $R^2$. First differences are often little more than a randomizing device; equation \((v)\)' bears witness to this.

Unfortunately, the use of first differences introduces so much "noise" into the system that very little can be inferred from these results concerning the structure of the reaction paths. In general, the standard errors are so large compared to the parameter estimates that the summation of the coefficients would not provide any useful information concerning the total effect of the exogenous variables on $R$, $I$, and $L$.

Neither the demand deposit variable nor the time deposit variable provide much new information. The random movements in $D$ were apparently so large that no pattern can be discerned from the coefficients. The lack of a pattern to the coefficients for the time deposit variables adds substance to the argument that the predictability of $T$ enables the banks to make their portfolio adjustments prior to the actual change in the level of the account.

The pattern of negative coefficients for changes in the capital account in equation \((iv)\)' and the pattern of positive coefficients in equation \((v)\)' for the same variable tend to substantiate the contention that these funds are transferred rather quickly from $R$ to $I$, but more slowly into $L$. The coefficients for changes in the yield on 3-5 year
securities provide little information. The loan equation does not show the substitution effect which was observed in equation (III). This tends to make the estimates for equation (III) somewhat suspect. There is no particular indication of lags in adjustment associated with variations in \( i \).

It is not obvious, however, that there are any substantial lags associated with changes in this variable. The movements in \( i \) from week to week are so small as to make the quick adjustment of the loan account quite possible. If the lags are short, the coefficient in equation (III) approximates the equilibrium value of the slope. This statement is subject to the qualification that equation (VI)' does not support the evidence of equation (III).

The most interesting coefficient estimates are those associated with changes in \( Y^* \). Here, the relationships appear to be fairly stable. There appears to be no particular tendency for the sizes of the coefficients in equations (IV)', (V)', and (VI)' to decline over the eight week period. This implies that the process of adjustment of \( R, I, \) and \( L \) to variations in the level of income is not completed at the end of eight weeks.

It is of some interest to note that when the variable \( Y^* \) is replaced by \( Y \), the relationships become less stable. The coefficients for changes in \( Y \) are given in table 3 below.
Table 3

<table>
<thead>
<tr>
<th></th>
<th>$\Delta R_t$</th>
<th>$\Delta I_t$</th>
<th>$\Delta I_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Y_{t-1}$</td>
<td>.003 (.003)</td>
<td>.001 (.002)</td>
<td>.000 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-2}$</td>
<td>.003 (.003)</td>
<td>.001 (.002)</td>
<td>.000 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-3}$</td>
<td>.003 (.003)</td>
<td>-.002 (.002)</td>
<td>.000 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-4}$</td>
<td>.001 (.004)</td>
<td>-.001 (.002)</td>
<td>-.000 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-5}$</td>
<td>-.001 (.004)</td>
<td>-.002 (.002)</td>
<td>-.001 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-6}$</td>
<td>-.008 (.004)</td>
<td>.001 (.002)</td>
<td>-.002 (.001)</td>
</tr>
<tr>
<td>$\Delta Y_{t-7}$</td>
<td>-.005 (.003)</td>
<td>.002 (.002)</td>
<td>-.001 (.001)</td>
</tr>
</tbody>
</table>

$R^2$: 0.547 0.258 0.482

$d$: 2.031 2.288 1.896
These estimates were obtained from a set of regressions which are the same as (IV)', (V)', and (VI)' except that changes in \( Y^* \) were replaced by changes in \( Y \). The greater instability of these estimates and the lower \( R^2 \) for the reserve asset and loan equations imply that there was a less than proportional response of \( L \) to variations in \( Y \). This is something which was not apparent from equations (I) - (III). It appears that the issue of nonlinearities is an open one. Further research is obviously needed to resolve the issue.

While the estimates of equations (IV)', (V)', and (VI)' strongly suggest that the lags in response are variable and relatively long for variations in \( D \), \( C \), and \( Y \), little more can be inferred from the evidence. It appears that more data are needed to make it possible to estimate the pattern and extent of the distributed lags. With the availability of data which span a longer period of time, it will be possible to take monthly averages of the weekly balance sheet items. Such average data should be much better behaved than the weekly data used here. It is quite conceivable that first differences of monthly data will yield more interesting results. Obviously, such data will also serve as a better basis for the estimation of equations (I) - (III).

An attempt must be made now to summarize the analysis and to link the discussion to monetary policy. This is the subject of the next and final chapter.
VI. CONCLUSIONS AND POLICY IMPLICATIONS

The analysis of this study provides some interesting, but tentative, conclusions concerning bank behavior. These conclusions, in turn provide some strong evidence concerning the importance of the influence of monetary policy on the level of commercial bank loans. In this chapter, the essential theoretical arguments are summarized and the conclusions drawn. The discussion then turns to the policy implications of the analysis.

It appears that the conceptual framework which underlies this study represents a useful analytic simplification. The separation of the total asset portfolio into a transactions balance, a security portfolio, and a loan portfolio appears to provide some interesting insights into bank behavior. The size of each asset group is conditioned by the behavioral decisions of both the bank and borrowers.

The behavior of the bank involves the balancing of the expected returns and estimated risks of each asset group against the returns and risks of the other two. While the supply of available funds constrains the bank's acquisition of total assets, the relative expected returns and risks of the three asset groups condition the share of each in the total asset portfolio. The expected return on loan is not independent of the size of the loan portfolio. The greater the bank's purchases of loans per unit of time, the lower the expected return on these new loans. The bank must reduce the stringency of its loan terms in order to attract new borrowers. This situation makes it necessary for the analysis to include the demand for bank loans. If the expected rate of return on new loans is influenced by the
demand for these loans, the relative attractiveness of the other two portfolio groups is likewise influenced.

A crucial issue in many debates over the effectiveness of monetary policy has been the importance of the demand for loans in the determination of actual volume of new loans which banks grant. A major conclusion of this study is that bank supply conditions are an important element in the determination of the size of the loan portfolio. In fact, during the period of time covered by the empirical analysis, these supply conditions were the dominant element in the determination of the size of \( L \). During this period, the most important elements in the determination of the relative sizes of the three portfolio groups were the size and composition of the total supply of funds and the market yield on investment assets. These factors are not related to the demand for bank loans.*

It is of some interest to note in this context that regressions for \( R_t \), \( I_t \), and \( L_t \) which omit the income and cyclical dummy variables of equations (I) - (III) of Chapter V explain .86\%, .73\% and .93\% of the variances of these portfolio groups, respectively.

This evidence is not meant to imply that demand factors should not be considered in an analysis of bank portfolio selection. It does imply, however, that banks do not passively react to variations in the level of loan demand.

* The small coefficients for \( D_{t-1} \) in equations (II) and (III) of Chapter V imply that deposits derived from bank acquisition of earning assets during a given week have left the bank by the end of that week.
In the period studied, the influence of \( Y \) on the three asset groups was relatively minor. The loan portfolio responded to variations in \( Y \) only during the second half of the expansion in the level of income. The relative importance of the cyclical shift coefficients in the equations may imply that at least part of the observed lack of association between \( Y \) and \( L \) is attributable to the poor qualities of the income variable, and to the inadequacies of the demand theory. The fact remains, however, that the estimates of the sizes of the three asset groups in the previous chapter do not stand or fall with the quality of either the income variable or the demand theory.

The second major conclusion of the analysis relates to bank demand for investment assets. It was argued that this portfolio becomes extremely attractive during the early stages of the decline in the level of income. The available empirical evidence tends to confirm this hypothesis. Apparently there is an incentive to shift funds into investment assets during the decline in \( Y \). It is perhaps worth pointing out that the lack of any relationship between \( Y \) and \( L \) during periods of declining income implies that the discussion at the end of Chapter IV has substance. The rate of loan repayment apparently cannot keep pace with the decline in the level of loan demand. This implies that banks are restricted in their ability to shift funds into investment assets. During the period studied, the actual substitution came out of reserve assets rather than loans. The evidence suggests that the rate of loan repayment helps to condition the rate of increase in security prices. The fact that banks cannot shift quickly out of \( L \) and
into I helps to explain why interest rates and security prices have managed to retain their cyclical characteristics.

It was stated in Chapter I, that the purpose of this study was to attempt to determine the influence of monetary policy on the behavior of commercial banks. With the statement of the two major conclusions of the analysis, the discussion now turns to the policy implications of this analysis. The discussion of the influence of policy is carried out in terms of the banking system as a whole. This approach requires the assumption that the analysis of the behavior of an individual commercial bank can be applied to banks in general. In terms of the discussion which follows, this does not appear to be a bad assumption.

For the banking system as a whole, both changes in reserve requirements and the use of open market operations work directly on available bank reserves.* Changes in reserve requirements alter the relationship between a fixed quantity of total reserves and the proportion of the total which must be held against deposits. Such changes initially alter the volume of excess reserves in the system. Changes in total reserves through open market operations have essentially the same initial effect. When the Federal Reserve System purchases securities on the open market, there is an immediate increase

* The framework of the three asset model does not allow an analysis of the effects of changes in the discount rate on bank behavior.
in the volume of reserves which equals the value of the securities purchased. The bulk of the increase in total reserves is in the form of excess reserves.

The use of either policy instrument produces an initial change in the excess reserves in the system. Each bank which is initially affected by the policy move experiences a change in its cash position. In terms of the analysis of this study, either a reduction in reserve requirements or a purchase of securities on the open market, produces an increase in the reserve asset portfolios of these banks above their final, equilibrium levels. This corresponds to the initial rise in $R_t$ depicted in figure 1a of Chapter V.*

Banks find themselves with reserve asset portfolios which are above their desired levels. They attempt to reduce the size of the cash balance through the purchase of earning assets. As these assets are acquired, new demand deposits are created. These deposits increase the reserve asset portfolios of other banks and the process continues. The total process of asset acquisition comes to an end when all banks have adjusted the relative sizes of their asset portfolios to the point where the relative rates of return on the three portfolios are commensurate with the risks involved in holding them. In the final equilibrium, all three asset groups are, of course, larger than before.

* It is assumed that banks treat both changes in their cash positions alike. It is assumed, for example, that banks react in the same manner to a $1.00 increase in excess reserves generated by a reduction of reserve requirements as they do to $1.00 of excess reserves produced by receipt of a new deposit account. As a first approximation, this is probably not a bad assumption.
In a loose sense, the Federal Reserve System is able to determine the total level of demand deposits in the economy through its control over the total level of reserves. While this control is not perfect, because of changing bank attitudes concerning the advisability of holding excess reserves, it is sufficiently close for purposes of the discussion at hand. Recent research indicates that the propensity of banks to hold excess reserves is predictable. To the extent to which this is true, the Federal Reserve System is capable of determining the level of demand deposits in the system, if it so chooses.

The ability of the Federal Reserve System to control the level of demand deposits implies that it is also able to exert a sizeable influence of the total level of bank held assets. The relationship between the level of demand deposits and the sizes of the three asset groups was not clearly established in the empirical work of Chapter V. This appears to have been the result of the behavior of the demand deposit variables during the period studied. The lack of regular, predictable movements in the levels of such deposits during the period made the estimation of their relationship with $I$ and $L$ difficult. This lack of predictability made the lags in response too long to allow accurate estimation of the equilibrium relationships between the level of demand deposits and the sizes of the three portfolio groups.*

* Presumably, an empirical analysis which could use monthly averages of the variables would yield estimates of the equilibrium relationships.
The very convincing results for the time and savings deposit variable strongly suggest that the supply of bank funds has an extremely important impact on the sizes of the three portfolios. This implies, in turn, that through its control over the level of demand deposits, the policies of the Federal Reserve System have an equally important impact on the sizes of the three portfolios.

It should be stressed that the evidence of this study does not provide an adequate basis for quantitative statements concerning the impact of policy on commercial bank behavior. The existence of lags in response and the nature of the data prevent this. It does appear possible to venture some qualitative statements, however. Through its open market operations and its control over reserve requirements, the Federal Reserve System can determine the total supply of funds available to banks within tolerably narrow limits. Its control over bank reserves enables the Federal Reserve to counteract any undesirable movements in the levels of the other variables in the model. This implies that the monetary authority exerts the final and dominant influence on the size of the aggregate loan portfolio.

The manner in which banks allocate their available funds among alternative assets depends upon such things as the strength and direction of movement of the level of loan demand, the existing composition of the supply of funds, the current yield on investment assets, and upon the expected future movements in security prices. The influence which variations in the level of demand deposits exert on the banks' loan portfolios is a matter of predictable, economic choice. The empirical evidence strongly suggests that the
size of the total supply of funds has had, and will have, an extremely
important influence of the size of the aggregate loan portfolio. The
Federal Reserve System is capable of determining the level of one of the
most important components of the supply of funds. The extent of the influence
of a given change in the level of demand deposits depends upon the behavior
of the other components of the supply of funds, and upon the relative rates
of return and estimated risks on the three asset groups. All these effects
are capable of prediction.

Banks do not blindly meter out a constant proportion of their available
funds to the three portfolio groups; and they do not passively react to
variations in the level of loan demand. The link between monetary policy
and commercial bank loans is a strong one; but the level of demand deposits
is only one factor among many in the portfolio decision. If the level of
the aggregate loan portfolio is a relevant target variable for policy, the
target can be hit only if these other factors are explicitly and care-
fully taken into account in the policy decision. As matters now stand,
there has not been enough information assembled to allow one to accurately
forecast the size of the impact of a given change in the level of demand
deposits on the commercial banking system. Only the direction of response
is clear.

The available evidence implies that the policies of the Federal
Reserve System have an important, and perhaps a potentially dominant, effect
on the level of loans in the banking system as a whole. The implications
of this conclusion are not clear. This study has been interested only in
partial relationships. There has been no attempt to link the behavior of other financial institutions to that of commercial banks. Given that a reduction in the level of demand deposits leads to a decline in the level of commercial bank loan portfolios, we are still left with the problem of determining what happens to the borrowers who no longer get adequate accommodation at their banks. To what extent are borrowers able to obtain loans from other financial institutions? The total reaction of the financial sector is equally unclear when the levels of deposits and commercial bank loans rise. To what extent does this rise come at the expense of other financial institutions? Until these questions can be answered, our knowledge of the effect of monetary policy on the economy is incomplete to say the least.

It is certainly true that there is no one-to-one correspondence between the level of new bank loans and the level of spending on final goods and services. The determination of the relationship between these loans and the level of income no doubt lies far in the future. This study makes no pretense of having isolated the monetary mechanism. It does appear to shed some new light on the behavior of an integral part of the total mechanism, however.

The results of this study strongly imply that further research in this area will pay high returns. The availability of more and better data will greatly increase the chances of making useful, quantitative statements concerning the impact of monetary policy on commercial banks. This study, which used weekly balance sheet data to form a time series which covers only a three year period, provided a rather surprising amount of information.
The use of smoothed data to form a time series which covers a longer period of time should provide the necessary elements for obtaining usable estimates of the coefficients of the model. Coefficients which not only estimate the immediate response of banks to variations in the exogenous variables, but also the equilibrium relationships involved. With these estimates will come some true indication of the influence which commercial banks exert on the real sectors of the economy.
APPENDIX A: INVESTOR EQUILIBRIUM

The purpose of this appendix is to complete the brief review of the basic elements of portfolio theory begun in Chapter I. Some characteristics of investor equilibrium are considered in the two asset case.

As in Chapter I, a risk avverting investor is considered whose preferences are represented by a utility function which is quadratic in return. The investor attempts to find the combination of assets $x_1$ and $x_2$ which maximize his expected utility. For simplicity, the expected returns on the two securities, $E_1$ and $E_2$, are assumed to be non-correlated.

The locus of efficient portfolios, i.e., the set of portfolios which minimize risk for a given expected return, is easily obtained. As before, the expected return, per dollar, on a portfolio is given by $E_T = w_1 E_1 + w_2 E_2$, where $w_1 + w_2 = 1$ and $w_1, w_2 < 1$. In the case of non-correlated returns, the variance of return, per dollar, is given by $\sigma^2_T = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2$.

The locus of efficient portfolios minimizes $\sigma^2_T$ subject to a given expected return. The expressions for the shares of $x_1$ and $x_2$ in the portfolios which are members of the efficient set are obtained from the expressions for $E_T$ and $\sigma^2_T$ using the method of Lagrange multipliers. The constrained minimization yields: $w_1 = \frac{E_T - E_2}{E_1 - E_2}$, $w_2 = \frac{E_T - E_1}{E_2 - E_1}$. These values of $w_1$ and $w_2$ are the shares of $x_1$ and $x_2$ in the total asset portfolio.
which minimize the risk of return associated with any given expected return on the portfolio.

The properties of the locus of such efficient portfolios are presented in terms of the following expressions:

\[
\sigma_T^2 = w_1^2 \sigma_1^2 + (1-w_1)^2 \sigma_2^2
\]

\[
E_T = w_1 E_1 + (1-w_1) E_2 = w_1 (E_1 - E_2) + E_2
\]

\[
w_1 = \frac{E_T - E_2}{E_1 - E_2}.
\]

The values of \( E_1, E_2, \sigma_1^2 \) and \( \sigma_2^2 \) are considered to be given. The problem is to determine the response of \( \sigma_T \) to variations in \( E_T \). Such variations result from changes in the relative shares of the two assets in the total portfolio.

\[
\sigma_t = \left[ w_1^2 \sigma_1^2 + (1-2w_1 + w_1^2) \sigma_2^2 \right]^{1/2}
\]

\[
\frac{\partial \sigma_T}{\partial E_T} = \left[ \frac{2w_1 \sigma_1^2 \frac{\partial w_1}{\partial E_T} - 2 \sigma_2^2 \frac{\partial w_1}{\partial E_T} + 2w_1 \sigma_2^2 \frac{\partial w_1}{\partial E_T}}{2[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2]^{1/2}} \right]
\]

\[
= \frac{[w_1 \sigma_1^2 - \sigma_2^2 + w_1 \sigma_2^2] \frac{\partial w_1}{\partial E_T}}{\sigma_T}, \quad \frac{\partial w_1}{\partial E_T} = \frac{1}{E_1 - E_2}
\]

and
\[
\frac{\partial \sigma_T}{\partial E_T} = \frac{w_1(\sigma_1^2 + \sigma_2^2) - \sigma_2^2}{(E_1 - E_2)\sigma_T} \geq 0 \quad \text{for} \quad w_1(\sigma_1^2 + \sigma_2^2) \leq \sigma_2^2.
\]

\[
\frac{\partial \sigma_T}{\partial E_T} = 0 \quad \text{for} \quad w_1(\sigma_1^2 + \sigma_2^2) = \sigma_2^2, \quad \text{i.e., for} \quad w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}.
\]

As \( w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \) is the minimum risk portfolio, \( \frac{\partial \sigma_T}{\partial E_T} \geq 0 \) for every point on the locus of efficient portfolios. The second derivative of the locus is also of interest.

\[
\frac{\partial^2 \sigma_T}{\partial E_T^2} = \frac{(E_1 - E_2)\sigma_T(\sigma_1^2 + \sigma_2^2) - [w_1(\sigma_1^2 + \sigma_2^2) - \sigma_2^2]^2}{[(E_1 - E_2)\sigma_T]^3} > 0 \quad \text{for} \quad E_2 > E_1.
\]

Both the first and second derivatives are positive. The investor can obtain increased expected return on his portfolio only by experiencing greater portfolio risk. The greater the share of the high risk asset in the portfolio, and hence the greater \( E_T \), the greater the increase in \( \sigma_T \) associated with a given increase in \( E_T \); the locus is convex from the origin.

In order to determine the actual shares of \( x_1 \) and \( x_2 \) in the portfolio which the investor chooses, it is necessary to specify the properties of his preference function. Under the assumption that the function is quadratic in return, his expected utility is given by \( E(u) = E_T + bE_T^2 + c\sigma_T^2 \).

---

* The utility scale is unique only up to an additive or a multiplicative constant.
The preference function is considered in terms of a set of indifference curves. Each indifference curve represents the combinations of $E_T$ and $\sigma_T$ which produce the same level of expected utility. In order to exclude the possibility of negative marginal utility, the quadratic utility function can be used only over the range $E_T < -\frac{1}{2b}$ for $b < 0$. The slope of an indifference curve is obtained by setting the total differential of expected utility equal to zero, which yields

$$\frac{\partial \sigma_T}{\partial E_T} = -\frac{1 + 2bE_T}{2b\sigma_T} > 0 \text{ for } b < 0,$$

$$E_T < -\frac{1}{2b}; \quad \frac{\partial^2 \sigma_T}{\partial E_T^2} = -\frac{2b\sigma_T - (1 + 2bE_T)}{2b\sigma_T^2} < 0.$$

The investor is willing to accept added risk only if he is compensated by increased expected return. At a given level of expected utility, the greater $\sigma_T$, the greater the increase in $E_T$ required to induce the investor to accept added risk. The indifference curve has a positive slope and it is concave from the origin.

Given the properties of both the utility function and the locus of efficient portfolios, it is possible to discuss the characteristics of investor equilibrium. Obviously, the investor will choose a portfolio which is a member of the set of efficient portfolios. He chooses the shares of assets $x_1$ and $x_2$ in the total portfolio which maximize his expected utility. The maximum is achieved at the point of tangency of an indifference curve and the locus of efficient portfolios.\(^1\)
Let \( m^* \) be the slope of the indifference curve, i.e., \( m^* = -\frac{1 + 2bE_T}{2b\sigma_T} \)

and let \( m \) be the slope of the locus of efficient portfolios,

\[
m = \frac{\frac{1}{2}(\sigma_1^2 + \sigma_2^2) - \sigma_2^2}{(E_1 - E_2)\sigma_T}.
\]

In equilibrium, \( m^* = m \) and

\[
w_1 = \frac{1 + 2bE_T(E_1 - E_2)\sigma_2}{2b\sigma_T[(\sigma_1^2 + \sigma_2^2) - \sigma_2^2]}.
\]

This equilibrium position represents the portfolio mix which maximizes the investor's expected utility given \( E_1, E_2, \sigma_1^2, \sigma_2^2 \). The actual expected return and risk per dollar of the equilibrium portfolio are determined by \( v_1 \) and \( w_2 \). The value of \( v_1 \) derived above specifies \( E_T \) and \( \sigma_T^2 \) in the two asset case.

Up to this point it has been assumed that the investor conceives the expected returns and risks on the two assets to be constant. The remaining problem to be considered is the investor's response to variations in the expected return on one of the securities, say, \( E_2 \). Of particular interest are the conditions under which an increase in \( E_2 \) induces the investor to increase the share of asset \( x_2 \) in the portfolio.

It is not completely obvious that an increase in \( E_2 \) will induce the investor to increase \( w_2 \). It is conceivable that at the new point of equilibrium, the investor will hold more, rather than less, of asset \( x_1 \).
Such a reaction will occur if he views the increase in $E_2$ as enabling him to "afford" to hold more of the less risky asset, $x_1$. An increase in $E_2$ implies that the investor can earn a greater $E_T$ at the same $\sigma_T$, or conversely, that he can earn the same $E_T$ at a lower risk. If the investor chooses to earn the same expected return at smaller risk, the response is the counterpart of a negative income effect overpowering the positive substitution effect in the theory of consumer choice. The discussion is interested in the conditions under which an increase in $E_2$ induces the investor to substitute $x_2$ for $x_1$, i.e., to decrease $w_1$.

This problem is approached by considering the reaction of $m^*$ and of $m$ to variations in $E_2$. The estimated risks of the two assets, $\sigma_1$ and $\sigma_2$, are assumed to remain constant along with $E_1$. An increase in $E_2$ enables the investor to obtain a higher expected return at the same portfolio risk. There is a new locus of efficient portfolios. This locus offers a tradeoff between $E_T$ and $\sigma_T$ which is more favorable to the investor than the former relationship. He is able to achieve a higher level of expected utility.

Given $\sigma_1$ and $\sigma_2$, $\sigma_T$ will rise only if there in an increase in $w_2$. The response of the shares of the two assets is a dollar of the total portfolio depends upon the reactions of $m^*$ and of $m$ to variations in $E_2$, given $E_1$. The investor will increase $w_2$ only if the slope of the locus of efficient portfolios at $\sigma_T$ is now less than the slope of the indifference curve at this point. If this condition holds at $\sigma_T$, the
investor can achieve equilibrium only if he increases $w_2$ and hence $s_T$.

The problem of the relative reactions of the two slopes is approached by considering the response of the ratio $\frac{m^*}{m} = M$ to variations in $E_2$.

If $\frac{\partial M}{\partial E_2} > 0$, $m^*$ has risen relative to $m$, and the investor will increase $w_2$.

$$M = \frac{m^*}{m} = -\frac{\frac{1}{2}b\sigma_T}{\nu_1(\sigma_1^2 + \sigma_2^2) - \sigma_2^2}$$

$$= -\frac{\frac{1}{2}b\sigma_T}{(E_1 - E_2)\sigma_T}$$

$$\frac{\partial M}{\partial E_2} = -\frac{1 + 2bE_1}{2b\nu_1(\sigma_1^2 + \sigma_2^2) - \sigma_2^2} > 0 \text{ for } b < 0, \quad E_T < -\frac{1}{2b}$$

The necessary and sufficient conditions for an increase in $E_2$ to produce an increase in $w_2$ are that the investor is a risk averter and that the equilibria be obtainable in the range of nonnegative marginal utility.

It is concluded that in the case described, a risk averting investor will substitute the high risk asset for the one with the lower risk whenever there is an increase in the expected return on the former. This conclusion completes the review of the elements of the theory of portfolio selection.
APPENDIX B: DESCRIPTION OF BALANCE SHEET DATA

The purpose of this appendix is to present the components of the aggregate asset and liability data used in the empirical analysis. The analysis of Chapter V used observations which were summed over all banks in the sample for each week (Wednesday) in the three year period. Each of the aggregate balance sheets appears as follows:

AGGREGATE COMMERCIAL BANK BALANCE SHEET

Assets:

1. Reserve Assets
   1. excess reserves
   2. vault cash *
   3. "due from" balances
   4. Federal funds sales (purchases a negative item)
   5. borrowing from Federal Reserve Bank (a negative item)
   6. loans for purchasing or carrying securities
   7. loans to financial institutions
   8. Treasury bills
   9. Treasury certificates of indebtedness
   10. Treasury notes and bonds maturing within one year

* Over the period studied, the banks in the sample were allowed to count vault cash as in reserves in the following manner: in excess of 2 and 1% effective December 3, 1959 and September 1, 1960, respectively. Effective November 24, 1960, all vault cash.¹
II. Investment Assets
   1. U.S. bonds maturing within 1 to 5 years
   2. U.S. bonds maturing after 5 years
   3. other securities

III. Loan Assets
   1. commercial and industrial loans
   2. agricultural loans
   3. real estate loans
   4. other loans (consumer, etc.)

Liabilities

I. Demand Deposits **
   1. individuals, partnerships, and corporations
   2. states and political subdivisions
   4. domestic interbank
   5. foreign

II. Time and Savings Deposits ***

Capital Accounts

** The total demand deposits account is considered net of cash items in process of collection. Each demand deposit component is considered net of required reserves.

*** Net of required reserves.
Three of the above balance sheet items require some explanation. The reserve asset item, "due from" balances, represents the aggregate deposits of the banks in the sample which are held by banks not in the sample. By a like token, purchases and sales of Federal funds represent transactions with banks not in the sample. Finally, the liability item, domestic interbank demand deposits, represents deposit "due to" banks not in the sample.

The balance sheet above indicates that sufficient data exist to allow analyses of the relative shares of the components in each of the three asset groups. Appendix C attempts to outline such an analysis for the determination of the composition of the reserve asset portfolio.
APPENDIX C: THE COMPOSITION OF THE RESERVE ASSET PORTFOLIO

The reserve asset portfolio is a transactions balance. It is held to enable the bank to smooth out the differences in the timing and size of the inflows and outflows of funds. While the portfolio is a transactions balance, there is no reason to suppose that it needs to be composed entirely of cash. The balance sheet reproduced in Appendix B clearly indicates that the portfolio is composed of many different types of liquid assets; cash is only one asset among many in the reserve asset portfolio. The purpose of this appendix is to outline briefly the prime determinants of the composition of the portfolio. The determinants of the share of excess reserves in the portfolio is of particular interest. For the analysis which follows it is useful to regard the level of the reserve asset portfolio as fixed. The discussion of the composition of the portfolio involves an analysis of the shares of the various assets in a dollar of the reserve asset portfolio.

The bank has access to assets which closely approach cash in their risk and liquidity features, but which yield a positive expected rate of return. The assets in R range in liquidity and expected return from cash to Government securities with up to 1 year maturity. The proportion of cash in the portfolio depends upon the expected size and pattern of transactions, the expected rates of return on competing assets, the transactions cost involved in the purchase and sale of securities, and upon the costs of borrowing. Here, the analogy between cash (excess reserves) in the reserve asset portfolio and the role of cash in the transactions balance of interest
theory is complete. The higher the volume of transactions, the higher transactions costs, and the lower the expected returns on near money assets, the greater the share of cash in the reserve asset portfolio. *

At any point in time, the desired composition of \( R \) depends, in large part, upon the patterns of movement of the levels of deposits and of loan demand which are expected to prevail over some period of time, say a year. Due to the seasonal characteristics of the levels of both deposits and of loan demand, the bank is able to structure the maturity of its reserve assets in a manner which provides funds through maturity at the times that they are expected to be needed. \(^2\) If the exact timing of the deposit and loan demand movements were known with certainty, there would be relatively little need for the bank to hold excess reserves. Maturing assets could provide much of the needed cash. In this case, the only constraint imposed on the purchase of earning assets is the brokerage cost. These movements are not known with certainty, however, and the bank is induced to hold more of its transactions balance in cash form in order to insulate it from unexpected short-term variations in the levels of deposits and loan demand.

Perhaps the easiest way to describe the determinants of the share of cash in the transactions balance is to point out that the arguments are

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* The treatment of excess reserves as a component of a transactions balance, or portfolio, differs from existing work in this area. Existing analyses of bank demand for excess reserves have attempted to relate this demand directly to total bank reserves or deposits, rather than to the assets with which excess reserves most directly compete. \(^1\)
essentially the same as those used to describe the share of \( r \) in the total asset portfolio. The bank places newly acquired funds into earning assets only if it is relatively sure that the funds will remain with it sufficiently long to allow the interest income to compensate it for the cost involved in the sale of the asset. For the components of \( r \) this is a very short time indeed.*

The bank has the option of borrowing rather than selling assets to meet deposit losses or increases in the level of loan demand. It also has the option of retiring existing debt with any newly acquired funds. The incentive to acquire or to retire debt in this manner depends upon the marginal costs of borrowing relative to the expected returns and risks on assets. If the marginal costs of borrowing are less than the expected marginal return on any asset relative to its risk, the bank has an incentive to borrow to acquire that asset. If the marginal costs of borrowing exceed the expected rate of return on assets relative to their risks, the bank has every incentive to sell its assets and to retire any existing debt.

Observed bank response to relative rates of interest would suggest that the sort of behavior depicted above does not exist. One can observe banks which acquire little or no debt even during times when the stated borrowing rates are below the current market yields on even the most risk-free assets. This sort of situation appears to exist because the actual

* For Federal funds sales this is usually a matter of a single day; for Treasury bills, approximately two days.\(^3\)
marginal costs of borrowing tend to be higher than the stated rates.

A bank can borrow either from other banks or from its Federal Reserve Bank. The actual marginal cost of such borrowing varies not only with the stated rates, but also with the length of time the bank has been in debt and with the size of the loan. Large Federal funds purchases can prove to be difficult to make, and continued large purchases may be unsure and expensive.\(^4\) The same sort of argument applies to the use of the discount window. If the bank finds itself in the position of needing borrowed funds for some time, it may encounter difficulty in obtaining continued use of the discount facility.\(^5\) It appears that both Federal funds purchases and loans from the Federal Reserve Bank have greatest relevance for short-term portfolio adjustments and/or for relatively small amounts of funds. Within this context, the bank is sensitive to relative rates of interest. The use of debt competes most directly with very short-term assets such as cash, sales of Federal funds, and Treasury bills. The share of cash in the reserve asset portfolio varies positively with the costs of borrowing.

In summary, the share of cash (excess reserves) in the reserve asset portfolio varies positively with the volume and lack of predictability of transactions, the costs of borrowing, and the costs associated with buying and selling earning assets. It varies negatively with the expected rates of return on competing assets in the portfolio. For banks with easy access to the money market, the proportion of cash in \( R \) should be at, or near, a minimum except during periods of extremely low interest rates. Over a wide range of such rates, the demand for excess reserves would appear to be more a function of ease of access to the instruments of the money market than of anything else.
REFERENCES

Chapter I


   and


   and


   and


Chapter II


and


2. For two recent analyses of the influence of deposit instability on bank behavior Cf., Porter, Op. Cit., pp. 328-334 and 346-349; and


7. See reference 5 above.

Chapter III


2. Such data are published daily in the New York Times.


5. Ibid.
   and

   also Cf.,


Chapter IV


Chapter V


and


and


Chapter VI


   and

Appendix A


Appendix B


Appendix C


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4. Ibid., pp. 37 and 75-78.

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