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Financial Intermediaries and the Effectiveness of Monetary Controls

James Tobin

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The essential function of banks and other financial intermediaries is to satisfy simultaneously the portfolio preferences of two types of individuals or firms. On one side are borrowers, who wish to expand their holdings of real assets -- inventories, residential real estate, productive plant and equipment, etc. -- beyond the limits of their own net worth. On the other side are lenders, who wish to hold part or all of their net worth in assets of stable money value with negligible risk of default. The assets of financial intermediaries are obligations of the borrowers -- promissory notes, bonds, mortgages. The liabilities of financial intermediaries are the assets of the lenders -- bank deposits, insurance policies, pension rights.

Financial intermediaries assume liabilities of smaller default risk and greater predictability of value than their assets. The principal kinds of institutions take on liabilities of greater liquidity too; thus bank depositors can require payment on demand, while bank loans become due only on specified dates. The reasons that the intermediation of financial institutions can accomplish these transformations between the nature of the obligation of the borrower and the nature of the asset of the ultimate lender are these: (1) administrative economy and expertise in negotiating, accounting, appraising, and collecting; (2) reduction of risk per dollar of lending by the pooling of independent risks, with respect both to loan default and to deposit withdrawal; (3) governmental guarantees of the liabilities of the institutions and other provisions (bank examination, investment regulations, supervision of insurance companies, last-resort lending) designed to assure the solvency and liquidity of the institutions.

The main instrument of monetary control of the economy is quantitative limitation of the total liabilities of one category of financial intermediary, commercial banks. The growth of other intermediaries has raised prominently the question whether their exemption from quantitative limitation makes control of commercial banks an empty gesture.* That question is the

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* This question, and the other general implications of nonbank intermediaries, were effectively brought before the profession in the celebrated article of Gurley and Shaw ("Financial Aspects of Economic Development," American Economic Review XLV, September 1955, 515-538), to which the present paper owes an obvious debt. In similar vein Warren Smith concludes his excellent article, "On the Effectiveness of Monetary Policy," American Economic Review, XLVI, September 1956, 588-606, by saying, "...it is possible that in future years, as the activities of financial institutions other than commercial banks become more important, the continued effectiveness of monetary controls will require that the Federal Reserve's authority be extended to cover some of the operations of these institutions."

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concern of this paper. In Part I the most relevant features of the markets in which financial intermediaries operate are discussed. These features are embodied in the model used in Parts II and III as the basis for comparing the present regime of control of commercial banks with an uncontrolled regime, on the one hand, and with extensions of control to cover other intermediaries, on the other. In Part II these comparisons are made verbally and diagrammatically, with the aid of the assumption that all intermediaries other than commercial banks can be lumped into a single category. In Part III the argument is algebraic, and this assumption does not have to be made. The model abstracts from many of the complexities of actual financial institutions and capital markets; it is hoped that it retains the features essential for the issue at hand.

The main conclusions can be briefly stated. The presence of other financial intermediaries does not mean that monetary control through commercial banks has no effect on the economy. Even if increases in the assets and liabilities of other intermediaries wholly offset enforced reductions in commercial bank deposits, or expansion of commercial banks means equivalent contraction by other intermediaries, the traditional instrument of control can still be effective. However, substitutions between the assets and liabilities of other intermediaries and those of commercial banks do diminish the effectiveness of the traditional instrument; a billion dollar change in the permitted volume of commercial bank deposits would have more effect on the economy if such substitutions were prevented. Whether it is important that monetary controls be more effective is another question, to which this paper is not addressed. When a given remedial effect can be achieved either by a small dose of strong medicine or a large dose of weak medicine, it is not obvious that the small dose is preferable. Extension of quantitative controls to other financial intermediaries involves considerations of equity and of public policy with respect to the scope of governmental authority, as well as the efficacy of instruments of economic stabilization.

In the interests of concise terminology, <u>banks</u> will refer to commercial banks, and <u>nonbanks</u> to other financial institutions, including savings banks. Moreover, <u>intermediary</u> will refer to an entire species, or industry, of financial institutions. Thus all commercial banks constitute one intermediary, all life insurance companies another, and so on. An <u>institution</u> will mean an individual member of the species, an individual firm in the industry -- a bank, or a life insurance company, or a retirement program.

Part I

The market structure of financial intermediaries and the special position of banks

Financial institutions fall fairly easily into distinct categories, each industry offering a differentiated product to its customers, both lenders and borrowers. From the point of view of lenders, the obligations of the various intermediaries are more or less close, but not perfect, substitutes. For example, savings deposits share most of the attributes of demand deposits; but they are not means of payment, and the institution has the right, seldom exercised, to require notice of withdrawal. Similarly there is differentiation in the kinds of credit offered borrowers. Each intermediary has its specialty -- e.g. the commercial loan for banks, the real estate mortgage for the savings-and-loan association. But the borrowers' market is not completely compartmentalized. The same credit instruments are handled by more than one intermediary, and many borrowers have flexibility in the type of debt they incur. Thus there is some substitutability, in the demand for credit by borrowers, between the assets of the various intermediaries.*

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* These features of the market structure of intermediaries, and their implications for the supposed uniqueness of banks, have been emphasized by Gurley and Shaw, loc. cit. An example of substitutability on the deposit side is analyzed by David and Charlotte Alhadeff, "The Struggle for Commercial Bank Savings," Quarterly Journal of Economics, LXXII, February 1958, 1-22.

There is also product differentiation within intermediaries, between institutions, arising from location, advertising, and the other sources of monopolistic competition. But this is of a smaller order than the differentiation between intermediaries. For present purposes, the products offered by the institutions within a given intermediary can be regarded as homogeneous.

Among financial intermediaries commercial banks occupy a special position both in the attention of economic analysis and as the fulcrum of monetary control. The usual rationale for this special position is that, alone among intermediaries, banks "create" means of payment. This rationale is on its face far from convincing. The means-of-payment characteristic of demand deposits is indeed a feature differentiating bank liabilities from those of other intermediaries. Insurance against death is equally a feature differentiating life insurance policies from the obligations of other intermediaries, including banks. It is not obvious that one kind of differentiation should be singled out for special analytical and legal treatment. Like other differentia, the means-of-payment attribute has its price. Savings deposits, for example, are perfect substitutes for demand deposits in every respect except as a medium of exchange. This advantage of checking accounts does not give banks absolute immunity from the competition of savings banks; it is a limited advantage that can be, at least in some part for many depositors, overcome by differences in yield.

Another customary distinction of commercial banks concerns the role of the asset preferences of depositors in determining aggregate liabilities. Total deposits are determined, not at all by the choices of depositors,* but

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* In the United States, this famous proposition regarding the creation of bank deposits is subject to a number of minor technical qualifications due to the influence of the public's demand for currency on the supply of bank reserves and to differences in reserve requirements for time and demand deposits.

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by the volume of assets that banks find it permissible, under legal reserve requirements, to acquire. For any other intermediary, virtually the reverse situation is held to prevail: the aggregate assets of the intermediary are limited to the funds entrusted to them by lenders or depositors. Nonbank intermediaries merely transmit to borrowers the asset preferences of the lenders. This distinction of banks is logically separate from their uniqueness as manufacturers of means of payment. The total liabilities, whether means of payment or not, of any intermediary subjected to effective reserve requirements would be determined by the supply of reserves rather than the preferences of lenders.

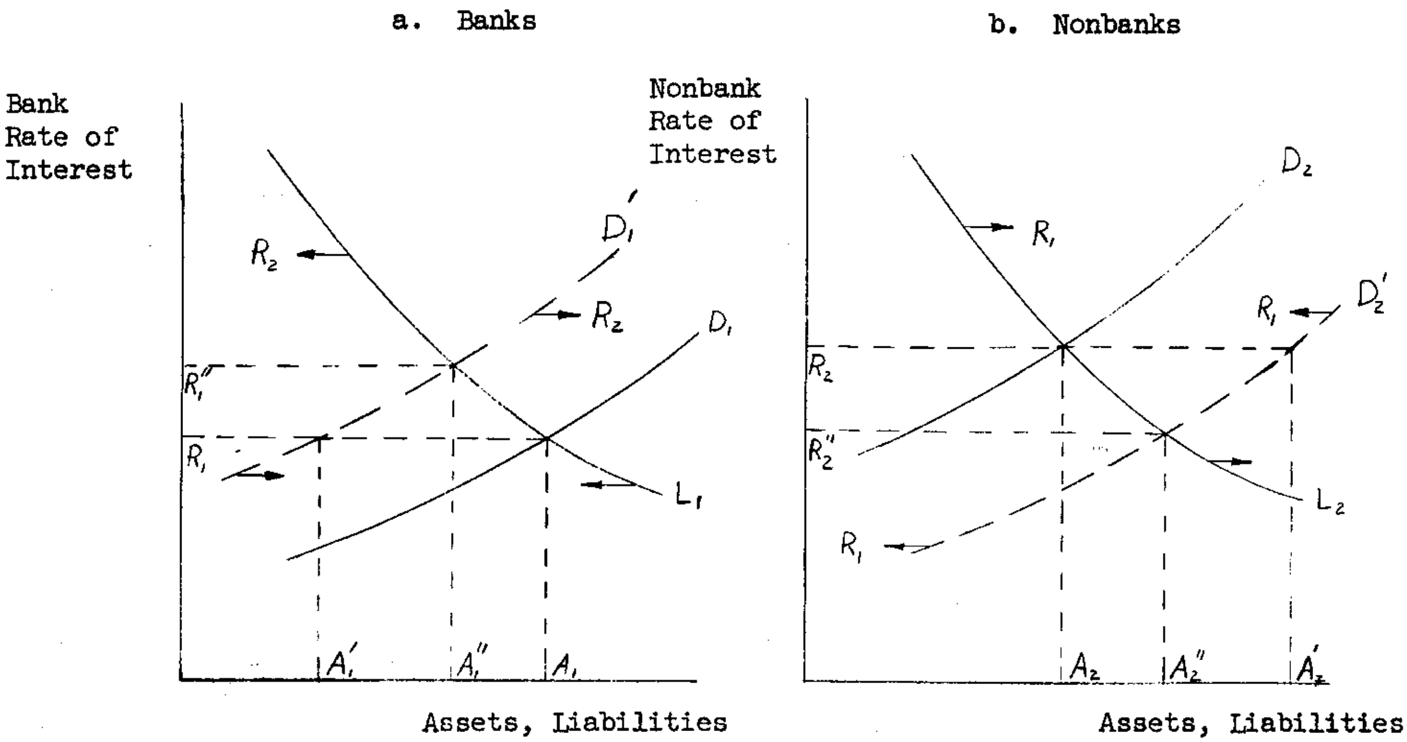
In an unregulated financial world, the preferences of depositors, as well as those of borrowers, would be a very relevant factor in determining the volume of bank deposits. The volume of assets and liabilities of every intermediary, banks as well as nonbanks, would be determined by a competitive equilibrium, in which the rate of interest charged borrowers by the institutions of that intermediary just balances the rate of interest paid their creditors. Some discrepancy between the two rates would exist in equilibrium, to compensate financial institutions for their risks and costs. But assuming that this normal premium is a constant or increasing function of the volume of the intermediary, no harm is done by ignoring it; the equilibrium condition for each intermediary can then be conveniently stated as equality between the borrower-rate and lender-rate in that intermediary. If the borrower-rate exceeds the lender-rate, competition among existing institutions and possibly entry of new firms will expand the aggregate liabilities and assets of the intermediary. To expand loans, it will generally be necessary to lower the borrower-rate. To expand deposits or other liabilities, it will generally be necessary to raise the lender-rate. Thus the expansion of the industry brings the two rates to equality.

The reverse process occurs for the opposite disequilibrium, when the lender-rate exceeds the borrower-rate.

In this unregulated competitive regime, suppose that depositors decide that at prevailing lender-rates they prefer to hold more savings accounts or other nonbank liabilities and less demand deposits. They transfer demand deposits to the credit of nonbanks, providing these intermediaries with the means to seek additional assets. Nonbanks, finding themselves able to attract more funds from the public even with some reduction in their lender-rates, offer better terms to borrowers and bid up the prices of existing earning assets. At lower borrower-rates commercial banks release some earning assets, because they no longer yield enough to pay the going rate on the banks' deposit liabilities. Bank deposits decline along with bank assets. In effect, the nonbank intermediaries favored by the shift in public preferences simply swap the deposits transferred to them for a corresponding amount of bank assets.

Figure 1 is an over-simplified illustration of this process. In Figure 1a, the lender- and borrower-rates of interest for banks are measured on the vertical axis, and aggregate bank assets or liabilities on the horizontal axis. The curve L_1 is the demand curve for loans: taking as given the rates charged by other intermediaries, banks as an industry will be able to make more loans the lower the rate they charge. The curve D_1 is the supply curve of deposits: taking as given the rates offered lenders by other intermediaries, banks as an industry will attract more deposits the higher the rate they offer. Competition among banks sets the equilibrium rate of interest at R_1, and the volume of assets or deposit liabilities at A_1. In Figure 1b nonbanks are for simplicity pooled into a single industry. The rate of interest charged or paid by this intermediary is measured vertically and the total volume of assets or liabilities horizontally. Curves L_2 and D_2 are analogous to the demand and supply curves for banks; L_2 and D_2 are drawn on the assumption that the bank rate is R_1. They determine an equilibrium rate of interest R_2, not necessarily equal to R_1, at a volume A_2. Now suppose there is a shift in lenders' (depositors') preferences away from banks towards nonbanks, as shown by the shifts of the supply curves to D_1' and D_2'. At existing rates R_1 and R_2, this means a shift of deposits of amount $A_1'A_1 = A_2A_2'$. Nonbanks will receive bank deposits of this amount. They will use them to acquire additional assets, lowering their rate of interest and reaching a new equilibrium at R_2'' and A_2''. At the same time, banks will release earning assets and raise rates, finding a new equilibrium at R_1'' and A_1''. Even this is not the end of the story, for so far cross-effects of rate changes have been ignored. Reduction of the rate of interest of nonbanks from R_2 to R_2'' can be expected to shift the bank curves D_1 and L_1 in the directions indicated by the arrows. Likewise, the increase in the bank rate from R_1 to R_1'' would shift the curves D_2' and L_2 as shown. But these adjustments would not alter the general qualitative comparison of the new equilibrium with the old: the change in lenders' preferences alters the volume of bank deposits as well as the volume of nonbank liabilities. Both of banks and of nonbanks, it can be said with truth that they dispose only of the funds that lenders entrust to them. This is, however, only a partial truth, because the borrowers' side of the market is equally relevant. The strength of the demand to borrow from a given intermediary determines how vigorously institutions of that type compete for the funds of lenders.

Figure 1

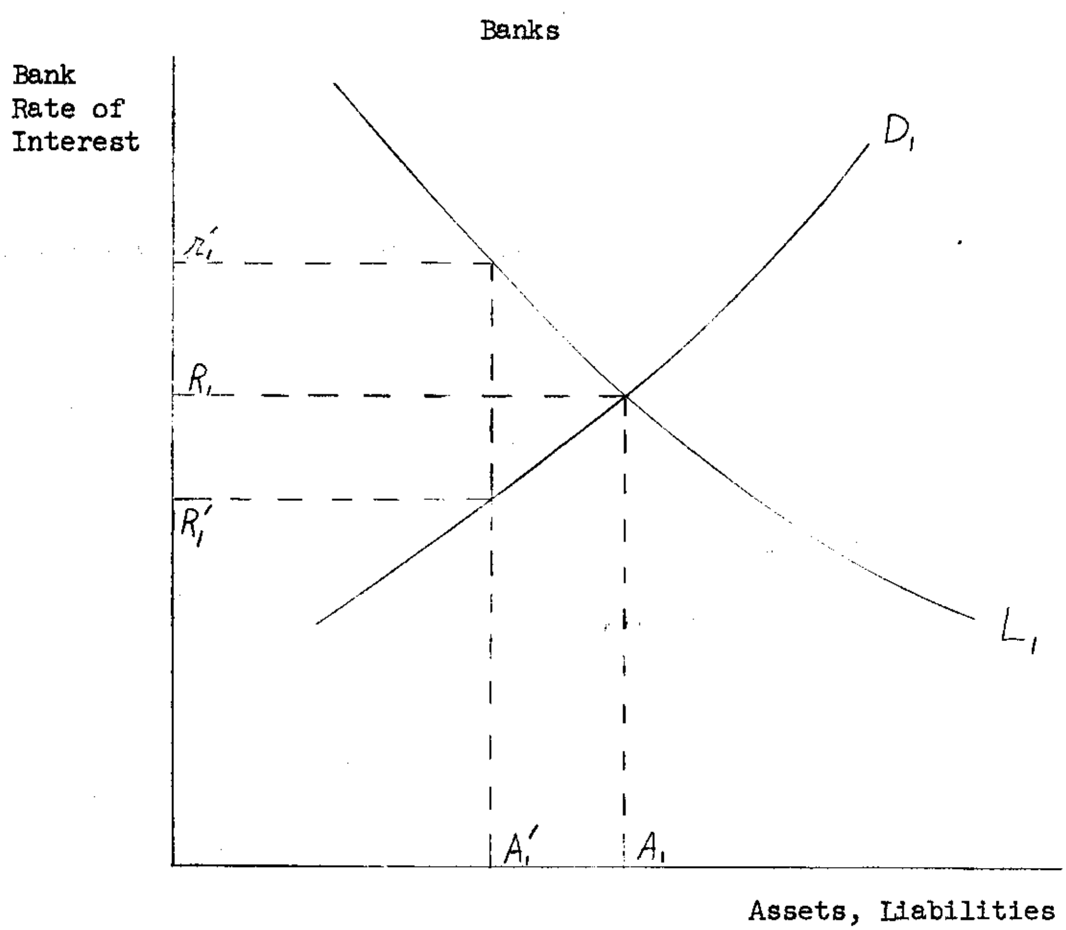


The regime of excess reserves in the 1930's was an approximation to the kind of world just described. The moral of the excursion into a world now so distant is the following: It is more accurate to attribute the special place of banks among intermediaries to the quantitative restrictions to which banks alone are subjected than to attribute the quantitative restrictions to the special character of bank liabilities.

Commercial banking is in fact an industry prevented by quantitative limitation from expanding to its equilibrium size, and prevented by restrictions on competition among banks for deposits from making its restricted size an equilibrium. At its restricted scale of total assets and liabilities, the rate that banks can charge their borrowers exceeds the rate they need to pay their depositors. This discrepancy is preserved partly by legal and institutional arrangements, among them restrictions on entry into the industry and prohibitions on payments of interest on demand deposits, and partly by product differentiation among banks, here left out of explicit consideration.

In Figure 2, as in Figure 1, the interest rates on loans and deposits are measured vertically, the outstanding volume of loans and deposits horizontally. The L_1 curve represents the demand of borrowers for bank loans at various interest rates, and the D_1 curve the total of deposits lenders are willing to hold at various deposit interest rates. Unrestricted competition would result in a volume of loans and deposits A_1 with a rate of interest R_1. However, reserve requirements limit the volume to A_1'. At this volume, the supply price of deposits is R_1', and the demand price of loans is r_1'. If the actual interest rate paid on deposits were to exceed R_1', lenders would wish to deposit more than banks could accept, and it would be necessary to ration deposits. If the actual interest rate charged on loans were to fall short of r_1', borrowers would wish to borrow more than banks could lend, and rationing of loans would be necessary. Since any one bank can always obtain additional reserves by bidding deposits away from its competitors, competition among banks would tend to make the two rates equal even when reserves are restrictive on the industry at large. The obstacles to competition mentioned above keep the two rates apart and permit bank shareholders to capture the rents arising from scarcity of reserves, rents that in competition would accrue to depositors or borrowers.

Figure 2



Part II

The Efficacy of Quantitative Controls of Commercial Banks

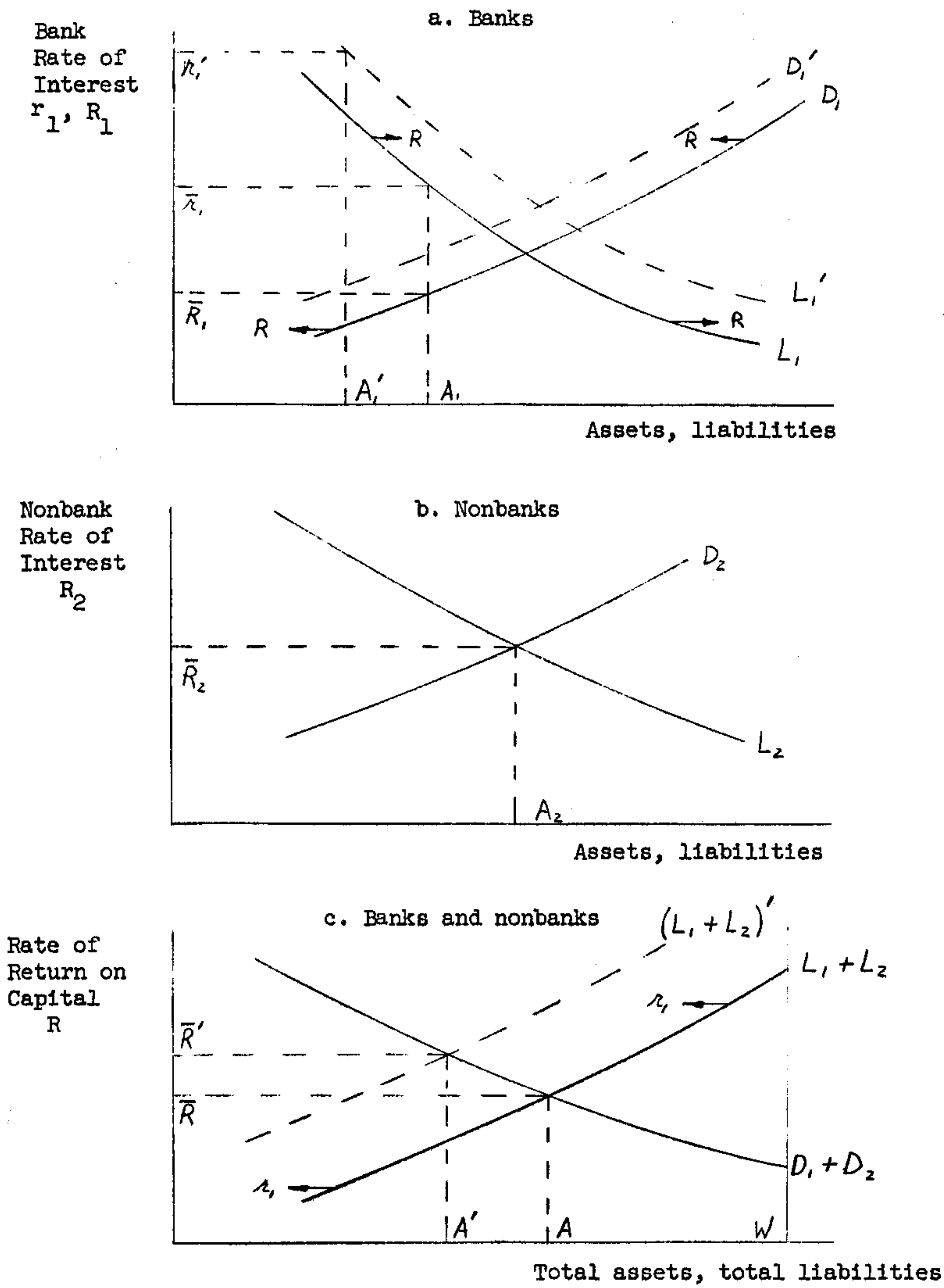
How do quantitative controls over commercial banks affect the demand for goods and services? Are these controls nullified, partly or wholly, by the freedom of borrowers and lenders to turn to other financial intermediaries? Would monetary controls be more effective if they were imposed on these institutions as well as on banks? These are the questions to which the model set forth in this paper is addressed.

A monetary control can be considered inflationary if it lowers the rate of return on ownership of real capital that the community requires to induce it to hold a given stock of capital, and deflationary if it raises that rate of return. (The words inflationary and deflationary are used merely to indicate the direction of influence; the manner in which the influence is divided between price change and output change depends on aspects of the economic situation that are not relevant here.) The value of the rate of return referred to is a hypothetical one -- the level at which owners of wealth are content to absorb the given stock of capital into their portfolios or balance sheets along with other assets and debts. In equilibrium, this critical rate of return must equal the expected marginal productivity of the capital stock, which depends technologically on the size of the stock relative to expected levels of output and employment. If a monetary action lowers the rate of return on capital that owners of wealth will accept, it becomes easier for the economy to accumulate capital. If a monetary action increases the rate of return on equity investments demanded by owners of wealth, then it discourages capital accumulation.

Let the value of the total stock of capital in the economy at a given moment of time be given. Let R be the rate of return on capital that induces the community to hold this stock. For the reasons given in the preceding paragraph, R will serve as a gauge of the direction and degree of monetary influence on the economy. On the basis of this expected rate of return on capital investments, certain individuals and firms desire to hold more capital than their own net worths permit. These constitute the borrowers' side of the market for loans from intermediaries, and for loans directly placed with lenders. Loans to these borrowers finance the holding of part of the existing stock of capital. The remainder must be held directly by its ultimate owners, either as the equities of borrowers or as parts of the portfolios of lenders. In lenders' portfolios equity in capital must compete for place with the obligations of borrowers and of financial institutions. The aggregate net worth of borrowers and lenders is identically equal to the value of the stock of capital. (In this account the other components of total private wealth -- internal government debt, basic monetary stock, and claims against foreign countries -- can be ignored.)

Continuing the simplification that there are only two intermediaries, banks and nonbanks, and no direct lending, the liabilities of these two kinds of institutions are for wealth owners the only alternatives to direct ownership of capital. Similarly the assets of these two intermediaries are the only two kinds of credit available to borrowers. The difference between the total value of capital and the total assets or liabilities of the two intermediaries represents the amount of capital that must be directly held as equities of individuals. Figure 3a shows the assets and liabilities of banks in relation to bank rates of interest, and Figure 3b is the same for nonbanks. The supply and demand curves in Figures 3a and 3b have the same interpretation as in Figure 1. For example, L_1 is the demand for bank loans in relation to the bank rate r_1, given the nonbank rate $\bar{R}_2$ and the rate of return on capital $\bar{R}$. Curve D_1 represents the supply of bank deposits in relation to the bank deposit rate R_1, given $\bar{R}_2$ and $\bar{R}$. In Figure 3c the downward-sloping curve shows the relationship of total deposits in banks and nonbanks together to the rate of return on capital R, given the rates on deposits

Figure 3


a. Banks
Bank
Rate of
Interest
r_1, R_1
$\bar{r}_1'$
$\bar{r}_1$
$\bar{R}_1$
R
D_1'
D_1
L_1'
L_1
A_1'
A_1
Assets, liabilities
b. Nonbanks
Nonbank
Rate of
Interest
R_2
$\bar{R}_2$
D_2
L_2
A_2
Assets, liabilities
c. Banks and nonbanks
Rate of
Return on
Capital
R
$\bar{R}'$
$\bar{R}$
$(L_1 + L_2)'$
$L_1 + L_2$
r_1
$D_1 + D_2$
A'
A
W
Total assets, total liabilities

$\bar{R}_1$ and $\bar{R}_2$. The higher the rate of return on capital, the less attractive are deposits in intermediaries compared to direct ownership of capital. The vertical line at W is the given amount of total wealth. Thus curve $D_1 + D_2$ can be read from right to left to show a positive relationship between lenders' demand for direct equity and its rate of return. The upward-sloping curve $L_1 + L_2$ indicates that, at given rates for loans from intermediaries, $\bar{r}_1$ and $\bar{R}_2$, borrowers will wish to borrow more the greater the prospects of profit from capital investment.

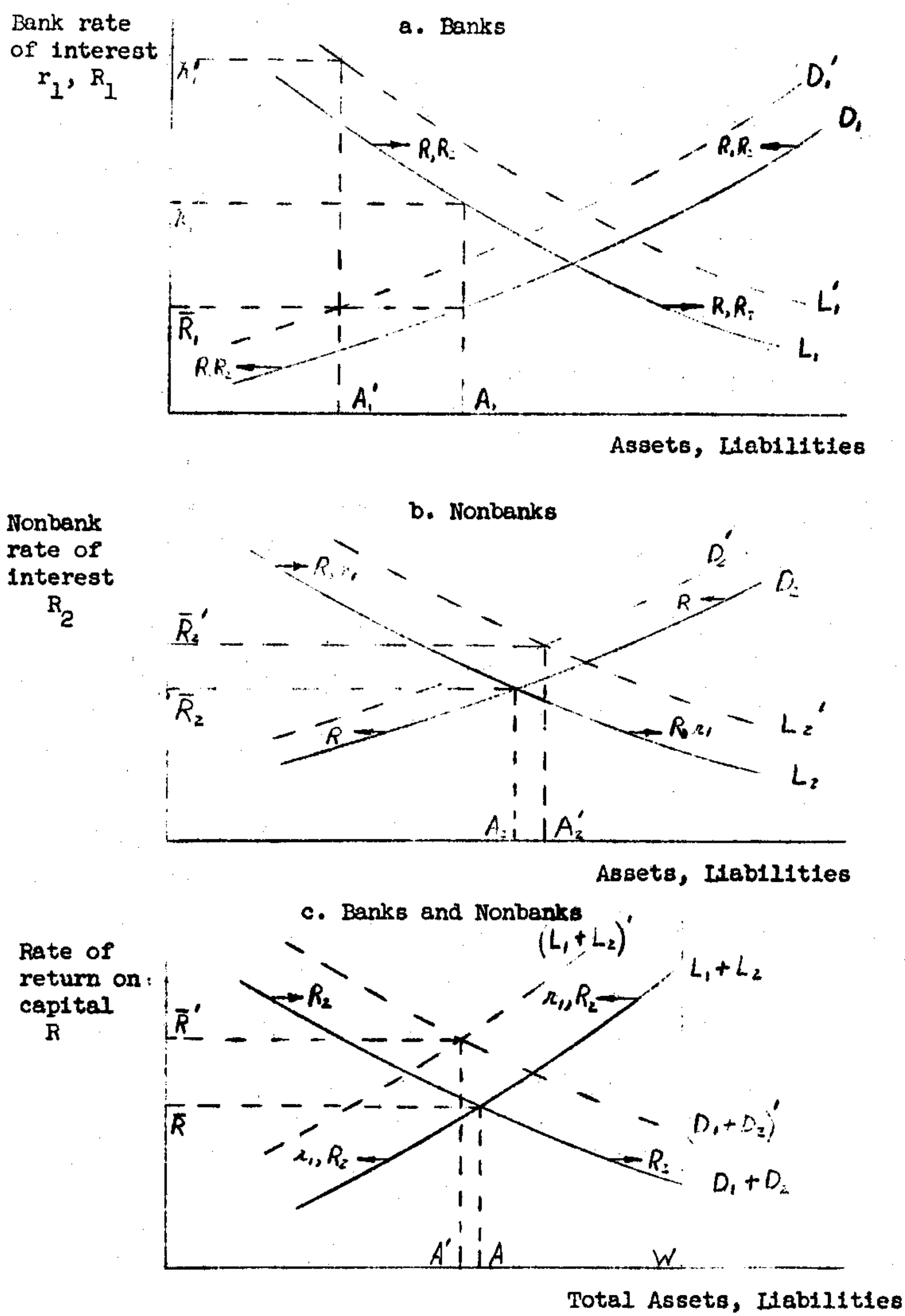
Suppose that, as a deflationary measure, the permissible volume of bank deposits is reduced from A_1 to A_1' . Assume, only tentatively, that nothing happens to nonbanks. The rate paid bank depositors is unchanged at $\bar{R}_1$. The bank loan rate if increased to $\bar{r}_1'$. The amount of wealth that must be directly held is increased by the amount AA' of the reduction in deposits. If individuals are to be induced to hold this addition, they must earn a higher return. The increase in R shifts backwards the supply curve of bank deposits, which must end up in such a position, D_1' , that lenders are satisfied to hold the lesser volume of deposits at the established interest rate $\bar{R}_1$. Meanwhile the rise in bank loan rate shifts the aggregate loan curve in Figure 3c, $L_1 + L_2$, backwards towards $(L_1 + L_2)'$. The new equilibrium is indicated by the curves and points marked with single primes The increase in the required rate of return R is the gauge of the success of the deflationary policy. In the diagrams, arrows indicate the direction of shift of a curve, and the symbols beside the arrows the variables whose change in value causes the shift.

The tentative assumption that nonbanks are unaffected by this process must be removed. Nonbanks provide another alternative, besides direct and self-financed equity in capital, for borrowers and depositors displaced from banks. As shown in Figure 4, which uses the same variables and symbols as Figure 3, the increase in bank loan rate due to the restriction of banks causes the demand for nonbank loans to increase. The resulting increase in the nonbank loan and deposit rate R_2 has several effects: It increases the demand for bank loans and reduces the supply of bank deposits. (Figure 4a). It increases the total supply of deposits; and it reinforces the increase in bank loan rate r_1 to reduce the total demand for loans. (Figure 4c.) The resulting increase in the required equity rate R shifts backwards the nonbank deposit supply curve D_2 and reinforces the other shifts in Figures 4a and 4b. In the new equilibrium R is higher than before, and this is indicative of the success of the deflationary policy.

Figure 4b shows an increase in nonbanks' assets and liabilities. This is the result normally to be expected from a curtailment of banks, but it is not inevitable. If the cross-effect of R on nonbank deposits were strong relative to the combined cross-effects of r_1 and R on nonbank loans, nonbank deposits and assets might fail to expand at all, or even contract.

In Figure 4 as drawn the curtailment of total intermediary assets and liabilities is considerably smaller than the reduction of bank deposits. In large measure the loans and deposits of nonbanks are substituted for those of banks. But this substitution by no means renders the restriction on banks ineffectual. Since borrowers are charged more and depositors are paid more, the demand for capital is reduced; it takes a higher return on a given stock to induce the community to hold it. Indeed it would be entirely possible for nonbank assets to expand to the full extent of the contraction of bank assets or more, without destroying the effectiveness of the policy. The new equilibrium A' in Figure 4c may be to the right or left of the old one A .

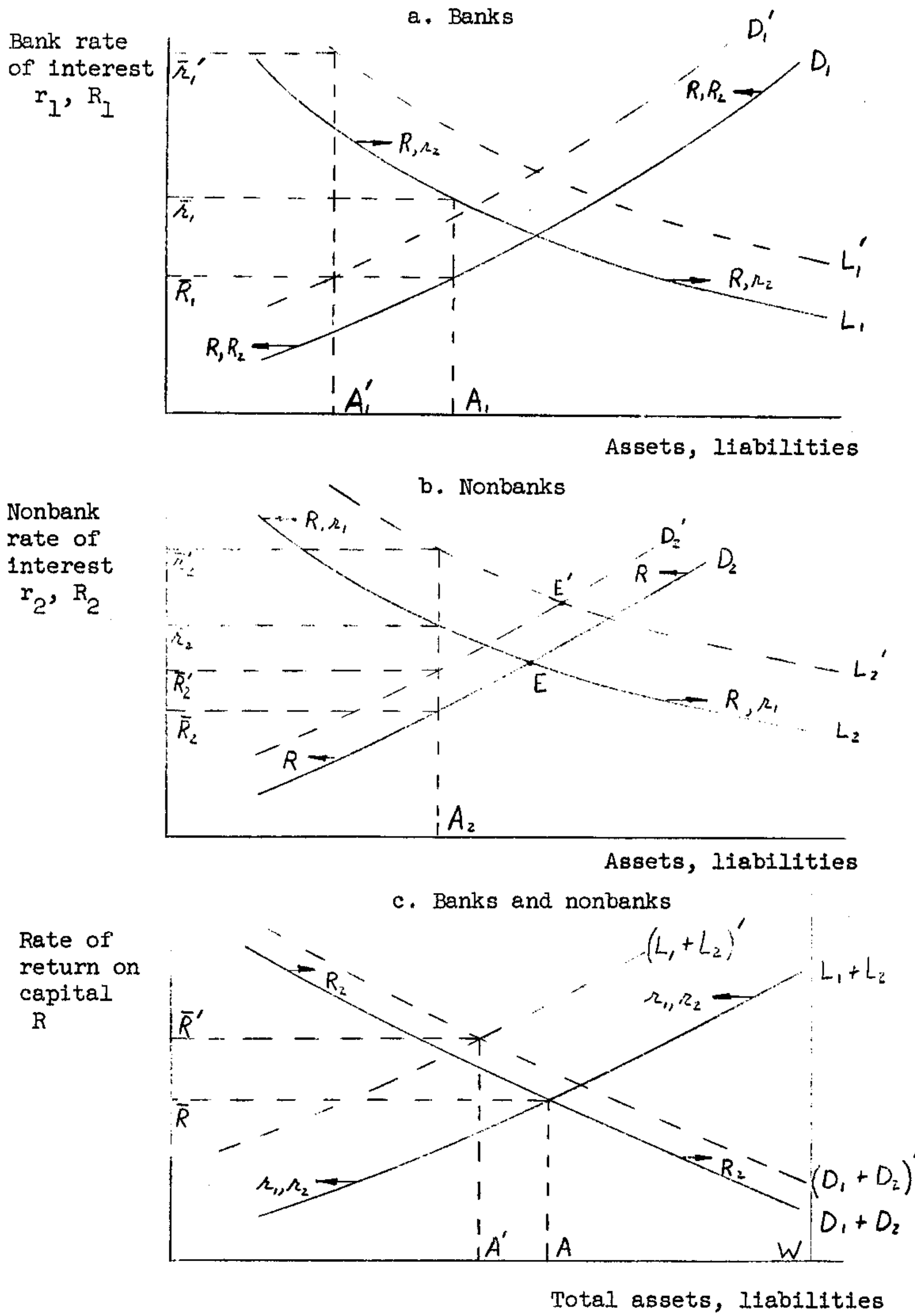
Figure 4


Bank rate of interest r_1, R_1
a. Banks
D_1'
D_1
L_1'
L_1
$\bar{R}_1$
A_1'
A_1
Assets, Liabilities
Nonbank rate of interest R_2
b. Nonbanks
D_2'
D_2
$\bar{R}_2'$
$\bar{R}_2$
L_2'
L_2
A_2
A_2'
Assets, Liabilities
Rate of return on capital R
c. Banks and Nonbanks
$(L_1 + L_2)'$
$L_1 + L_2$
$\bar{R}'$
$\bar{R}$
$(D_1 + D_2)'$
$D_1 + D_2$
A'
A
W
Total Assets, Liabilities

Although substitution of nonbank assets and liabilities for those of banks does not *per se* mean that monetary controls over banks alone are ineffective, these controls might be more effective if such substitution were prevented. "More effective" in this context means a larger increase in the required rate of return on capital R for a given restriction of commercial banks. Figure 5 follows the same model as Figures 3 and 4. However in Figure 5b nonbanks are subject to an effective size limit. The mechanism for enforcing such a limit is not a present concern. It could be done by some kind of reserve requirement (with reserves different from those of commercial banks and controlled as to quantity by a central authority) or by administrative fiat. As in the case of banks, an inevitable byproduct would be a discrepancy between the loan rate $\bar{r}_2$ and the deposit rate $\bar{R}_2$. Again, some anti-competitive mechanism is needed to enforce this discrepancy. But the deposit rate for nonbanks, unlike that for banks, must be assumed to be flexible. Otherwise the system is overdetermined; there would have to be some rationing mechanism for accepting deposits.

In this regime restriction of bank operations raises the demand for nonbank loans; since the volume of nonbank loans is prevented from expanding, the only outcome is an increase in r_2. The increased costs of borrowing, from both banks and nonbanks, shifts the $L_1 + L_2$ curve backwards, as shown in Figure 5c. The rise in R shifts the deposit supply curve for nonbanks and increases R_2. By higher rates on nonbank deposits as well as on direct equity, the public is reconciled to the lower available volume of bank deposits. In the new equilibrium R will have to be enough higher to accomplish an increase in direct ownership of capital exactly equal to the reduction of bank deposits.

Figure 5


a. Banks
Bank rate of interest r_1, R_1
Assets, liabilities
b. Nonbanks
Nonbank rate of interest r_2, R_2
Assets, liabilities
c. Banks and nonbanks
Rate of return on capital R
Total assets, liabilities

The interesting question is whether this increase in R is larger than the increase that occurs from the same restriction on banks in the absence of control over nonbanks. An answer can be given on the assumption that all the relationships can be regarded as linear in the relevant ranges. If in the regime of Figure 4 -- no limitation on nonbanks -- the volume of nonbank assets and liabilities would be increased, then the increase in R is greater in the regime of Figure 5 -- effective limitation on nonbanks. This is the situation actually depicted in Figures 4 and 5; note that the "shadow equilibrium" in Figure 5b moves to the right, from E to E' . If in the first regime there would be no substitution of nonbank assets and liabilities for those of banks, then limitation of nonbanks would not enhance the deflationary effects of reduction of bank loans and deposits. Indeed if E' were to the left of E in Figure 5b, but still to the right of A_2 , there would be less deflationary effect in the second regime than in the first. These statements are not easy to demonstrate without the mathematics of Part III. But it is at least plausible that a greater rise in R is required the larger the necessary increase in direct ownership of capital.

Once a limit on nonbanks existed, its variation would be an instrument of monetary control supplementary or alternative to variation of the quantitative limit on banks. The effectiveness of varying the limit on nonbanks is a separate matter from the question discussed above, the effect of the existence of this kind of limit on the strength of the control over banks. Reducing the permissible volume of nonbank assets and liabilities increases the required return on capital R , acting in much the same manner as the analogous restriction of banks illustrated in Figure 5. The situations are not wholly symmetrical, however, since the nonbank deposit rate is assumed to be flexible and the bank deposit rate fixed.

Part III

The General Model

Assume that there are n types of financial assets that owners of wealth can hold. The first is commercial bank deposits. An n+1 asset, direct equity in capital, is designated by subscript 0 . Let D_i be the total amount held in the ith asset (i=0, 1, 2, ..., n). The sum of the D_i must be equal to W , the fixed value of total wealth, the value of the capital stock. Let R_i be the rate of interest offered owners of the ith asset; R_0 is simply the R of Part II. Each D_i may be taken to be a function of all the R_i . There are n independent functions to distribute a fixed total wealth W into n+1 categories. Thus,

$$(1) \qquad D_i = D_i(R, R_1, R_2, \ldots, R_n) \qquad (i=1,2,\ldots, n)$$

The assets are assumed to be gross substitutes, so that the effect of a reduction in the jth interest rate, other rates remaining constant, is to diminish D_j and to increase or at least to leave unchanged each of the other asset holdings including D_0 . Similarly, it is assumed that the effect of a reduction in R is to increase or leave unchanged every financial asset holding. Using the notation D_{ij} to represent the partial derivative of the function D_i with respect to the jth rate, these assumptions are as follows:

$$(2) \qquad \begin{array}{ll} D_{ij} > 0 \quad (i=j) & \\ D_{ij} \leqq 0 \quad (i \neq j) & \\ \sum_{i=1}^{n} D_{ij} > 0 & \qquad (i,j = 1,2,\ldots, n) \\ D_{i0} \leqq 0 & \end{array}$$

The demands of borrowers for loans of each type depend jointly on the rate of return on capital, R, and on the n different borrowers' interest rates r_i :

$$(3) \qquad L_i = L_i(R, r_1, r_2, \ldots, r_n) \qquad\qquad (i = 1,2, \ldots, n)$$

Debts of different types are assumed to be gross substitutes, so that the effect of a reduction in the jth borrowers' rate, other rates remaining constant, is to increase L_j and to diminish or leave unchanged both all other debts and L_0 $(=W-\sum_{i=1}^{n} L_i)$, capital directly owned and self-financed. The effect of a reduction in R is to diminish or leave unchanged borrowers' demands for each type of loan. These assumptions are as follows:

$$(4) \qquad \begin{array}{ll} L_{ij} < 0 & (i=j) \\ L_{ij} \geqq 0 & (i \neq j) \\ \sum_{i=1}^{n} L_{ij} < 0 & \\ L_{i0} \geqq 0 & \end{array} \qquad\qquad (i,j = 1,2, \ldots, n)$$

In equilibrium lending must equal borrowing in each intermediary:

$$(5) \qquad D_i - L_i = 0 \qquad\qquad (i = 1,2, \ldots, n)$$

This equality ignores as inessential to the present purpose the other items on the balance sheets of financial institutions. The most important omissions are capital accounts and reserves. To allow for these would substitute more complicated balance equations for (5).

Given the behavior described by (1) to (5), a number of alternative regimes are conceivable, depending on which variables, if any, are taken as parameters and which are determined by the system. The values of parameters are determined otherwise than by the behavior of financial institutions, borrowers, and lenders; they are generally the instruments of control by the monetary authorities. The values of the other variables are determined by the interactions of intermediaries, borrowers, and lenders. A list of some of the more interesting regimes is given in Table 1, but it is far from exhaustive. In regime I there is no monetary control; this is the unregulated competitive world discussed in Part I above and illustrated by Figure 1. Regime II corresponds roughly to the present situation in the United States and other countries where controls are limited to commercial banks. In regimes III and IV there are, in different ways, greater degrees of control than United States monetary authorities now have.

Table 1

Regime	Parameters	Variables Determined	Remarks
I. Uncontrolled	One: R, rate of return on capital	n: R_i, lender-and borrower-rates for all intermediaries.	No discrepancies between lender- and borrower-rates.
II. Control of banks.	Two: $\bar{D}_i$, the volume of bank deposits; $\bar{R}_1$, the rate paid bank depositors.	n+1: R, the required rate of return on capital; r_1 the rate charged bank borrowers; R_i, the rates paid lenders by the n-1 nonbank intermediaries	No discrepancies between lender- and borrower- rates except for banks.
III. Individual controls over all intermediaries	n+1: $\bar{D}_i$, the volume of liabilities of each intermediary; $\bar{R}_1$ the rate paid bank depositors.	2n: R, the required rate of return on capital; r_i, the rates charged borrowers by all n intermediaries; R_i the rates paid lenders by the n-1 nonbank intermediaries.	Discrepancies between lender- and borrower-rates in all intermediaries.

IV. Control of aggregate intermediary liabilities	Two: $\sum_{i=1}^{n} D_i$, the sum of intermediary liabilities; $\bar{R}_1$, the rate paid bank depositors.	n+1: R_1, the required rate of return on capital; r_1, the rate charged bank borrowers; R_i, the rates paid lenders by the n-1 nonbank intermediaries.	No discrepancies between lender- and borrower-rates except for banks.

Regime I

Lending and borrowing rate will be equal for all intermediaries in an unregulated competitive regime:

$$R_i = r_i \qquad (i = 1,2, \ldots, n) \tag{I.1}$$

(As mentioned above, it has been assumed for simplicity that competition among financial institutions within a given intermediary brings these rates into equality. This assumption could be relaxed to permit a premium to compensate for administrative costs and risks of default and illiquidity, without essential difference so long as the premium is a constant or increasing function of the total volume of assets and liabilities of the intermediary.)

Given these equalities of borrower- and lender-rates and given R , the n rates are determined by the n equations (5). In each intermediary there will be an equilibrium like that depicted for banks, Figure 1a. These interconnected equilibria must sum to a total demand for financial assets that does not exceed the total amount of wealth W . As pointed out in Part I, the relative sizes of the various intermediaries, including banks, would then depend on the preferences of borrowers and lenders, as expressed in the functions L_i and D_i . In an unregulated competitive regime of this kind, there would be no room for monetary control.

Regime II

Reserve requirements restrict the liabilities of banks to $\bar{D}_1$, and the rate paid to depositors is institutionally fixed at $\bar{R}_1$. The following equation is added to system (5):

$$\text{(II.1)} \qquad D_1(R, \bar{R}_1, R_2, \ldots, R_n) = \bar{D}_1$$

Equations (I.1) apply to all intermediaries except banks, whose loan rate is free to exceed $\bar{R}_1$. Thus the n+1 equations (5) and (II.1) determine n-1 lender-rates R_2 , ..., R_n and one borrower rate r_1 , given $\bar{R}_1$ and $\bar{D}_1$.

The effects of relaxing or tightening the quantitative restriction on banks may be found by differentiating the system (5) and (II.1) with respect to $\bar{D}_1$, giving the set of linear equations (II.2). The derivatives that solve this system are marked with subscript II to indicate that they refer to regime II.

$$\text{(II.2)} \quad \begin{bmatrix} D_{10}-L_{10} & D_{12}-L_{12} & \cdots & D_{1n}-L_{1n} & -L_{11} \\ D_{20}-L_{20} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} & -L_{21} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ D_{n0}-L_{n0} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} & -L_{n1} \\ D_{10} & D_{12} & \cdots & D_{1n} & 0 \end{bmatrix} \begin{bmatrix} (\delta R/\delta \bar{D}_1)_{II} \\ (\delta R_2/\delta D_1)_{II} \\ \cdot \\ \cdot \\ \cdot \\ (\delta R_n/\delta \bar{D}_1)_{II} \\ (\delta r_1/\delta \bar{D}_1)_{II} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

By assumptions (2) and (4), the first column of the matrix of coefficients is composed entirely of non-positive elements. In the last column, the first element is positive and the rest are non-positive. In the remaining columns, the diagonal elements are positive and the rest non-positive. Moreover, the sum of the first n elements in every column but the first is positive. It is shown in the Appendix that the determinant of such a matrix is positive and that all the cofactors of the last row are negative. All the derivatives that solve (II.2) are negative. An increase in the permitted volume of bank deposits will lower borrower-rates at all intermediaries and will lower the acceptable return on direct equity. This result was illustrated in Figure 4 for the case n=2. The result applies, of course, only to reserve restrictions that keep $\bar{D}_1$ low enough so that r_1 exceeds $\bar{R}_1$. The regime of excess reserves is regime I.

As a byproduct system (II.2) will give the effect of a change in bank deposits $\bar{D}_1$ on the liabilities of every intermediary $(\delta D_i/\delta \bar{D}_1)_{II}$. (Of course $(\delta D_1/\delta \bar{D}_1) = 1$.)

$$\text{(II.3)}\quad D_{i0}(\delta R/\delta \bar{D}_1)_{II} + D_{i2}(\delta R_2/\delta \bar{D}_1)_{II} + \dots + D_{in}(\delta R_n/\delta \bar{D}_1)_{II} = (\delta D_i/\delta \bar{D}_1)_{II} (i=1,2,..n)$$

Summing (II.3) from 1 to n gives the corresponding derivative for aggregate intermediary liabilities:

$$\text{(II.4)}\quad \sum_1^n D_{i0}\left(\frac{\partial R}{\partial \bar{D}_1}\right)_{II} + \sum_1^n D_{i2}\left(\frac{\partial R_2}{\partial \bar{D}_1}\right)_{II} + \dots + \sum_1^n D_{in}\left(\frac{\partial R_n}{\partial \bar{D}_1}\right)_{II} = \left(\frac{\partial \sum_1^n D_i}{\partial \bar{D}_1}\right) = 1 - \left(\frac{\partial \sum_2^n D_i}{\partial \bar{D}_1}\right)$$

Thus, on the assumptions of the model, controls over a single intermediary, banks, are effective. How does it happen that the tail can wag the dog? The crucial assumption is that the assets and liabilities of the various intermediaries are incomplete substitutes for each other, either for lenders or for borrowers. If lenders reacted to a reduction in the ith rate by increasing their holdings in other intermediaries by as much as they decrease the ith asset

<u>and</u> if borrowers react to a reduction in the ith rate by reducing their other kinds of debt by as much as they increase their debt to the ith intermediary, then regulation of a single intermediary could not be effective. (In the matrix of coefficients in (II.2) the last n column sums would be zero rather than positive, and this would make zero the first cofactor of the last row.)

It may be noted in passing that the bank deposit rate $\bar{R}_1$ is another possible instrument of control. Differentiating the system (5) and (II.1) with respect to $\bar{R}_1$ gives a set of equations similar to (II.2), with the same matrix of coefficients:

$$\text{(II.5)} \quad \begin{bmatrix} D_{10}-L_{10} & D_{12}-L_{12} & \cdots & D_{1n}-L_{1n} & -L_{11} \\ D_{20}-L_{20} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} & -L_{21} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ D_{n0}-L_{n0} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} & -L_{n1} \\ D_{10} & D_{12} & \cdots & D_{1n} & 0 \end{bmatrix} \begin{bmatrix} (\partial R/\partial \bar{R}_1)_{II} \\ (\partial R_2/\partial \bar{R}_1)_{II} \\ \cdot \\ \cdot \\ \cdot \\ (\partial R_n/\partial \bar{R}_1)_{II} \\ (\partial R_1/\partial \bar{R}_1)_{II} \end{bmatrix} = \begin{bmatrix} D_{11} \\ -D_{21} \\ \cdot \\ \cdot \\ \cdot \\ -D_{n1} \\ -D_{11} \end{bmatrix}$$

The denominator of the expression for $(\delta R/\delta \bar{R}_1)_{II}$ is the determinant of the matrix of coefficients, which is positive. The numerator can also be shown to be positive. It is the negative of the determinant of the matrix:

$$\begin{bmatrix} D_{11} & D_{12}-L_{12} & \cdots & D_{1n}-L_{1n} & -L_{11} \\ D_{21} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} & -L_{21} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ D_{n1} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} & -L_{n1} \\ D_{11} & D_{12} & & D_{1n} & 0 \end{bmatrix}$$

Subtracting the last row from the first and interchanging the first and last columns gives a matrix with positive diagonals and non-positive elements elsewhere, and with columns whose sums are positive. Such a matrix is shown in the Appendix to have a positive determinant. By similar argument all the derivatives that solve (II.5) are positive.

The usefulness of the bank deposit rate as a control instrument is limited in practice by a factor omitted from the present model. Bank deposits must compete not only with the liabilities of other intermediaries but also with interest-free currency. It is not inconceivable that currency could bear interest, either positive or, as Silvio Gesell suggested, negative; the currency rate, along with the deposit rate, could be a lever of monetary control. But even without such a fundamental departure from convention, there may well be sufficient looseness in substitutions between currency and deposits to permit the deposit rate to be a useful instrument.

Regime III

Although quantitative control of bank deposits is effective so long as the assets and debts of other intermediaries are incomplete substitutes for commercial bank loans and deposits, the partial substitution of the competing assets and debts may dilute the impact of monetary control. To take the extreme case, suppose that effective quantitative limits $\bar{D}_i$ are imposed on all n intermediaries. Let $r_i' = r_i - R_i$ $(i=1,2, \ldots, n)$ be the excess of the borrower-rate over the lender-rate arising as a result of the quantitative limit. It is assumed that institutional arrangements preserve these discrepancies in every intermediary. The following set of 2n equations then determines R, the n-1 rates $R_2, \ldots, R_n$ paid to lenders, and the n premiums r_i':

$$(\text{III.1}) \quad \begin{aligned} &(a) \quad D_i(R, \bar{R}_1, R_2, \ldots, R_n) - L_i(R, \bar{R}_1 + r_1', R_2 + r_2', \ldots, R_n + r_n') = 0 \\ &(b) \quad D_i(R, \bar{R}_1, R_2, \ldots, R_n) = \bar{D}_i \qquad (i = 1, 2, \ldots, n) \end{aligned}$$

The n-1 lender-rates $R_2, \ldots, R_n$ and the required rate of return on capital equity R are determined by the n equations (III.1b). Differentiating these with respect to $\bar{D}_1$ gives the following set of n linear equations:

$$(\text{III.2}) \quad \begin{bmatrix} D_{10} & D_{12} & \cdots & D_{1n} \\ D_{20} & D_{22} & \cdots & D_{2n} \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ D_{n0} & D_{n2} & \cdots & D_{nn} \end{bmatrix} \begin{bmatrix} (\partial R/\partial \bar{D}_1)_{III} \\ (\partial R_2/\partial \bar{D}_1)_{III} \\ \cdot \\ \cdot \\ \cdot \\ (\partial R_n/\partial \bar{D}_1)_{III} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

(Differentiating with respect to any other restriction $\bar{D}_i$ yields the same set, except that the 1 in the right hand vector is in the ith position rather than the first.) In the matrix of coefficients, the first column is entirely non-positive. In other columns the diagonals are positive, the other elements non-positive, and the column sums positive. The Appendix shows that the determinant of such a matrix is negative and that every cofactor of the first column is positive. Consequently both $(\delta R/\delta \bar{D}_1)_{III}$ and every other $(\delta R/\delta \bar{D}_i)_{III}$ are negative. Every restriction has a negative effect on R

The interesting question is whether $(\delta R/\delta \bar{D}_1)_{III}$, as determined by (III.2) is larger in absolute value than $(\delta R/\delta \bar{D}_1)_{II}$, as determined by (II.2). Subtract the n equations (II.3) from (III.2):

$$\text{(III.3)}\quad \begin{bmatrix} D_{10} & D_{12} & \cdots & D_{1n} \\ D_{20} & D_{22} & \cdots & D_{2n} \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ D_{n0} & D_{n2} & \cdots & D_{nn} \end{bmatrix} \begin{bmatrix} (\partial R/\partial \bar{D}_1)_{III} - (\partial R/\partial \bar{D}_1)_{II} \\ (\partial R_1/\partial \bar{D}_1)_{III} - (\partial R_1/\partial \bar{D}_1)_{II} \\ \cdot \\ \cdot \\ \cdot \\ (\partial R_n/\partial \bar{D}_1)_{III} - (\partial R_n/\partial \bar{D}_1)_{II} \end{bmatrix} = \begin{bmatrix} 0 \\ -(\partial D_2/\partial D_1)_{II} \\ \cdot \\ \cdot \\ \cdot \\ -(\partial D_n/\partial \bar{D}_1)_{II} \end{bmatrix}$$

Accordingly:

$$\text{(III.4)}\quad (\delta R/\delta \bar{D}_1)_{III} - (\delta R/\delta \bar{D}_1)_{II} = -\sum_{i=2}^{n} (\delta D_i/\delta \bar{D}_1)_{II} (\delta R/\delta \bar{D}_i)_{III}$$

Since all $(\delta R/\delta \bar{D}_i)_{III}$ are negative, the expression will be negative if all the $(\delta D_i/\delta \bar{D}_i)_{II}$ are negative; this is a sufficient, not a necessary, condition. If in regime II the liabilities of nonbanks are substituted for those of banks when banks are further restricted, then the effectiveness of restricting banks will be increased by preventing this substitution.

Regime IV

A simpler form of control would be an aggregate limit on all intermediaries. For example, all, banks and nonbanks, would compete for the same fixed quantity of reserves. But banks would retain the special characteristic of having an institutionally determined deposit rate, with a loan rate above it. The equation system for this regime is:

$$\text{(IV.1)}\quad \begin{aligned} & D_i(R, \bar{R}_1, R_2, \ldots, R_n) - L_i(R, \bar{R}_1 + r_1', R_2, \ldots, R_n) = 0 \quad (i=1,2, \ldots, n) \\ & \sum_{1}^{n} D_i(R, \bar{R}_1, R_2, \ldots, R_n) = \bar{D} \end{aligned}$$

Differentiating (IV.1) with respect to $\bar{D}$ gives:

$$(IV.2)\quad \begin{bmatrix} D_{10}-L_{10} & D_{12}-L_{12} & \cdots & D_{1n}-L_{1n} & -L_{11} \\ D_{20}-L_{20} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} & -L_{21} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ D_{n0}-L_{n0} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} & -L_{n1} \\ \sum_1^n D_{i0} & \sum_1^n D_{i2} & \cdots & \sum_1^n D_{in} & 0 \end{bmatrix} \begin{bmatrix} (\partial R/\partial \bar{D})_{IV} \\ (\partial R_2/\partial \bar{D})_{IV} \\ \cdot \\ \cdot \\ \cdot \\ (\partial R_n/\partial \bar{D})_{IV} \\ (\partial r_i/\partial \bar{D})_{IV} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

By familiar argument $(\delta R/\delta \bar{D})_{IV}$ is negative.

The comparison of interest is between this aggregative control and the control over banks alone (regime II). Consider the n+1 equations formed by the first n equations of (II.2) and the single equation (II.4). Subtract these equations from (IV.2):

$$(IV.3)\quad \begin{bmatrix} D_{10}-L_{10} & D_{12}-L_{12} & \cdots & D_{1n}-L_{1n} & -L_{11} \\ D_{20}-L_{20} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} & -L_{21} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ D_{n0}-L_{n0} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} & -L_{n1} \\ \sum_1^n D_{i0} & \sum_1^n D_{i2} & \cdots & \sum_1^n D_{in} & 0 \end{bmatrix} \begin{bmatrix} (\partial R/\partial \bar{D})_{IV} - (\partial R/\partial \bar{D}_1)_{II} \\ (\partial R_2/\partial \bar{D})_{IV} - (\partial R/\partial \bar{D}_1)_{II} \\ \cdot \\ \cdot \\ \cdot \\ (\partial R_n/\partial \bar{D})_{V} - (\partial R_n/\partial \bar{D}_1)_{II} \\ (\partial r_i/\partial \bar{D})_{V} - (\partial R_i/\partial \bar{D}_1)_{II} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ -(\partial \sum_2^n D_i/\partial \bar{D})_{III} \end{bmatrix}$$

Thus:

$$(IV.4) \quad (\delta R/\delta \bar{D})_{IV} - (\delta R/\delta \bar{D}_1)_{II} = - (\delta \sum_2^n D_i/\delta \bar{D}_1)_{II} \ (\delta R/\delta \bar{D})_{IV}$$

This will be negative if $(\delta \sum_2^n D_i/\delta \bar{D}_1)_{II}$ is negative. If in regime II nonbank liabilities and assets are substituted for those of banks, the effectiveness of control is increased by applying a single limit to all intermediaries.

Appendix

1. Let A be a non-singular square matrix with non-positive off-diagonal elements, positive diagonal elements, column sums $\sum_i a_{ij}$ positive. To prove that det(A) is positive.

Consider the matrix B where $b_{ij} = -a_{ij}/a_{jj}$ for $i \neq j$ and $b_{jj} = 0$. B is then a matrix of non-negative elements with column sums $\sum_i b_{ij} < 1$. det(A) will be positive if det(I-B) is positive, for $\det(I-B) = (1/\prod_j a_{jj})\det(A)$.

Proof that $\det(I-B) > 0$:*

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* For this proof, I am much indebted to Martin Beckmann.

Suppose $\det(I - B) \leqq 0$. Since for sufficiently large λ, $\det(\lambda I - B) > 0$, there must exist a root $\lambda_o \geqq 1$ with $\det(\lambda_o I - B) = 0$. The equation system $[\lambda_o I - B]'x = 0$ has a solution vector $x \neq 0$. Let x_j be the element of largest absolute value $|x_j| > 0$. $|x_j| \leqq |\lambda_o x_j|$. By the jth equation of the system, $|\lambda_o x_j| = |\sum_i b_{ij} x_i|$. $|\sum_i b_{ij} x_i| \leqq \sum_i b_{ij} |x_i| \leqq \sum_i b_{ij} |x_j| < |x_j|$

This contradicts that $\lambda_o \geqq 1$. Hence $\det(I - B) > 0$.

2. Consider a non-singular matrix A_1 formed by substituting for the first column of A a vector of non-positive elements. The proposition is that $\det(A_1)$ is negative. Proof by induction:

If the proposition is true for n x n square matrices, then it is true for n+1 x n+1 square matrices. Add to $A(n)$ a new first row and first column so that the resulting matrix is $A_1(n+1)$. Expand A_1 by the first row. The first cofactor is $\det(A(n))$, which is positive according to note 1 above. The cofactors of the remaining elements of the first row all involve $n \times n$ minors of which the first column consists entirely of non-positive elements, while the remaining columns come from $A(n)$. The minor of the second element is $A_1(n)$ and by assumption negative; thus the second cofactor is positive. The minor of the third element can be made into an $A_1(n)$ by placing the third row at the top of the minor. This interchange alters the sign; hence the minor and cofactor are both positive. In general, the minor of the ith element can be made into an $A_1(n)$ by the i-2 interchanges necessary to place the ith row at the top of the minor. The minors will be positive for i odd and negative for i even; therefore all the cofactors are positive. Since all the elements of the first row are non-positive, $A_1(n+1)$ is negative.

The proposition is true for $n = 2$. $A_1(2) = \begin{vmatrix} - & - \\ - & + \end{vmatrix}$.

If the first and last rows of the matrix of coefficients in (II.2) are interchanged, the resulting matrix is $A_1(n+1)$. Hence the matrix in (II.2) has a positive determinant, with negative cofactors for the last row.