COWLES COMMISSION DISCUSSION PAPER: STATISTICS 382

Comment on Goodman's Discussion Paper (Statistics 381) "A Comment on Consistent Estimation In an Econometric Shock Model"

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Dear Professor Goodman:

With great interest I have read your paper "A comment on consistent estimation in an econometric shock model" which I have received as Cowles Commission Discussion Paper: Statistics 381. Finding that the theorem of your paper is related to some of my own work, I gather that my approach is more general, whereas yours is more specific and therefore capable of giving more precise results. Reference will be made to three of my publications, viz. "A study in the analysis of stationary time series", Uppsala 1938 /ref.1/, "On prediction in stationary time series", AMS 1948 /ref.2/, and "Demand Analysis", Stockholm 1952 and New York 1953 /ref.3./.

In ref.1 I gave a general device for the estimation of the parameters in models of type

(1) $$x_i = u_i + a_1 u_{i-1} + \dots + a_h u_{i-h}$$ /your notation/.

In the case $h = 1$ my estimate coincides with yours. The consistence of my estimate is a consequence of the ergodicity of the process x_i, a consequence which is immediate in view of Birkhoff-Khintchine's theorem /page 35 in ref.1/, and which was elaborated in some detail in ref.2. A more detailed treatment, including a proof of the ergodicity of process x_t, is found in ref.3 /see Chs. 9.4, 10.2 and 11.1/. My proof of the consistence is based upon the general property of ergodicity, while your proof makes use of the specific structure of the process x_t. It would be possible to carry your analysis further, so as to obtain the standard error of the estimate z, a type of result which cannot be derived from the ergodicity /see page 35 in ref.1/.

It may be added that the estimates in question do not have the optimum properties of least-square or maximum-likelihood estimates. In his recent thesis, "Hypothesis testing in time series analysis", Uppsala 1951, Dr. P. Whittle has given a general method for the deduction of least-square estimates of the parameters in model(1). In the particular case h = 1 his estimate of parameter $a = a_1$ is obtained from the relation

$$a - 2a^2r_1 + 2a^3r_2 - 2a^4r_3 + \ldots = (1-a^2)\ (r_1 - 2ar_2 + 3a^2r_3 - 4a^3r_4 + \ldots),$$

where r_1, r_2,... are the autocorrelation coefficients of the process x_t /see also ref.3, Exercise 24 in Part III/.

Yours sincerely,

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