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Remarks on the Rationale of Leadership

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Remarks on the Rationale of Leadership*

What is the rationale of leadership, in an organization where all members have a common goal? If the goal of an industrial enterprise is the maximum profit for its owner it is clear that such a goal will be fulfilled more efficiently if those loyal to the goal (the owner and his family) have authority over those who have different goals (the workers). Yet, a system of leaders who give orders to sub-leaders, etc. - the system of hierarchy, a tree-like organization chart - is applied also in cases where, at least ideally, the loyalty is to the same goal for all: cases are the army, or any government agency, the organization on board ship, etc. It is therefore worthwhile to analyze leadership by first abstracting it from any possible divergence of interests between the leader and his subordinates of various degrees. The question is: given the goal of the team, what are the conditions under which "subordination" is preferred to "coordination", and "orders" are preferable to "exchange of information" through mutual "reports", conferences, etc.

We may recall from previous discussions that the team problem consists in finding simultaneously the best "network and code" and the best "decision rules." Let $p(x)$ be the probability that the external world is in the state x. Let $a_1 \ldots\ldots a_n$ denote the actions of the n members of the team. The payoff to the team, depending on the actions and on the external state, can be denoted by $\varphi(a_1 \ldots\ldots a_n, x)$. Then the goal is to maximize the expected payoff U, where

$$(1) \qquad U = \sum_x \varphi(a_1, \ldots\ldots, a_n, x)p(x) \ .$$

Now the action of each, say the i-th, member will depend on: 1) the information - say, $\eta_i(x)$ - that he received when the state of the world is x; 2) the decision rule that tells him which action a_i he should take

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in response to his information $\eta_i(x)$. Thus the decision rule is a function, α_i, such that

$$(2) \qquad a_i = \alpha_i \left[\eta_i(x) \right] \quad .$$

The n-tuple $\alpha_1, \ldots\ldots, \alpha_n$ can be replaced by a single symbol α expressing the "set of decision rules". Similarly, the n-tuple of functions $(\eta_1, \ldots\ldots \eta_n)$ can be denoted summarily by η. Obviously η expresses the "information structure" within the organization. It expresses "who knows what sort of thing" and it depends on how information is collected and spread through the organization. It depends on the "network" (observation posts, communication lines) and the "code" (language: e.g. the degrees of precision). For each given information structure there will be some cheapest network and code; let its cost be $k(\eta)$. Now replacing in (1) each a_i by its expression in (2) we obtain an expression for the expected profit U that will depend on α and η only (apart, of course, from the <u>givens</u>, i.e., the payoff function φ and the probability distribution p). From U we subtract the organization cost $k(\eta)$ to obtain the <u>net</u> expected payoff which is

$$(3) \qquad U(\alpha, \eta; \varphi, p) - k(\eta).$$

We see again: the problem is to find the best functions α and η, given the functions φ, p and k.

The team member may communicate with another by giving him a "report" about the state of the world, i.e., by telling him that x is within a certain subset X, say. Or he can give an "<u>order</u>". In this case, the sender of the messages, e.g., the President of U.S., has, in the light of his information, made a decision (e.g., to declare war) and informs his subordinate that the latter has now to choose, in the light of <u>his</u> information, the best decision (e.g., the general strategy of war); his subordinates (the Joint Chiefs of Staff, say) in turn, takes the decision and passes on to the next lower echelon (e.g., the chief of a certain theatre

of operations) the task of choosing the best possible among the several sub-decisions that now have become necessary.

Formally, let the state of the world be described by two variables: x_1 ("general" operation) and x_2 ("local" situation), and let the action variables be a_1 (for the leader) and a_2 (for the subordinate). The payoff function is, accordingly, $u(a_1,a_2,x_1,x_2)$. The leader observes that $x_1 = x_1^o$, say; he decides upon $a_1 = a_1^o$, say. This reduces the payoff function to a more special one:

$$u(a_1^o,a_2,x_1^o,x_2) = u^o(a_2,x_2), \text{ say}$$

which may be called the (local) task, or sub-goal. And the message from the leader to the subordinate consists in stating this sub-goal, i.e. - the special, "local" payoff function u^o.

We see, then, that a "report" consists in communicating a statement about x, the state of the world; and order consists in communicating the task, u^o. Under what conditions can there be any advantage in this latter form of communication?

I don't know the answer today. Perhaps it depends on the logical form of certain goals. In some cases, a "serial" coupling is preferable to a "parallel" coupling. The work of Rimoldi on possible sequences of questions in medical enquiries suggests a "serial" coupling, in cases when the answer to one question determines what kind of second question should be asked. Most "20 Questions" games are of this kind. Serial coupling makes hierarchy appropriate. On the other hand, there are cases when simultaneously two or more questions can be asked, the answers

being independent.* In general both methods will be appropriate, to different parts of the task. For example, if a team has to guess a card, one part of the team can take care of determining the suit (spades, hearts, etc.), while another, co-ordinate part of the team can try to determine whether the card is an ace, king, etc. Within the latter sub-team there may be hierarchy; the leader starts by asking whether the card is a picture (king, queen, Jack) or a number (though a better method is to dichotomize into equal parts); the leader may then turn his attention to other tasks, while his sub-ordinate goes into further details.

This formal, logical aspect of a team that may call for hierarchy is probably not the only justification for leadership in teams - i.e. for the existence of "orders" instead of "reports". But I am not ready to try any fuller analysis today.

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* After having given the talk, I have come across a paper by Christie and Luce, prepared for the Control Systems Laboratory, University of Illinois in 1954, in which very similar ideas have been developed, although not in relation to organization form but, rather, in relation to the decision processes of a single individual.