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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

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Comments on "Interindustry Economics" by Chenery and Clark*

Alan S. Manne

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by

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Abstract

In publishing "Interindustry Economics," Hollis Chenery and Paul Clark have furnished the profession with a uniquely useful work - both a readable textbook for the beginner, and at the same time a systematic treatise for the input-output practitioner. It is unfortunate, however, that the authors have leaned over backwards to find merits in Leontief's original model as compared with the activity analysis approach to interindustry economics. Activity analysis provides a convenient framework for handling certain kinds of "conceptual" difficulties - import substitution, processing substitution, labor-capital substitution, and the output of by-products - difficulties that have arisen in any of the empirical applications proposed by Chenery and Clark. On balance, their work provides impressive evidence against the presumption that the collection of data for a square matrix is cheaper than for a rectangular one.

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In publishing "Interindustry Economics,"* Hollis Chenery and

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* John Wiley and Sons, New York, 1959.

Paul Clark have furnished the profession with a uniquely useful work. Within the same covers, they have succeeded in providing both a readable textbook for the beginner, and at the same time a systematic treatise for the input-output practitioner. Two chapters are to be singled out for special mention: chapter 6, "Testing the Validity of Input-Output Assumptions" and chapter 7, "A Survey of Interindustry Research." Together, these chapters provide a long-overdue survey of the literature on empirical implementation of Leontief's model of the economy. Much of this literature has appeared only in mimeographed form, and has not hitherto been generally accessible.

So much for the nice things that deserve to be said about this volume, and now for my chief complaint: In comparing Leontief's original input-output formulation of interindustry economics with a linear programming approach to the same class of substantive problems, the authors lean over backwards to find merits in the "square" tableau as compared with the "rectangular" one. For example:

> The main obstacle to the ready adoption of linear programming for national planning is the heavy demand that it makes for data. The construction of an input-output table of 40-50 sectors, with a corresponding breakdown of final demand, is a strain on the statistical resources of most countries, and the adoption of activity analysis in a very detailed form is not a feasible task at the present time. (p. 299)

What the authors seem to be saying here - and in other passages as well (e.g., pp. 82, 128, 129) - goes something like this: Undoubtedly the rectangular matrix form of linear programming provides a certain type of theoretical flexibility not inherent in the square Leontief model. (Alternative processes of production are ruled out in the latter model and admitted in the former.) However, it is more expensive to perform the numerical analysis in the case of linear programming, and it is also more expensive to collect the data.

In other words, Chenery and Clark have scrupulously refrained from claiming that the square matrix is defensible on the basis of its theoretical or its predictive merits. Instead, they rely upon the very pragmatic argument of apparent cost. They have also refrained from pointing out that some of the best thinking of input-output practitioners has been devoted to patching and tinkering with the basic framework - improvisations which could have been largely avoided through activity analysis. Because of the constant need for this patching and tinkering, it is to be seriously doubted whether the activity analysis formulation is really more expensive from the viewpoint of data collection. Indeed - at any given level of commodity detail - it is likely that just the reverse is true. Now to cite some evidence for these rather dogmatic assertions - evidence which is to be found directly within the covers of the Chenery-Clark book:

(1) Some of the most interesting applications of interindustry analysis to problems of economic development consist of the problem of choice between importing a given commodity and producing it at home. (Chapters 10 and 11 contain a fascinating series of such "make-or-buy" problems based upon the authors' own experience in Italy.) When the authors attempted to apply the square tableau formulation to a 1951-56 projection of the entire Italian economy, they found it

necessary to engage in a substantial "auxiliary analysis" of imports - in effect, trying to outguess the solution of their model.

> For each of the 200 product classes in the table, a marginal import parameter ... was estimated. Some of these import parameters were in fact based on the average import proportions of the base year, others on a more recent year, still others on marginal trends in imports from year to year, but in principle each was an independent estimate of import substitution possibilities for a particular product class. (p. 254)

By contrast, when Chenery and Clark turned to the use of a programming model for their pilot study of Southern Italy, they found it possible to handle the problem of imports by means of the very straightforward device of stipulating world prices, and then permitting the model itself to adjust the quantity of imports in the light of relative scarcities throughout the system. (pp. 290-298) Which of these two approaches necessitated a greater amount of preliminary data processing? It is fairly clear that the advantage lies with the activity analysis formulation.

(2) In the 1951-56 projection for the Italian economy, it was necessary to make an "auxiliary analysis" of the iron and steel industry. "... as a result of blast furnace expansion under the Marshall Plan, the steel industry was using more pig iron relative to scrap, and appropriate adjustments were made in these coefficients, as well as in the related input coefficients for iron ore and pyrite ashes." (p. 253)

Would it not have been more straightforward and <u>less</u> time-consuming to stipulate several alternative processes for steel production - some entailing a large amount of pig iron inputs and others a small amount - and then permit the model itself to choose among these, on the basis of relative scarcities?

(3) In this same Italian projection, it was found desirable to

make a detailed sub-study for the major fuels.

> Here it was clear that a significant technological change was in progress, centered around widespread substitution of domestic natural gas for imported coal. The situation was complicated further by interrelations with fuel oil and domestic low-grade coal, ... as well as by differences in the ability of different consuming industries to make fuel substitutions. (p. 253)

This example of fuels substitution should serve as additional evidence in favor of using a rectangular matrix as the basic analytical format in interindustry studies.

(4) The problem of by-products constitutes an unmitigated nuisance within a square tableau.

> For example, demand for hides by the leather industry may be portrayed as stimulating the livestock industry to increase its production of meat, rather than the other way around. A much better way to handle a by-product is to treat it as a fictitious intra-industry sale (e.g., hides to the livestock industry) and to calculate a regular input coefficient related to the output of the industry itself. (pp. 145, 146)

That this device will occasionally work may be seen in the following numerical example (positive outputs and negative inputs):

	before fictitious sale		after fictitious sale	
	domestic production of livestock	import of hides	domestic production of meat	import of hides
hides	+1	+1.	0	+1
meat	+1	0	+1	0
foreign exchange	-1.	-.80	-.20	-.80

If, however, the foreign exchange coefficient for importing hides changes from -.80 to -1.20, this method breaks down. The domestic production of meat will then be accompanied by a positive by-product of +.20 worth of foreign exchange! Again, this is a "conceptual" difficulty that could have been sidestepped by the use of an activity analysis format.

To conclude: Activity analysis does little to solve some of the major problems of interindustry econometrics - specifically, the projection of final demand, the forecast of technological advances, and the estimation of capacity in labor-paced processes. However, activity analysis does provide a convenient framework for handling certain kinds of difficulties - import substitution, processing substitution, labor-capital substitution, and the output of by-products - difficulties that have arisen in any of the empirical applications proposed by Chenery and Clark. On balance, their work provides impressive evidence against the presumption that the collection of data for a square matrix is cheaper than for a rectangular one.

As for the relative costs of computing, it is a fair bet that advances in the art of electronic calculations have already rendered this an irrelevant issue. (See the Appendix for an estimate of these relative costs.) The "decomposition" principle of Dantzig and Wolfe* has opened up the possibility of dealing with models containing literally thousands of equations and of activities. With such tools at our command, it seems a rather dubious use of expensive labor to insist upon fitting interindustry studies into square matrices. The most likely course of

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* "A Decomposition Principle for Linear Programs," RAND Corporation, P-1544, November 10, 1958.

evolution will be a progressive increase in the number of sectors analyzed by means of several alternative activities and a corresponding decrease in the number of sectors for which our models contain just a single producing activity. It would be a serious error to underestimate the speed at which this evolution is taking place.

Appendix - Relative Costs of Computation

To my knowledge, there is no experimental evidence currently available on the relative computational effort needed in order to solve a Leontief matrix and a linear programming problem. (The 200-sector matrix for 1947 was inverted on the Univac I, but very little experience on this machine with general-purpose linear programming has ever been reported.) The following theoretical comparison ignores many factors quite significant in actual numerical analysis, but should at least serve as a starting-point.

In this comparison, it will be assumed that both the Leontief matrix and the linear programming matrix have the same number of rows, _m_ . The number of columns in the former case will also be _m_ , and in the latter case will be denoted by _n_ . $(n > m)$ To facilitate this comparison, we shall make one highly unrealistic assumption - that all coefficient entries in the original matrices are non-zero. (Sparseness in these matrices will obviously lower the absolute costs of computing, but should not grossly distort the relative costs.)

In the case of linear programming, we shall suppose that the numerical analysis is performed by means of the revised simplex method, and that the basis-inverse is continually maintained in its explicit form. Since a linear programming solution remains feasible only within the immediate neighborhood of a particular vector of input availabilities and of "final demands," it can be argued that such a computation is analogous to a specific solution of a Leontief matrix. However, since an inverse is obtained by the simplex method and since this inverse can be employed for purposes of parametric programming, it can also be argued that the inversion of a Leontief matrix - rather than a specific solution - corresponds more closely to a linear programming optimization. This second viewpoint will be the one adopted here. That is, we shall compare the work involved in a linear programming solution with that involved in inverting a Leontief matrix.

The comparison will be conducted solely in terms of the number of multiplications required at each of the major steps in the process. The number of multiplications is generally regarded as a good indicator of the total volume of arithmetic and of logical operations.

Under these ground-rules, we utilize Evans' estimates for input-output calculations. According to Evans - and contrary to popular impressions - it is more efficient to invert a Leontief matrix by means of the old-fashioned "elimination method" rather than "expansion-in-powers."* By use of elimination methods, the required number of multiplications can be kept down to only m^3 , rather than something of the order of the $5m^3$ involved in the power series technique.

Turning now to the linear programming algorithm, one major difficulty is that there is no easy way to predict the exact number of basis changes needed in order to reach an optimal solution. For this reason, it is convenient to regard the number of these iterations as a parameter r , and to begin by calculating the number of multiplications required at each iteration.

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* See W. D. Evans, "Input-Output Computations" in The Structural Interdependence of the Economy, T. Barna (ed.), Wiley, 1954, especially pages 77-81.

Also by Evans, "Letter to the Editor," Econometrica, January 1958. For a step-by-step explanation of the elimination method, see S. I. Gass, Linear Programming, McGraw-Hill, 1958, pages 27-31.

Now at each such step, it will be necessary to make $3m^2$ high-precision multiplications: one set of m^2 multiplications to determine the implicit prices, one set to obtain the "representation" of the new activity in terms of the previous basis, and one set to transform the previous basis-inverse into the new inverse.* In addition

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* For more details on this estimate, see H. M. Wagner, "A Comparison of the Original and Revised Simplex Methods," Operations Research, June, 1957. Also see W. Orchard-Hays, "Evolution of Linear Programming Computing Techniques," Management Science, January 1958.

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to these high-precision multiplications, there will be one group of low-precision operations needed in order to determine which of the n possible activities ought to be the new one introduced into the basis. Since these $m \cdot n$ multiplications may be performed with low-precision arithmetic, they are likely to be the equivalent of only $\frac{n \cdot m}{4}$ high-precision operations. The total number of equivalent high-precision multiplications for a linear programming solution then works out to be:

$$\text{(number of iterations)} \quad \times \quad \begin{pmatrix}\text{number of multiplications} \\ \text{per iteration}\end{pmatrix}$$

$$(r) \quad \times \quad (3m^2 + \frac{m \cdot n}{4})$$

Furthermore, RAND's experience suggests that $r \approx 2m$, and that $n \approx 4m$. Inserting these very rough numbers, the total works out as follows:

$$(r) \quad (3m^2 + \frac{m \cdot n}{4}) = 8m^3$$

Summing up: This comparison suggests that the amount of computing effort needed in order to optimize a linear programming problem lies around eight times that required for the inversion of a Leontief matrix. It is noteworthy that the relative costs are unaffected by the absolute size of the matrices involved. It is also noteworthy that advances in the art of computing would today make it possible to invert in less than 15 minutes the 200-order Leontief matrix of the U. S. - an operation requiring 48 hours in 1952.