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Long Term Consequences of the New View of Investment

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In 1956 appeared the first in a long series of papers [1, 2, 4, 5, 6, 7, 9, 11] disputing the traditional thesis that capital deepening is the major source of productivity gains and conjecturing that we owe our economic growth to our progressive technology.

Thesis and antithesis were synthesized by 1960. Investment has been married to Technology. In the new view, investment is prized as the principal carrier of technological progress [8, 12, 16] .

No effort is made here to appraise empirically the "new view." Nor do we question the urgent need for accelerated investment, public and private. Instead this note is directed at certain implications of the new role which has been attributed to investment. In what sense does the new role make investment more important? Does the new view of investment present any new reasons, any new incentives -- if added ones were needed -- for faster capital accumulation? To narrow the inquiry still further, we shall confine our analysis to the long-run, ultimate or asymptotic consequences of stationary investment-thrift policies of the simple "proportionality" variety. The results of the inquiry are summarized at the conclusion of the paper.

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* I am grateful to Arthur Okun for uncovering several mistakes in the original draft. He is innocent of any new mistakes which may have crept in.

Early Work

The empirical work cited above spans a great variety of analytical methods and empirical materials. One of the best known papers is that by Professor Solow [11] . A number of other investigators followed a similar approach.

Solow assumed the constancy of returns to scale and the "neutrality" of technical progress. He expressed aggregate output, Q_t , as the following function of capital, K_t , and employment, N_t :

$$Q_t = A(t)\ F(K_t, N_t) \tag{1}$$

This assumes that technical progress is essentially organizational.

By virtue of the assumptions, it follows that

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} + a_t \frac{\dot{k}_t}{k_t} \tag{2}$$

where y_t denotes output per worker, k_t denotes the capital-labor ratio and a_t is the elasticity of output with respect to capital, that is $\frac{F_K(K_t, N_t)K_t}{Q_t}$. With an estimate or assumption about a_t it was possible from time series of output, capital and labor to estimate the "index" of technology $A(t)$ which was implied.

A number of results were obtained from this approach but the foremost conclusion was the following: Less than twenty per cent of the increase in output per worker in the last quarter century was due to the increase in capital per worker which occurred.*

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* In the absence of additional information, we can only regard the eighty per cent as the unexplained increase in productivity. However the attribution of this residual has been a rich field for conjecture and unwarranted conclusions. Some have regarded it as proof of the high average and even marginal productivity of research outlays!

As a statement about economic history this statement could not be clearer. As a statement about the productivity of additional investment, if it was ever intended as such, it remains open to two quite contrasting interpretations. Does it mean, as many have taken it to mean, that capital deepening is ineffectual? Or does it mean that in recent history very little capital deepening has taken place?

It is, after all, the effectiveness of additional capital deepening which is of primary interest. On that score, the papers cited have been tentative and casual. Solow used the share of output going to capital in year t as an "outside" estimate of a_t . Later, Massell [7] used the average share of output going to capital over the period of the sample as an estimate of a . Since the conditions under which theory would justify such a procedure are not in fact met (e.g., competitiveness) this approach cannot be regarded as very satisfactory.

This method of estimating the capital-elasticity of output was bound to generate "elasticity pessimism," quite apart from the results of a regression like (2) . In the same way that (2) is obtained, one can derive from (1) the relation

$$\frac{dQ_t}{dt} \cdot \frac{1}{Q_t} = \frac{\dot{A}}{A} + a_t \frac{dK_t}{dt} \cdot \frac{1}{K_t} + (1-a_t) \frac{dN_t}{dt} \cdot \frac{1}{N_t}$$

The share of income accruing to capital is generally estimated to be smaller than one third. Therefore the above relation states that a one per cent increase in the capital stock -- each such increase takes three per cent of a year's national income -- will increase output by only one third of one per cent. The capital-output ratio in this country is a little less than 3 . Hence, an increase in the investment-income ratio from .10 to .20 would increase the growth rate by no more than one per cent (and would increase it less if the capital-output ratio rose in response to the heightened thrift). Professor Solow has remarked of just such a calculation: "This seems like a meager reward for what is after all a revolution in the speed of accumulation of capital." [12].

The New View

At a time when the reputation of investment seemed at low ebb there appeared the first signs of a new tide. The Economic Commission for Europe [14] argued in late 1959 that European population growth had stimulated productivity by necessitating a high rate of gross investment - thus bringing about a younger and more modern capital stock. On the same grounds, PEP's diagnosis of the British economy [8] discounted Britain's comparatively high investment per worker because her population growth was small.

The alacrity with which this new view of investment has taken hold is remarkable. The President's Economic Message to Congress [15] in January, 1961 states:

> "Expansion and modernization of the Nation's productive plant is essential to accelerate economic growth and to improve the international competitive position of American industry. Embodying modern research and technology in new facilities will advance productivity, reduce costs, and market new products."

Expansion and modernization are put on equal footing and the latter is stressed. A statement by the Council of Economic Advisers before the Joint Economic Committee on March 6, 1961 [16] amplifies this view:

> "One of the reasons for the recent slowdown in the rate of growth of productivity and output is a corresponding slowdown in the rate at which the stock of capital has been renewed and modernized... As has been confirmed by more recent research, the great importance of capital investment lies in its interaction with improved skills and technological progress. New ideas lie fallow without the modern equipment to give them life. From this point of view the function of capital formation is as much in modernizing the equipment of the industrial worker as in simply adding to it. The relation runs both ways: investment gives effect to technical progress and technical progress stimulates and justifies investment."

The dismal spell of the early investigations has been decisively broken.

The fundamental theory on which the new view of investment is now based is due to Professor Solow. In a 1960 piece [12] which is already a classic, he pointed out that a production function like (1) makes old and new capital share alike in technological progress while expressing the belief that "many if not most innovations need to be embodied in new kinds of durable equipment before they can be made effective." He then constructed a model which accords to investment this new role.

Solow says of this model: "[It] redresses the balance somewhat and attributes *greater importance* to capital investment. The reason is, of course, that capital formation is a vehicle for carrying technical change into effect" [12, p. 97; italics mine] .

We shall now sketch this model and then inquire into the nature of increased importance which the new view attributes to investment.

Unlike the earlier model of production, which permitted much more generality, Solow assumes that the index of technology, $B(t)$, advances exponentially at the constant relative rate r . Every capital good embodies the latest technology at the moment of its construction but it does not participate in subsequent technical progress. "Capital" thus becomes a continuum of heterogeneous vintages.

It is assumed that the output $Q_v(t)$ produced at time t by equipment $K_v(t)$ of vintage v is given by the Cobb-Douglas function:

$$(3) \qquad Q_v(t) = B_o e^{rv} K_v(t)^a \; N_v(t)^{1-a}$$

Since technical progress is neutral, the elasticity parameter a is the same for capital of all vintages.

If we are to estimate capital of vintage $\underline{v}$ still existing at time $\underline{t}$ from a gross investment time series, we must make an assumption about depreciation. Solow assumes a constant "force of mortality," δ , to which capital is continuously exposed. Hence,

$$(4) \qquad K_v(t) = K_v(v)\ e^{-\delta(t-v)} \equiv I(v)\ e^{-\delta(t-v)}$$

where $I(v)$ is gross investment -- unconsumed output -- at time $\underline{v}$. This makes the average life of a capital good about $1/\delta$ years.

The last step is to determine the distribution of the labor force over the vintages of capital. This he does by introducing the condition that the marginal productivity of labor be everywhere equal. Then aggregate output -- the sum of the homogeneous outputs of the various vintages of capital -- is given by

$$(5) \qquad Q_t = B_o\ e^{-a\delta t}\ N_t^{1-a}\ J_t^a$$

$$\text{where} \qquad J_t = \int_{-\infty}^{t} e^{(\delta + \frac{r}{a})v}\ I(v)dv$$

Since the new model uses the Cobb-Douglas assumption, let us specialize (1) in the same way to facilitiate comparisons. If all technology is organizational, then

$$(6) \qquad Q_t = A_o\ e^{rt}\ K_t^a\ N_t^{1-a}$$

or equivalently

$$(7) \qquad Q_t = A_o N_t^{1-a} \left[\int_{-\infty}^{t} e^{\frac{r}{a} t} K_v(t)\, dv \right]^a$$

We can bring (5) into the same form but with a crucial difference:

$$(8) \qquad Q_t = B_o N_t^{1-a} \left[\int_{-\infty}^{t} e^{\frac{r}{a} v} K_v(t)\, dv \right]^a$$

The basis for the optimism engendered by (8) can be illustrated by the following example.

Suppose that existing machines are of just two vintages, v_1 (old) and v_2 (new), and that there are an equal number of machines of the two vintages.

According to (7) a two per cent increase in the number of machines of the current vintage, v_2 , will bring about a one per cent increase in the value of the bracketed expression in (7); we are weighting a two per cent and a zero increase equally.

Consider the case in equation (8). The bracketed expression is the weighted sum of the machines of the two vintages with the weight for the contemporary machines, namely e^{rv_2} , being greater. Consequently a two per cent increase in the number of machines of current vintage will produce a proportionate increase in the value of the bracketed expression in (8) in excess of one per cent. Hence, current investment increases output per man partly through decreasing the average age of the capital stock.

Now we broaden our view and ask what happens as the program of capital accumulation continues. Pretty soon we will be confronted by a new situation; large investments today will present us with a great number of old machines tomorrow. To achieve that one per cent increase in the value of the bracketed expression tomorrow, a greater absolute increase in the number of new machines will then be required. Of course the weights accorded the old machines in the bracket in (8) are small and are smaller the older the vintage. This contrasts with (7) where all vintages get the same weight. But the essential point is that investment must grow in order to maintain a constant average age of capital. Hence, the long run consequences of a permanent change in investment policy are not so clear as the immediate consequences of a change in investment. The remainder of this paper is devoted to a study of the long run effects of different investment policies as predicted by the "old view" and the "new view" of the relation between investment and technology.

Implications for Long Run Investment Policy

We have seen that the two views of investment do indeed have different implications; the difference between the two views is therefore operational. But do the two views have different implications for the effects of, and therefore the choice of, investment policy in the long run?

That is a deliberately vague question to which we cannot give a complete answer. We shall confine our analysis to long-run investment policies of the following simple form: For the indefinite future investment shall be some fixed fraction of total output. The choice of an investment policy in this case reduces to selecting the investment-output ratio s . Hence *

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* If consumption is a constant fraction of gross output then, in exponential growth, it is also a constant fraction of net output. Therefore it is immaterial that we work with gross rather than net investment and output in the present context.

$$I(t) = s\,Q(t) \tag{9}$$

The second critical restriction is the assumption that the labor force grows exponential at the constant relative rate n :

$$N_t = N_0\, e^{nt} \tag{10}$$

These assumptions yield a growth process having a nice property [10, 13, 3] . Starting from the initial position, the path of growth will be asymptotic to a balanced-growth, "golden-age" equilibrium growth path along which path production, consumption, investment and the capital stock (of all ages) all grow exponentially at the same rate. Our principal concern will be with the relation between the investment ratio chosen and the asymptote -- the limiting growth path -- which the economy will gradually approach. In particular, we ask: Does the new view of investment impute to the investment ratio a greater influence upon the limiting growth path than can be attributed to it under the old view of investment? Does the new view of investment, in other words, imply a larger long-run asymptotic effect from a given increase of the investment ratio?

Let us first construct the growth model corresponding to the <u>old</u> view of investment and technology. We make the exponential depreciation assumption which corresponds to (4) :

$$K_t = \int_{-\infty}^{t} e^{-\delta(t-v)} I(v)\, dv \tag{11}$$

As a consequence, $\dot{K}_t = I(t) - \delta K_t$.

In order to make the "old" model of investment and growth comparable with the new model formulated by Solow we shall make the same Cobb-Douglas assumption; that is, we shall work with the production function in equation (6) . Differentiating Q_t in (6) with respect to time yields

$$\frac{dQ}{dt} = rQ + (1-a)\, Ae^{rt} N^{-a} K^{a} \frac{dN}{dt} + aAe^{rt} N^{1-a} K^{a-1} (I(t) - \delta K) \tag{12}$$

where we have used the relation $\dot{K}_t = I(t) - \delta K_t$ by virtue of (11) . Using (9), (10) and (11) (to express K^{a-1} in terms of Q and N) we obtain the fundamental differential equation corresponding to the "old view" of investment:

(13) $$\frac{dQ}{dt} = c_1 Q + c_2 Q^{c_3} e^{c_4 t}$$

where
$$c_1 = r + (1 - a)n - a\delta$$
$$c_2 = a s A^{\frac{1}{a}} N_o^{\frac{1-a}{a}}$$
$$c_3 = \frac{2a - 1}{a}$$
$$c_4 = \frac{r + (1 - a)n}{a}$$

According to the new view of investment, production takes place according to equation (5). Differentiating that equation with respect to time yields

(14) $$\frac{dQ}{dt} = - a \delta Q + (1-a)Be^{-a\delta t} N^{-a} J^{a} \frac{dN}{dt} + aBe^{-a\delta t} N^{1-a} J^{a-1} e^{(\delta+\frac{r}{a})t} I(t)$$

where we have used the relation $\dot{J} = e^{(\delta + \frac{r}{a})t} I(t)$.

Using (9), (10) and (5) (to write J^{a-1} in terms of Q and N) we obtain the fundamental differential equation corresponding to the new view of investment:

(15) $$\frac{dQ}{dt} = c_1' Q + c_2' Q^{c_3'} e^{c_4' t}$$

where
$$c_1' = (1 - a)n - a\,\delta$$

$$c_2' = a\,s\,B^{\frac{1}{a}}\,N_0^{\frac{1-a}{a}}$$

$$c_3' = \frac{2a - 1}{a}$$

$$c_4' = \frac{r + (1 - a)n}{a}$$

These two growth models -- which differ only in respect to the embodiment or non-embodiment of technical change in capital goods -- exhibit differential equations having the same form. There exists an explicit solution for the growth path $Q(t)$ resulting from such a differential equation [3] .

Let us focus on the limiting or asymptotic solution to the equation. This is

(16) $$Q(t) = \bar{Q}_0\, e^{\frac{c_4}{1 - c_3} t}$$

In the limit, growth is exponential at the relative rate $\frac{c_4}{1-c_3}$.

On the balanced-growth equilibrium path, the "initial point" is arbitrary; but the height of the path depends upon the existing labor force at whatever initial point is selected <u>and</u> depends upon the investment ratio. This <u>equilibrium</u> value of Q at time zero, $\bar{Q}_0$, is to be distinguished carefully from the actual value of output, Q_0 , at time zero; the two will be equal only if the initial capital-output ratio happens to equal that ratio which the chosen investment ratio will ultimately bring about.

$\overline{Q}_0$ is given by

$$\overline{Q}_0 = \left[\frac{(1 - c_3)\, c_2}{c_4 - (1 - c_3)\, c_1} \right]^{\frac{1}{1 - c_3}} \tag{17}$$

We are at last in a position to examine the consequences of the new view for this limiting or golden age mode of growth -- especially those consequences relating to the importance of the investment ratio.

Since $c_3 = c_3'$ and $c_4 = c_4'$ it is plain that the limiting growth rates under the two systems are identical. It is a "natural" growth rate in the usual sense that it is independent of the investment ratio. The growth rate is:

$$g = \frac{r + (1 - a)n}{1 - a} \tag{18}$$

The fact that the limiting growth rate of the old-style Cobb-Douglas model is independent of the investment ratio is well known. It is not surprising that the limiting solution of the new Cobb-Douglas model exhibits the same property. Associated with any exponential mode of growth is a certain unchanging age distribution of capital. Capital which is $(t - v)$ years old will grow at the rate $\underline{g}$ like most everything else; the proportion of capital which is $(t - v)$ years old or less is constant over time. The fact that capitals of different vintages get different technical weights is immaterial in the determination of the exponential equilibrium growth rate.

In what way, then, does the new view attribute more importance to investment? While the limiting growth rate in both models is independent of the investment ratio, the height of the equilibrium growth path will depend upon that ratio. We might well ask therefore if the new view imputes to the equilibrium growth path - in short, $\bar{Q}_0$ - a greater sensitivity to the investment ratio than is implied by the old view. This is a conjecture concerning the elasticity of $\bar{Q}_0$ with respect to $\underline{s}$.

Equations (13) and (17) yield the equilibrium output rate corresponding to an investment ratio $\underline{s}$ at some arbitrary zero point in time according to the old model (6) :

$$(19) \qquad \bar{Q}_0 = s^{\frac{a}{1-a}} \left[\frac{(1-a)\, A_o^{\frac{1}{a}}\, N_o^{\frac{1-a}{a}}}{r + (1-a)\,(n+\delta)} \right]^{\frac{a}{1-a}}$$

Equations (15) and (17) yield the equilibrium output rate associated with an investment ratio $\underline{s}$ at the arbitrary zero point according to the new model based on (5) :

$$(20) \qquad \bar{Q}'_0 = s^{\frac{a}{1-a}} \left[\frac{(1-a)\, B_o^{\frac{1}{a}}\, N_o^{\frac{1-a}{a}}}{\frac{r}{a} + (1-a)\,(n+\delta)} \right]^{\frac{a}{1-a}}$$

Equations (19) and (20) disprove the conjecture. The elasticity of $\bar{Q}_0$ with respect to $\underline{s}$ is $\frac{a}{1-a}$ in both equations. Whether one takes the new view or the old, it follows that, in the long run, a one per cent increase in the investment ratio will yield asymptotically a rate of output which is $\frac{a}{1-a}$ per cent in excess of what asymptotically it would otherwise have been (i.e., had a lower fixed investment ratio prevailed).

This result seems at first appearances to be in flat contradiction to the observations of the previous section. The explanation of the puzzle lies in the behavior of the average age - or more precisely, the age distribution - of capital. The trivial but easily overlooked fact is that, in exponential growth, the age distribution of capital depends upon the rate of growth and the rate of depreciation and upon nothing else. Since both rates are, in the long run; independent of the investment ratio, a once-for-all change in that ratio can have no persistent long-run influence on the age distribution of capital. Consequently, in the long run, any increase in thrift must rely for its effectiveness upon the prosaic mechanism of capital deepening - of an equiproportionate deepening of capital of every age.

This is easily proved. Consider a point far in the future of the kind of economy discussed here in which the effects of any aberrant investment policies of the distant past are no longer felt; the economy has been growing smoothly at the rate $\underline{g}$, along the growth path corresponding to the chosen fixed investment ratio, for quite some time.

The time path of gross investment traced out by such an economy is the upper exponential curve shown in Figure 1 . We are looking backward from a point in time at which, with no loss in generality, $t = 0$.

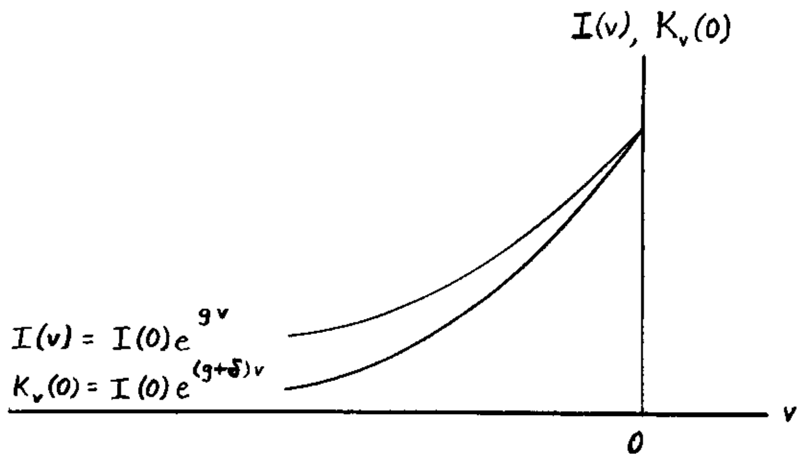

Figure 1

In order to obtain the amount of capital of vintage v still in use at $t = 0$, $K_v(0)$, we have to multiply $I(v)$ by $e^{\delta v}$. This gives the lower curve.

The lower curve is an exponential curve but not the curve of statistical theory with unit area under the curve. To obtain the mean age and the other moments of the age distribution of capital, it is necessary to normalize the curve so that its area will equal one. This requires dividing $K_v(0)$ by $I(0)/(g+\delta)$ for all v .* The normalized age-distribution curve is therefore

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* $I(0)/(g+\delta)$ is the total area under the $Kv(0)$ curve, by the familiar "capitalization" formula.

(21) $$f(v) = (g + \delta)\, e^{(g+\delta)v}$$

The mean age of capital, $-\bar{v}$, in this case is

(22) $$-\bar{v} = \int_{-\infty}^{0} (g + \delta)\, e^{(g+\delta)v} (-v)\, dv$$

$$= \left. \frac{-v(g + \delta) e^{(g+\delta)v}}{g + \delta} \right]_{-\infty}^{0} + \int_{-\infty}^{0} \frac{(g + \delta) e^{(g+\delta)v}}{(g + \delta)}\, dv$$

$$= 0 + \left. \frac{e^{(g+\delta)v}}{g + \delta} \right]_{-\infty}^{0}$$

$$= \frac{1}{g + \delta}$$

Other moments of the distribution can be derived in the same manner. But (21) shows simply that the asymptotic age distribution will depend only upon g and δ, whatever the investment ratio.

Has the new view of investment no significance for the selection of the fixed investment ratio? This depends upon the nature of the investment policy, of the decision rule employed.

Suppose that we wished to achieve a particular equilibrium growth path. Which model - new or old - predicts the smaller investment ratio which is required asymptotically to achieve it? Or suppose we are concerned with the absolute increase in output (or consumption), rather than the relative increase, which results from a one per cent increase in the investment ratio. Which model predicts the greater absolute effect of a given increase in the investment ratio? These two questions are the same. They both ask whether the coefficient T' of the investment ratio term in equation (20) is greater than the corresponding coefficient T in equation (19). If the answer is yes, then it would seem reasonable to assert that the new view makes investment policy more important than does the old view. However it could not be said, in this event, that the new view provides additional incentives to increase the long-run investment. (Such incentives would then have to be based upon the transient behavior of the system "before the asymptote is reached.")

Looking at (19) and (20) we can see that $T' > T$ if and only if

$$(23) \qquad \frac{B}{A} > \frac{\frac{r}{a} + (1-a)(n+\delta)}{r + (1-a)(n+\delta)} \equiv \psi$$

Now $\psi > 1$ since $r > 0$ and $0 < a < 1$. Can we make any a priori deductions about $\frac{B}{A}$?

First of all, note that at any point in time, $\underline{t}$, Q_t , N_t and $K_v(t)$ are data. Anyone subscribing to the "old view" must estimate A_o in such a way that his production function, equation (7), fits the facts. Similarly, anyone adhering to the "new view" will estimate B_o such that his production function, equation (8), fits those same facts. If we are to make a meaningful comparison of the two models, we must estimate the parameters of each of them in such a way that both models are admissible theories of the same economy. Equations (7) and (8) must "predict" the same current rate of output, given current resources. It is worth adding that the other parameters, a , r , n and δ , must be assigned the identical values in the two models.

From this consideration and (7) and (8) we find

$$(24) \qquad \frac{B_o}{A_o} = \frac{\int_{-\infty}^{t} e^{\frac{r}{a}t} K_v(t)\, dv}{\int_{-\infty}^{t} e^{\frac{r}{a}v} K_v(t)\, dv} \geq 1$$

It is apparent from (24) that $\frac{B_o}{A_o} > 1$ -- unless all capital is brand new. B_o must exceed A_o by an amount which is necessary to "compensate" for the drag on productivity which the new view attributes to old capital. A new-view man might say that an old-view man's estimate of A_o was really an estimate of the <u>average</u> level of technology, whereas B_o was the latest or best-practise level of technology, which was embodied only in the newest capital goods.

The above deduction is the only *a priori* statement that can be made about B_o/A_o . It is insufficient to satisfy condition (24). We have to consider the current age distribution of capital to determine the exact ratio of the two estimates of the level (as distinct from the rate of advance) of technology.

Let us consider an age distribution whose only claim to our attention is its simplicity. Suppose that gross investment has been growing steadily at the rate *h* for a long time. Let the present time constitute the zero point, for convenience; then $K_v(0) = 0$ for all $v > 0$ since future capital vintages have not yet been built. From the growth rate assumption, the depreciation assumption in (4) and (11), and equation (24) one obtains

$$(25) \qquad \frac{B_o}{A_o} = \frac{\int_{-\infty}^{0} e^{\delta v}\, I(v)dv}{\int_{-\infty}^{0} e^{\frac{r}{a}v}\, e^{\delta v}\, I(v)dv}$$

$$= \frac{\int_{-\infty}^{0} I(0)e^{(\delta+h)v}\, dv}{\int_{-\infty}^{0} I(0)e^{(\frac{r}{a}+\delta+h)v}\, dv}$$

$$= \frac{\frac{r}{a} + \delta + h}{\delta + h} \ .$$

Let us write $h = g + u$ where u may be positive or negative and g is defined in (18). Then, from (25)

$$(26) \qquad \frac{B_o}{A_o} = \frac{\frac{r}{a} + (1 - a)(n + \delta + u)}{r + (1 - a)(n + \delta + u)}$$

If $u = 0$ then, by (26), $\frac{B_o}{A_o} = \psi$ so that $T = T'$. In other words, if the distribution of capital by vintage happened to be described by the exponential curve $I(0)e^{(\delta+g)v}$, then the two models would yield the same coefficient in (19) and (20).*

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* This result could have been anticipated. Suppose the economy had been traveling along the exponential balanced-growth path corresponding to the prevailing investment ratio. Both models would have to predict the ruling growth path; A_o would have to be estimated such that $\bar{Q}_o = Q_o$ and B_o would have to be estimated such that $\bar{Q}'_o = Q_o$. Hence $T = T'$ in this very special case.

If $u < 0$, then the previous growth rate is smaller than the natural rate. $\frac{B_o}{A_o} > \psi$ by virtue of (26) and the assumptions on r and a .

In this case, $T' > T$.

Correspondingly, if $u > 0$, then $T' < T$.

These cases are symmetrical so that it is necessary to consider only the former in any detail.

This result can be explained quite simply. If growth has been slow, then the average age of capital exceeds the mean age which would be produced by a policy of proportional investment. The mean age at $t = 0$ is $\frac{1}{h+\delta}$ while equilibrium balanced growth must produce asymptotically a mean age of capital equal to $\frac{1}{g+\delta}$, as we have seen. Therefore, once we gear the economy to a fixed investment ratio - no matter what that ratio is! - the average age of capital will decline in the limit. According to the old view, this shift in the age distribution is of no significance; but the new view implies that such a change in the age structure will make a separate contribution to growth by raising the average level of technology embodied in the capital structure.

The exponential age distribution may have been worth considering for purposes of insight. But any empirical age distribution would be bound to show many irregularities. From (25) it is possible to derive the general condition on the time path of past gross investment such that $\frac{B_o}{A_o} > \psi$. This is

$$(27) \qquad \int_{-\infty}^{0} e^{\delta v} I(v) \left[1 - \psi e^{\frac{r}{a}v}\right] dv > 0$$

Let $\psi e^{\frac{r}{a} v_o} = 1$ define v_o. Then we can say that gross investment in the interval $-v_o \leq t \leq 0$ receives negative weight in (27); gross investment prior to $-v_o$ receives positive weight. It is still true to say, therefore, that $\frac{B_o}{A_o} > \psi$ is more likely to be satisfied the greater the average age of capital.

If the preceding analysis is correct, there is nothing a priori in the new view of investment which makes thrift more important or desirable so far as its ultimate, asymptotic consequences are concerned. However, we need not blind ourselves to the data. Is there evidence that capital in this country has grown so very old that, in terms of the model here, $\frac{B_o}{A_o} > \psi$?

Only in the case of certain simple distributions like the exponential does the average age of capital give us all the information we need about the age distribution in (27). But the mean age is a useful statistic.

The Council of Economic Advisers [16] has recently exhibited estimates of the mean age of equipment in the U. S. in 1959 and 1952-55 . In this interval the mean age of equipment declined from 8.5 years to 9.0 years. However, this is still short of the ripe old age of postwar equipment, 10.6 years in 1945. In 1948, 12.5% of plant and equipment was 5 years old or less; in 1959-60, it was only 9.4% .

This evidence was presented to show that a deceleration of investment will increase the average age of capital. An acceleration of investment can be expected to reduce the mean age, at least for a while. The analysis here suggests that, in the long run, the mean age of capital will depend, for any given investment ratio, only upon the rate of depreciation and the limiting rate of growth. To show that the present mean age of capital is in excess of the equilibrium mean age would require further analysis. It is interesting that if $g = .04$ and $\delta = .06$ then $\frac{1}{g + \delta} = 10$ years. In that case, capital could be expected ultimately to get older, not younger, if a fixed investment ratio were established and if the present model were taken as descriptive of the economy.

Conclusions

We have constructed two growth models which differ only in the underlying assumptions about the embodiment of new technology in old capital. An investigation and comparison of the limiting solutions of these models revealed that

(1) the models display the same limiting growth rate

(2) the elasticity of the limiting exponential growth path with respect to the investment ratio is the same in both models

(3) in the limit, the age distribution of capital depends only on the rates of growth and depreciation

(4) the height of the limiting exponential growth path produced by a given investment ratio may be predicted differently by the two models

(5) the difference will depend upon the difference between the initial age distribution of capital and the limiting equilibrium distribution; if capital is initially very old, then the new view will predict a higher equilibrium growth path corresponding to any fixed investment ratio than will the old view; this is because the change in the age distribution which occurs (in the limit) is favorable to productivity in the new view.

It should be emphasized that the transient effects of investment, as forecast by the two models, may be extremely important; they have been altogether disregarded here. An investigation of transient behavior would require an analysis of the stability of the two models, an analysis which we have avoided here at our peril. It may be that, even if both models were to predict the same exponential balanced-growth path corresponding to every investment ratio, the approach to the limiting path might be faster in the new model than in the old. Cheerfulness may yet break in.

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