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A TEST OF THE SPECIFICATION OF THE AGGREGATE PRODUCTION FUNCTION

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In 1957 Robert M. Solow published a study [7] in which the concept of the aggregate production function was applied to the American non-farm, private sector for the years 1909-1949 in an effort to measure the importance and pace of technological change. Initially, the only property postulated for this aggregate production function was that it was linear homogeneous (possessed constant returns to scale); however, after making a test, Solow concluded that one might make the further assumption of only neutral technological change. Solow concluded that his $A(t)$ series (which may be interpreted as measuring the impact of neutral technological change) rose at the rate of approximately 1.5 per cent per annum and that the bulk of the total growth in output per man-hour (roughly 90 per cent) was attributable to technical change, rather than to increased capital per man-hour worked.

Two more recent articles, in which the form of the aggregate production function was more fully specified, have largely confirmed the broad outlines of Solow's 1957 study. Benton F. Massell, working with data from the U.S. manufacturing sector for 1919-1955, also found that technical

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change accounted for roughly 90 per cent of the growth in output per man-hour in this sector [5]. Murray Brown and Joel Popkin, analyzing data from the U. S. non-farm, domestic sector, concluded that, over the period 1890-1958, neutral technological change was responsible for the bulk (roughly 60 per cent) of the growth in total output. [3]. Both Massell and Brown-Popkin assumed that the aggregate production function had the well-known Cobb-Douglas form.* Massell also assumed constant

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* In the case of Massell's production function, this form was implicit in the assumption that the share of capital in total output was a constant, except for random disturbances, over the period studied. With the additional assumptions of constant returns to scale and the equality of marginal products to real factor prices, the solution of the implied partial differential equation entails a production function of the Cobb-Douglas form.

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returns to scale, while Brown and Popkin allowed returns to scale to vary, even though they in fact found constant returns to scale in their data in two out of three sub-periods ("epochs") studied.**

** Brown and Popkin also developed a technique which allowed them to measure non-neutral technological change between, though not within, these sub-periods or "epochs."

In 1961, Solow, along with Kenneth Arrow, Hollis B. Chenery, and Bagicha Minhas, suggested an alternative specification of the aggregate production function, one in which the elasticity of substitution is an unspecified constant. [1]. (The economic interpretation requires this constant to be a positive number.) Arrow, Chenery, Minhas, and Solow also assumed constant returns to scale, in their derivation of this class of production functions, which are called constant-elasticity-of-substitution or CES production functions. The special cases are interesting: if the elasticity of substitution is zero, one obtains the familiar fixed proportions or "Leontief" production function and if the elasticity of substitution is infinite, the production surface degenerates into a plane and hence the isoquants become straight lines. If the elasticity of substitution is unity, the CES production function is one of the Cobb-Douglas variety, with the exponents of the labor and capital inputs (the elasticities of output with respect to labor and capital) also summing to unity (because of the constant returns to scale assumption).* Arrow,

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* That the unitary elasticity of substitution case should turn out to give a Cobb-Douglas production function is hardly surprising. It is true that, with a Cobb-Douglas production function and competitive factor pricing, cost minimization results in constant factor shares, i.e. the factor shares will be independent of relative factor prices. (This proposition is developed in the next section.) However, it is also true that with competitive factor pricing and cost minimization, the existence of constant factor shares implies a unitary elasticity of substitution. (See, e.g., Sidney Weintraub, [9], p. 70.) Thus a Cobb-Douglas production function must have everywhere a unitary elasticity of substitution. The authors of [1] have in effect proved the converse of this proposition (for the case of constant returns to scale).

Chenery, Minhas, and Solow estimated the elasticity of substitution by means of an international cross-section of specific industries: in general, they found that this parameter lies between zero and unity and that the difference from either extreme is statistically significant. The authors also fitted an aggregative CES production function to data from the American non-farm, private sector over the period 1909-1949, which were taken from Solow's 1957 article. [7]. Here, the elasticity of substitution was found to be roughly .6 and significantly different from either zero or unity. It would appear that the CES specification of the aggregate production function is superior to the Cobb-Douglas.

Before accepting this conclusion, one should notice that these tests indicated that a three parameter (actually four parameters, with an additional technology parameter) CES production function is superior to a two (or three) parameter Cobb-Douglas function. This seems to load the comparison against the Cobb-Douglas form. In this paper, the aggregative CES production function is tested against an aggregative Cobb-Douglas function with an equal number of parameters. An additional parameter is introduced into the Cobb-Douglas production function by permitting variable returns to scale; thus, in effect, we have a test as to whether constant returns to scale or a unitary elasticity of substitution is a more restrictive simplifying assumption. A secondary purpose of this paper is the attempt to quantify the influence of increasing returns to scale. In particular, one can inquire about the contribution of increasing returns to the observed increase in total output and ask how the existence of this phenomenon influences one's estimates of the importance of, and the rate of growth attributable to, "pure" technical change.

1. Theoretical considerations

Let V be total output, L the labor input, and K the (utilized) capital input. (This notation follows [1], as does the notation for the parameters of the CES production function.) The variable t denotes calendar time (in the applications, measured in years and equal to zero in 1929) and hopefully serves as a proxy for that important but little-known phenomenon, technical change. The u_i's are stochastic disturbances. The most general expression for the aggregate production function, under these circumstances, is:

$$(1) \qquad V = F(L,K,t;u_1) .$$

If we restrict ourselves to only neutral technological change* and write

* Neutral technological change is such that total output from a given combination of labor and capital inputs increases, while leaving the marginal rate of substitution between these two inputs (numerically equal to the ratio of their marginal products) unchanged. On this point, see Solow [7].

the error term in a multiplicative form, the aggregate production function may be represented by the following expression:

$$(1a) \qquad V = A(t)f(L,K)u_2 \ .$$

The CES production function, which is a further specification of the functional form of (1a), may be written:

$$(2) \qquad V = \gamma_0 \, 10^{\lambda t}\left[\delta \, K^{-\rho} + (1 - \delta)L^{-\rho}\right]^{-\frac{1}{\rho}} u_3 \ .$$

(Here, $\rho = \frac{1}{\sigma} - 1$, where σ is the elasticity of substitution.) The logarithmic variant of (2), which is the form used to compute the estimated standard deviation of residuals and other measures involving the computed residuals, is:

$$(2a) \qquad \log V = \log \gamma_0 + \lambda t - \frac{1}{\rho}\log\left[\delta \, K^{-\rho} + (1 - \delta)L^{-\rho}\right] + u_4 \ ,$$

where $u_4 = \log u_3$. (All logarithms are on the base 10, in this paper.) Equation (2a) was fitted by a two stage process; the parameters σ (or ρ) and λ were estimated from a regression relationship emanating from an equality between the marginal product of labor and the real wage. The remaining parameters can be estimated from equation (2a). As the estimation procedure is given explicitly in [1], the details of this discussion are not repeated here. In one statistical variant of (2a), a procedure identical to that outlined in [1] was followed. In a second variant, the procedure was followed exactly except for the constant term, which was obtained by forcing the computed residuals to sum to zero.

The Cobb-Douglas specification of equation (1a) is:

$$(3) \qquad V = A_o\, 10^{ct}\, L^{\alpha}\, K^{\beta}\, u_5\ .$$

(As with the CES production function, Greek letters denote parameters; A_o and c are parameters, also.) The logarithmic variant of (3) is:

$$(3a) \qquad \log V = \log A_o + ct + \alpha \log L + \beta \log K + u_6\ ,$$

where $u_6 = \log u_5$. It would appear easy to estimate the parameters of equation (3a), through the use of the standard techniques of regression analysis. In fact, a straight regression procedure gave nonsensical results, possibly because of the intercorrelation between the labor and the capital series. (See equation (3c) below.) Hence one is forced to search for an alternative method of parameter estimation.

Let w be the money wage and P_k the money price of a unit of capital services. It will be assumed that these factor markets are competitive, so that entrepreneurs take these prices as given (unaffected by their hiring decisions). The condition for cost minimization is:

$$(4) \qquad \frac{V_K}{V_L} = \frac{P_k}{w} \quad ,$$

where V_K denotes the partial derivative of V with respect to K and V_L has an analogous interpretation. It may be observed that equation (4) does not require one to assume that the product market is competitive nor that entrepreneurs actually carry profit maximization beyond cost minimization. Let us assume that the tendency to cost minimization is incomplete and subject to a stochastic disturbance. (The short run fixity of the capital stock might make this an appealing assumption.) Also, when the aggregate production function has the form (3), we have:

$$(5) \qquad V_K = \beta \frac{V}{K} \text{ and } V_L = \alpha \frac{V}{L}$$

Introducing a random disturbance into (4) in multiplicative form and also substituting relations (5), one obtains:

$$(6) \qquad \frac{\beta \frac{V}{K}}{\alpha \frac{V}{L}} = \frac{P_k}{w} u_7 \quad .$$

Algebraic manipulation yields:

$$(7) \qquad \frac{P_k K}{wL} = \frac{P_k K/PV}{wL/PV} = \frac{\beta}{\alpha} u_8 \, ,$$

where P is the price level of final output and $u_8 = 1/u_7$. The logarithmic variant of (7) is:

$$\text{(7a)} \qquad \log\left(\frac{P_k K/PV}{wL/PV}\right) = \log\frac{\beta}{\alpha} + \log u_8 .$$

If the expected value of the logarithm of u_8 is postulated to be zero, an appropriate method of estimating the ratio of β to α (the ratio of the elasticity of output with respect to capital to the labor elasticity of output) is:

$$\text{(8)} \qquad \text{est of } \frac{\beta}{\alpha} = \text{antilog}\left[\frac{1}{N}\Sigma \log\left(\frac{P_k K/PV}{wL/PV}\right)\right] ,$$

where N is the total number of observations and the summation is performed over all observations. In words, equation (8) states that an appropriate estimator is the geometric mean of the ratio of the capital share to the labor share. In the final estimation of the parameters of (3a), equation (8) was used to force the coefficients β and α to be in a given ratio and then regression techniques were used to estimate the remaining parameters. This is equivalent to estimating a regression of the form:

$$\text{(3b)} \qquad \log V = \log A_o + ct + \alpha\left[\log L + \theta \log K\right] + u_6 ,$$

where θ is equal to the ratio of β to α and is already known before performing the regression calculations.

2. The data and the problem of measuring the flow of capital services.

The data employed are those utilized in [1]; the immediate source of these data is Solow's 1957 article ([7], p. 315). The concept of real output is real private, non-farm GNP, i.e. both the government and the agricultural sectors have been excluded. The capital stock, which includes land and mineral deposits, is measured in constant dollars, while the labor input is measured in man-hours. Solow gives a series for the property share, the complement of which may be taken as the labor share.* Solow's more complete discussion indicates his sources and other details; there is, in general, no need to repeat that discussion here.

* Thus, the property share is column (4) of Solow's Table 1, and the wage share is obtained by subtracting this series from 1. The (utilized) capital series is column (3) of this table. The labor series can be unscrambled by dividing column (6) into column (3). V can be obtained by multiplying the labor input series just calculated by column (5).

One problem is, however, worth some consideration before the statistical results are presented. The capital data are most easily obtained as a series of the stock of available capital, while the notion most relevant to the production function concept is that of the flow of capital services. Solow's solution to this problem was to distinguish between available capital and capital actually employed; he assumed that capital and labor were equally underemployed during slack times and so his estimate of employed capital for a particular year was the stock of available capital multiplied by the proportion of the labor force unemployed during that year.

Solow's approach to this problem has been under criticism, and indeed he himself seemed to regard it as only a rough approximation that was better than no adjustment at all. Solow points out that this method does not indicate a change in the capital input when the flow of capital services changes from a given amount of capital actually utilized, as would occur if the length of the work week changes. Massell ([5], pp. 184-185) argues that Solow's method will not allow the substitution of capital for labor during a downturn to show up, which he (Massell) expects is likely to occur. Brown and Popkin point out ([3], p. 411) that this method is likely to increase the intercorrelation between explanatory variables, thus decreasing the reliability of regression estimates of the separate effects of these explanatory variables.

Massell and Brown-Popkin handled this problem in a different way. Although their methods differ from each other, there is one point of similarity between them: both articles ([5] and [3]) use a two stage process to obtain a measure of utilized capital, and in each the first stage uses a crude estimate of the production relations under study. (This is not a criticism, as an iterative approach to these problems is surely legitimate.) Massell first calculates a "normal" capital-output ratio as a linear function of time, using regression analysis with observations for "normal" years only. (Thus, the war years and the years of high unemployment are excluded.) Massell obtains a declining trend; the implied rise in the average product of capital may be interpreted as one measure (a crude one) of technical change. Massell's second stage is to multiply the "normal" capital-output ratio by the actual output-labor ratio, thus obtaining the capital-labor ratio he deems appropriate. Brown and Popkin's first stage was to fit trend lines to the peaks of the output series they employed.* They then assumed that years of peak output

* The resulting series of trend values might be interpreted as a series measuring the development of "potential output." Brown and Popkin's "first stage" appears related to Arthur M. Okun's second method of measuring potential output (that of "trial gaps"). See [6]. The Brown-Popkin series of trend values computed from peak year output levels moves one a step closer to estimating the production function, as this series is one measure (presumably a rough one) of the output which could have been produced if all the available labor and capital had been fully utilized.

were years of full utilization of the capital stock, while for other years it was assumed that the capital stock was underutilized in the same proportion by which actual output fell short of the trend value computed in the first stage (in one interpretation, the proportion by which actual output fell short of its potential level). Thus, the Brown-Popkin procedure gives, as an estimate of the utilized stock of capital in a non-peak year, the product of the available stock times the ratio of actual output to its "potential" level.

Unfortunately, both of these methods possess their shortcomings, also. Massell's method insulates the estimation of the production function from year-to-year variations in the actual stock of capital. (That it does not use all the information may be seen by the consideration that Massell needs only two observations of the capital stock, provided they occur in "normal" years. It is true, however, that he might desire more "normal" year observations, to reduce sampling variability, which cannot even be estimated with only two observations.) Furthermore, Massell's method requires that actual labor inputs be in a "normal" relationship to actual output for all years, which hardly seems reasonable. Aside from stochastic variation, some labor inputs might be expected to display a "fixity" in the face of a downturn in business; employers might tend to "stockpile" labor which is temporarily not needed, due to the costs of recruiting and training new labor. (For a development of this argument, see Okun, [6], section on "Productivity.") This line of reasoning is consistent with the observed downturn in Massell's capital-labor ratios during 1932-1933, which, as he notes, is contrary to his theoretical expectations.* ([5], p. 185).

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* The related phenomenon that output-labor ratios (the "productivity of labor") rise during upturns might also explain another feature of Massell's capital series: the capital-labor ratios are quite high during the early part of World War II, which would seem to be inconsistent with the abnormally <u>low</u> capital-output ratios that he cites earlier for the wartime period. ([5], p. 185). Presumably all, or nearly all, of the available capital would be utilized during a period of high demand such as the war period. An alternative explanation is that the flow of capital services from the utilized stock of capital was abnormally high, because of the temporarily long number of hours in the average work week.

A number of deficiencies in the Brown-Popkin method can also be listed. Most of these have been pointed up by the authors themselves ([3], p. 411), and again a citation will save space. There is, in my view, an additional shortcoming of their method. It will be recalled that, for non-peak years, the estimate of utilized capital is proportional to the actual level of output, the factor of proportionality being the ratio of the actual capital stock to the "potential" level of output. As the utilized stock of capital is used as an explanatory variable of the level of output, this method of correcting for underutilization of capital might tend to overstate the influence of the capital input.* What would be useful, as Brown and Popkin point out, would be an <u>independent</u> estimate of the degree of utilization of the total stock

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* Hence I would expect the Brown-Popkin elasticities of output with respect to capital to be upward-biased, rather than downward-biased, as they suggest. ([3], p. 411). The moderately large capital elasticities which they obtain for their last two "epochs" would appear to confirm this view.

Parenthetically, it may be remarked that even if there is bias present in this method of handling the capital stock problem, this does not necessarily mean that it is inferior to an alternative method. It is possible that this method could be more efficient than its alternatives, particularly if this bias were small.

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of capital, built up from the separate industrial sectors. Unfortunately, the Wharton School index of capacity utilization [4] is unavailable for the years before 1946.

We are left, then, with no perfectly satisfactory method of resolving this problem. A perfectionist would not have written this paper. In fact, I selected the Solow method, principally to maintain continuity with the estimates of the CES function given in [1]. Another advantage of this choice was that the capital stock data were already adjusted, an advantage not to be scorned. One might also defend this choice on the grounds that it uses all the information and probably has no pronounced biases, but perhaps this is an instance where one who lives in a statistical glass house should view casting stones with extreme reticence.

3. The statistical results

An application of the method outlined in [1] yielded the following estimates* of the CES variant of the aggregate production function:

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* A comparison of the present estimates with those given in [1] will indicate slightly different results. (The second and higher significant figures do not agree.) I have checked and rechecked my computations (the crucial parameters were estimated more than once both at a desk calculator and on an electronic computer). It is possible that the discrepancies may reflect rounding errors. In any case, it is hoped that correspondence with the authors of [1] will clear up this problem.

$$\text{(2b)} \qquad \log V = -0.2274 + 0.007844\ t - 1.587 \log [0.4884\ K^{-0.6302} + 0.5116\ L^{-0.6302}] ,$$

$$\hat{R}^2 = 0.9703, \quad \bar{S}_u = 0.02760, \quad \frac{\delta^2}{S^2} = 0.2948 .$$

(Here, $\bar{S}_u$ is the estimated variance of residuals (crudely corrected for degrees of freedom), $\hat{R}^2 = 1 - \dfrac{\Sigma u_4^2}{\Sigma(\log V)^2 - \frac{1}{N}(\Sigma \log V)^2}$, which may be interpreted as the fraction of "explained" variance, and $\dfrac{\delta^2}{S^2}$ is the von-Neumann-Hart statistic.) As the authors of [1] pointed out, the fit is moderately tight. The implied estimate of σ, the elasticity of substitution, is 0.613. The low value of the von-Neumann-Hart statistic leads to the conclusion that the residuals are autocorrelated, at any reasonable level of statistical significance.

As suggested in the earlier discussion, one can estimate the constant term by requiring the sum of the estimated residuals to be zero. As this seemed a minor and not inappropriate modification, it was carried out. The results are:

(2c) $$\log V = -0.2258 + 0.007844\, t - 1.587 \log [0.4884\, K^{-0.6302} + 0.5116\, L^{-0.6302}],$$

$$\hat{R}^2 = 0.9704,\ \bar{S}_u = 0.02754,\ d = 0.2876.$$

(Here, d is the Durbin-Watson statistic, which now becomes reasonable to use, as the estimated residuals now sum to zero. The von-Neumann-Hart statistic is, of course, the same as it is for (2b).) The estimate of the constant term does not change much; this tends to confirm the appropriateness of the parameter estimation methods outlined by Arrow, Chenery, Minhas, and Solow. (In fact, the mean of the computed residuals of (2b) does not differ significantly from zero.) This modification yields little improvement of the fit. The Durbin-Watson test also suggests a highly significant degree of autocorrelation of the residuals.

When the parameters of the Cobb-Douglas form are estimated freely, the result is:

$$(3c) \qquad \log V = -1.053 + \underset{(0.03341 \times 10^{-2})}{0.6963 \times 10^{-2}t} + \underset{(0.05370)}{0.03463} \log K$$

$$+ \underset{(0.0505)}{1.167} \log L, \quad R^2 = 0.9931, \; \overline{S}_u = 0.01352.$$

(The numbers below the parameter estimates are standard errors.) (3c) is a most unsatisfactory approximation to the aggregate production function. Although the coefficient of multiple determination is rather tight, the properties of this fitted relationship in terms of economic theory are most disturbing. The capital input has a statistically insignificant coefficient, suggesting that this is an irrelevant variable in this relationship! The coefficient on the labor input suggests (if one takes the associated standard error at face value) that not only are there increasing returns to scale but also increasing returns to labor as a variable factor! Rather than reject economic theory, we reject this relationship as a statistical artifact. The intercorrelation between the labor and the capital series (alluded to earlier) may be part of the explanation of these nonsensical results.

Accordingly, the coefficients of the logarithms of the labor and capital inputs were constrained to be in a fixed ratio to each other, as explained in section 1. Calculation of the constant θ from share data gave the estimate that the coefficient of the logarithm of the capital input should be 0.5177 that of the labor input. Substituting this number into regression (3b), one obtains the following result:

$$\text{(3d)} \quad \log V = -1.657 + \underset{(0.04682 \times 10^{-2})}{0.5640 \times 10^{-2}}\, t + \underset{(0.02578)}{0.4404} \log K$$

$$+ \underset{(0.04980)}{0.8506} \log L, \quad R^2 = 0.9813, \; \bar{S}_u = 0.02189, \; d = 0.5429.$$

This is far more satisfactory: the capital elasticity is now highly significant, while the labor elasticity is now significantly less than 1 (if one takes the associated standard error at face value). The fit is not quite so tight as before, but this seems a very reasonable price to pay for results that are consistent with the theoretical formulation of the problem. The Durbin-Watson test indicates a highly significant degree of autocorrelation of residuals, which, of course, vitiates standard statistical tests. The sum of the labor and capital elasticities is 1.29, suggesting the presence of increasing returns to scale. If one ignores the presence of autocorrelated residuals, this sum is significantly greater than unity.

The principal point of this analysis is to compare a CES aggregate production function with one of the Cobb-Douglas form having an equal number of estimated parameters. Comparing equations (2c) and (3d), one notices that the fit is slightly tighter for the Cobb-Douglas form. However, calculation of an F ratio of the estimated variances of residuals of the two relations yields the value 1.583. With 38 degrees of freedom in both the numerator and the denominator,* this is not statistically significant at the 5 per cent

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* There are 41 observations, and there are only three (free) parameters estimated in equation (3d). Hence there are 38 degrees of freedom in the denominator. The number of degrees of freedom associated with the numerator is a bit difficult to ascertain, because of the involved estimation procedure. However, there are four parameters to estimate and these are estimated with the series appearing in (2c) and essentially one extraneous series (the factor share data). Hence it seems reasonable to argue that three degrees of freedom (four minus one) are lost in the estimation procedure, and so the numerator has 38 degrees of freedom also. The F ratio is not close to the margin of significance, and so the conclusion of the text would not be affected by an alternative treatment of this question.

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level (provided the standard assumptions apply). As this test is also vitiated by the autocorrelation of residuals, each reader must decide for himself how much confidence to place in the conclusion that neither formulation has a marked superiority. If accepted, this implies that the assumption of constant returns to scale and that of a unitary elasticity of substitution involve roughly the same degree of oversimplification.

Before leaving the Cobb-Douglas variant of the aggregate production function (equation (3d)), it is interesting to analyze the growth in real output over the period studied with the aid of the parameters of this function.

Over the period 1909-1949, real output increased 215.6 per cent (i.e., was 3.156 times as large in 1949 as in 1909), while the labor input increased 54.2 per cent and the capital input increased 102.1 per cent. Using "normalized" values* of the labor and capital elasticities as weights, the

* The "normalized" values of the labor and capital elasticities are proportional to the computed values, but are scaled down so that the sum of the two is unity. The method of computing the contribution of increasing returns to scale then utilizes the difference between the actual elasticities with respect to labor and capital and their respective "normalized" values.

weighted increase of the two inputs is 70.5 per cent. This is the contribution of the increase in the inputs to the increase in output, under the assumptions of a constant technology and of constant returns to scale. Using the Brown-Popkin method of computing the influence of increasing returns to scale (See [3], p. 404 and n. 15, p. 409), one can estimate that 20.5 percentage points (about 9.5 per cent) of the observed increase in output is attributable to this phenomenon. If the residual increase in output (the increase in output after subtracting the "pure" effect of an increase in the inputs and the contribution of increasing returns to scale) is attributed to the "pure" effect of technological change (the standard treatment), the contribution of technological change is 124.6 percentage points, which is 58 percent of the total increase in output.** Like the other studies cited, these results lead to

** This figure agrees very closely with that calculated from the work of Brown and Popkin [3]. In view of the fact that the period of study, the coverage of the economy, the sources of the data, and the method of adjusting the capital series all differ somewhat, this agreement is encouraging!

the conclusion that "technical change" is responsible for the greater part of the observed increase in real output. Finally, it may be recalled that the labor input increased by only 54.2 per cent over the period, while the weighted average increase of the labor and capital inputs was 70.5 per cent. By a slight extension of the residual technique, it can be argued that the difference between these two percentages (16.3 percentage points) is an estimate of the "pure" effect on output of increased capital intensity (higher capital-labor ratios), after abstracting from increasing returns to scale and technological change. Thus the "pure" effect of increased capital intensity appears to be less important than the effects of either technical change or increasing returns to scale, which is consistent with the analyses of Solow and Massell. The standard caveat applies: an increasing stock of capital may be a carrier of technical change, and it is obviously needed (when the labor force grows) in order that increasing returns to scale may be fully utilized.

One may look at the rate of growth of "technical change" itself (the $A(t)$ series of section 1); this shows how rapidly output will grow over time, with constant labor and capital inputs. It will be recalled that in his 1957 article, Solow estimated this rate of growth to be 1.5 per cent per annum. Equation (2c) suggests that real output will grow at the rate of 1.82 per cent per annum, if the labor and capital inputs are held constant. From equation (3d), the rate of growth of "technical change" can be estimated to be 1.31 per cent per annum. (A look at the accompanying standard errors suggests that the difference between this rate and the other two is either statistically significant or nearly so, although a precise test is vitiated by the presence of autocorrelated residuals.) Thus, when one allows for increasing returns to scale, the estimates of the pace of technical progress diminish somewhat. This seems entirely reasonable, since over the period of the study the labor and capital inputs were in fact increasing, and some of the increase in total output attributed to technical change on the assumption of constant returns to scale would, on the alternative assumption, represent the influence of increasing returns to scale.*

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* Using slightly different data, Walters [8] also concluded that the existence of increasing returns to scale (if tentatively accepted) decreased the estimate of the pace of technological progress.

Another issue is the question of whether technical change proceeds at a constant percentage rate of growth or whether it shows some acceleration. This possibility may be tested by introducing a t^2 term in the regression representing the Cobb-Douglas production function. The results, which are still obtained by constraining the labor and capital elasticities to be in the fixed ratio indicated by the share data, are:

$$(9) \qquad \log V = -1.288 + \underset{(0.03774 \times 10^{-2})}{0.6013 \times 10^{-2}} t + \underset{(0.02230 \times 10^{-3})}{0.1092 \times 10^{-3}} t^2$$

$$+ \underset{(0.02104)}{0.4143} \log K + \underset{(0.04063)}{0.8002} \log L \, ,$$

$$R^2 = 0.9887, \ \bar{S}_u = 0.01728, \ d = 0.8061 \, .$$

The introduction of the t^2 term improves the fit, while the coefficient of this variable is almost five times the associated standard error. Thus Solow would appear to be corroborated in his original suggestion ([7], p. 316) that "technical change" proceeds at an accelerating pace, at least for the period of his (and this) study.* As before, the labor and capital elasticities

* But, see Massell's criticism of Solow on this point ([5], p. 187), which would also apply here.

are many times their associated standard errors; there appears to be diminishing returns when either factor is increased alone; and the sum of the labor and capital elasticities is significantly greater than 1, subject to the qualification that the residuals exhibit a significant degree of autocorrelation.

4. Qualifications and summary

Several limitations of the present study deserve explicit recognition. First, I have unashamedly worked with the "aggregate production function," although this is a concept that a purist would not accept. Presumably the concept of a production function has greatest applicability at the firm, the industry, or at most the homogeneous sector level. For those who cannot accept this notion, this study will be of limited interest. At best, I have blithely ignored some very difficult problems of aggregation. Secondly, even if one accepts the framework of an aggregative model, what is presented here is some alternative parameter estimates of one (or possibly two) relations of a complete system. It is well known that such a procedure leads to single equation bias, if the explanatory variables are themselves endogenous variables in the full system. Thirdly, the problems involved in adjusting the stock of available capital to a capital-in-use concept have already been indicated; this is an area in which one feels less than completely satisfied regardless of which procedure one selects, especially in a study using pre-World-War-II data.* Finally, it may be unnecessary to choose between the two restrictive

* I regard the use of an index of capacity utilization as holding forth great promise, provided this index is built up from series relating to individual industries at a detailed level of disaggregation.

assumptions of constant returns to scale and a unitary elasticity of substitution. A recent paper [2] by Murray Brown and John S. deCani generalizes the CES family of production functions so that linear homogeneity is not required. (Brown and deCani do assume homogeneity of a constant degree, however.) Of course, one must expect to pay some price for relaxing both of these assumptions; the estimation procedure, particularly if one introduces neutral technological change, will presumably become still more difficult.

Subject to these qualifications, the major conclusions of this study may be indicated. First, it would appear that the simplifying assumption of a unitary elasticity of substitution is not appreciably more nor less restrictive than that of constant returns to scale. Thus, the two alternative specifications of the aggregate production function subjected to a test fitted the data almost equally well. Secondly, even with increasing returns to scale explicitly taken into account, technical change appears to be still responsible for the greater part of the observed growth in total output over the period of this study, and the "pure" effect of increased capital intensity appears to be smaller than the contribution of increasing returns to scale. The existency of increasing returns would appear to reduce the estimates of the speed of technological progress, however. Finally, there is some suggestion that technological progress accelerated over the period under examination.

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