COWLES COMMISSION DISCUSSION PAPER: STATISTICS No. 377

Spurious Correlation: A Causal Interpretation

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March 20, 1953

March 6, 1953

SPURIOUS CORRELATION: A CAUSAL INTERPRETATION

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Even in the first course in statistics, the slogan "Correlation is no proof of causation!" is imprinted firmly in the mind of the aspiring statistician or social scientist. It is possible that he leaves the course (and many subsequent courses) with no very clear ideas as to which is proved by correlation, but he never ceases to be on guard against "spurious" correlation, that master of imposture who is always representing himself as "true" correlation.

The very distinction between "true" and "spurious" correlation appears to imply that while correlation in general may be no proof of causation, "true" correlation does constitute such proof. If this is what is intended by the adjective "true," are there any operational means for distinguishing between true correlations, which do imply causation, and spurious correlations, which do not?

A generation or more ago, the concept of spurious correlation was examined by a number of statisticians, and in particular by G. U. Yule (6). More recently, important contributions to our understanding of the phenomenon have been made by Hans Zeisel (7) and by Patricia L. Kendall and Paul F. Lazarsfeld (1). Essentially, all these treatments deal with the three variable case—the clarification of the relation between two variables by the introduction of a third. Generalizations to n variables are indicated but not examined in detail.

Meanwhile, the main stream of statistical research has been diverted into somewhat different (but closely related) directions by Frisch's work on confluence analysis and the subsequent exploration of the "identification problem" and of "structural relations" at the hands of

Haavelmo, Koopmans, Marschak, and many others.[1] This work has been carried on at a level of great generality. It has now reached a point where it can be used to illuminate the concept of spurious correlation in the three-variable case. The bridge from the identification problem to the problem of spurious correlation is built by constructing a precise and operationally meaningful definition of causality--or, more specifically, of causal ordering, among variables in a model.[2]

1. Statement of the Problem

We begin with a set of observations of a pair of variables, x and y. We compute the coefficient of correlation, r_{xy}, between the variables, and we wish to know, provided that this coefficient is not zero, what we can conclude as to the causal relation between the two variables. If we are suspicious that the observed correlation may derive from "spurious" causes, we introduce a third variable, z, that, we conjecture, may account for this observed correlation. We next compute the partial correlation, $r_{xy.z}$, between x and y with z "held constant," and compare this with the zero order correlation, r_{xy}. If $r_{xy.z}$ is close to zero, while r_{xy} is not,

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1/ See Koopmans (2) for a survey and references to the literature.

2/ Simon (4) and (5). I should like, without elaborating it here, to insert the caveat that the concept of causal ordering employed in this paper does not in any way solve the "problem of Hume" nor contradict his assertion that all we can ever observe are covariations. If we employ an ontological definition of cause--one based on the notion of the "necessary" connection of events--then correlation cannot, of course, prove causation. But neither can anything else prove causation, and hence we can have not basis for distinguishing "true" from "spurious correlation. If we wish to retain the latter distinction (and working scientists have not shown that they are able to get along without it), and if at the same time we wish to remain empiricists, then the term "cause" must be defined in a way that does not entail objectionable ontological consequences. That is the course we shall pursue here.

we conclude that either: (a) z is an intervening variable--the causal effect of x on y (or vice versa) operates through z; or (b) the correlation between x and y results from the joint causal effect of z on both those variables, and hence this correlation is spurious. It will be noted that in case (a), we do not know whether the causal arrow should run from x to y or from y to x (via z in both cases); and in any event, the data do not tell us whether we have case (a) or case (b).

The problem may be clarified by a pair of specific examples adapted from Zeisel.[3]

1. The data consist of measurements of three variables in a number of groups of people: x is the percentage of members of the group that is married, y is the average number of pounds of candy consumed per month per member, z is the average age of members of the group. A high (negative) correlation, r_{xy}, was observed between marital status and amount of candy consumed. But there was also a high (negative) correlation, r_{yz}, between candy consumption and age; and a high (positive) correlation, r_{xz}, between marital status and age. However, when age was held constant, the correlation, $r_{xy.z}$, between marital status and candy consumption was nearly zero. By our previous analysis, either age is an intervening variable between marital status and candy consumption; or the correlation between marital status and candy consumption is spurious, being a joint effect caused by the variation in age. "Common sense"--the nature of which we will want to examine below in detail--tells us that the latter explanation is the correct one.

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[3] Zeisel (7) pp. 192-195. Reference to the original source will show that in this and the following example we have changed the variables from attributes to continuous variables for purposes of exposition.

II. The data consist again of measurements of three variables in a number of groups of people: x is the percentage of female employees who are married, y is the average number of absences per week per employee, z is the average number of hours of housework performed per week per employee.[4] A high correlation, r_{xy}, was observed between marital status and absenteeism. However, when the amount of housework, z, was held constant, the correlation $r_{xy.z}$ was virtually zero. In this case, by applying again some common sense notions about the direction of causation, we reach the conclusion that z is an intervening variable between x and y: that is, that marital status results in a higher average amount of housework performed, and this, in turn, in more absenteeism.

Now what is bothersome about these two examples is that the same statistical evidence, so far as the coefficients of correlation are concerned, has been used to reach entirely different conclusions in the two cases. In the first case we concluded that the correlation between x and y was spurious; in the second case that there was a true correlation, mediated by the intervening variable z. Clearly, it was not the statistical evidence, but the "common sense" assumptions added afterwards, that permitted us to draw these distinct conclusions.

In order to solve our problem, we shall have to state it more precisely. In another paper I have shown that causal relations can be unambiguously defined only if the system of equations we postulate to hold among the variables has as many equations as variables--that is, if the system is "self-contained."[5]

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[4] Zeisel (7), pp. 191-192.

[5] Simon (5), Section 5.

Since we are concerned with the explanation of the relation between two variables by the device of introducing a third variable, we will need to consider two such self-contained systems: a two-equation system in x and y (which we will designate by (II)), and a three equation system in x, y, and z, (which we will designate by (III)). The two systems will be represented thus:

$$\text{(II)} \qquad \begin{aligned} a_{11}x + a_{12}y &= u_1 \\ a_{21}x + a_{22}y &= u_2 \end{aligned}$$

$$\text{(III)} \qquad \begin{aligned} b_{11}x + b_{12}y + b_{13}z &= v_1 \\ b_{21}x + b_{22}y + b_{23}z &= v_2 \\ b_{31}x + b_{32}y + b_{33}z &= v_3 \end{aligned}$$

The a's and b's are constant coefficients, and the u's and v's are "shocks" of a character to be described more fully below.

2. Causal Relations

Now let us see what we might mean by a causal relation between x and y in system (II). Suppose that u_1 and u_2 are variables that can be controlled at will by an experimenter. Then the pair of values (x,y) that will be observed is a function of the pair of values (u_1,u_2) of the experimental variables that is selected. A change in u_1 will, in general, produce a change in both x and y; and likewise for a change in u_2. Suppose, however, that $a_{12}=0$. Then a change in u_2 will produce a change in y, but not in x; while a change in u_1 will produce a change in both. Then we may say that y is causally dependent on x in (II). We may also say that the first equation of (II), in this case, represents the mechanism determining x, while the second equation represents the mechanism determining y—that is, we set up a correspondence between individual equations and individual variables.

Suppose further that although the experimenter can hold the u's constant or permit them to vary, he cannot observe their values—the only observables are the x's and y's. Then, so long as $a_{12}=0$, we can still estimate the parameter a_{21}/a_{22}. For let u_1 vary, but hold u_2 constant. Then:

$$(2.1) \quad a_{21}/a_{22} = -\Delta y/\Delta x,$$

where Δy and Δx are the variations in the observed y's and x's. In particular, $\Delta y = 0$ if and only if $a_{21} = 0$.

Let us return now to our assumption that $a_{12}=0$. If we have a set of n equations in one-to-one correspondence with a set of n variables, if none of the diagonal coefficients vanish, and if the non-diagonal coefficient a_{ij} does <u>not</u> vanish, then we may say that the j^{th} variable <u>influences</u> the i^{th} variable. by setting $a_{12}=0$ in our example, we are saying that y does not influence x. We propose to show that the "common sense" arguments by means of which in the earlier examples we drew causal inferences from observed correlations were based upon the following Hypothesis:

<u>Hypothesis of the Direction of Influence</u>. If x and y are two variables depending on the state of a system at two different points in time, and if the time of x precedes the time of y, then y does not influence x (i.e., $a_{ij}= 0$, where a_{ij} is the coefficient of y in the equation determining x.).

Stated simply, the hypothesis means that later events cannot influence earlier events—a hypothesis implicit in the postulates of most dynamical systems.

Note that if we have a self-contained system in n variables, all referred to different times, and if the system satisfies the hypothesis, then the matrix of coefficients can be arranged in such a way that all the elements above and to the right of the main diagonal are zero. For, arrange the variables in order of their occurence in time, and the equations in corresponding order. Then, by the hypothesis, $a_{ij}=0$ if $i<j$.

3. The Stochastic Case

Next, suppose that u_1 and u_2 are not experimentally controlled, but are random variables with an unknown probability density function $P(u_1,u_2)$. Then, if $a_{12}=0$ in (II), and if, in addition $E(u_1u_2)=0$ where E is the expected value, we can still estimate a_{21}/a_{22}. For multiplying the first and second equations of (II), we get

$$(3.1) \quad a_{11}(a_{21}x^2 + a_{22}xy) = u_1u_2$$

$$(3.2) \quad a_{21}E(x^2) + a_{22}E(xy) = \frac{E(u_1u_2)}{a_{11}} = 0 \text{ , whence}$$

$$(3.3) \quad a_{21}/a_{22} = \frac{E(xy)}{E(x^2)}$$

Since x and y are observables, the quantities on the right-hand side of (3.3) can be estimated from samples of observations. Note that if $E(u_1u_2)\neq 0$, or if $a_{12}\neq 0$, it is not possible to estimate a_{21}/a_{22}. This is, in its simplest form, the familiar problem of identifiability.

We see that, provided $a_{12}=0$ and $E(u_1u_2)=0$, the fact that $r_{xy}\neq 0$ implies, by (3.3) that $a_{21}\neq 0$, and hence that y is causally dependent on x. The assumption $a_{12}=0$ will follow from the Hypothesis of the Direction of Influence if we have independent evidence that x precedes y in time. Validation of the assumption that $E(u_1u_2)=0$ leads us to the problem of spurious correlation.

4. Spurious Correlation

If we suspect that $E(u_1u_2) \neq 0$, we may suppose instead that u_1 and u_2 have a common part dependent on a new variable, z. If we let:

$$(4.1) \quad u_1 = v_1 - b_{13}z$$

$$(4.2) \quad u_2 = v_2 - b_{23}z$$

then we obtain the first two equations of system (III), with $b_{ij} = a_{ij} (i,j = 1,2)$. Now, if we are prepared to assume (tentatively) that z accounts for <u>all</u> the systematic part of u_1 and u_2, then we can complete the system (III) by adding the third equation and the three assumptions:

$$(4.3a) \quad E(v_1v_2)=0; \quad (4.3b) \quad E(v_1v_3)=0; \quad (4.3c) \quad E(v_2v_3)=0.$$

We are interested in particular in the question of whether certain coefficients in the matrix of (III) do or do not vanish. Multiplying pairs of equations of III, taking expected values, and making use of the assumptions (4.3), we obtain three relations between the coefficients and the observables x, y, and z—to wit:

$$(4.4) \quad b_{11}(b_{21}E(x^2)+b_{22}E(xy)+b_{23}E(xz)) + b_{12}(b_{21}E(xy)+b_{22}E(y^2)+b_{23}E(yz)) + b_{13}(b_{21}E(xz)+b_{22}E(yz)+b_{23}E(z^2)) = E(v_1v_2)=0$$

and two similar equations.

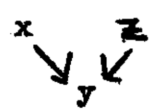
Now we wish to determine whether or not the off-diagonal elements of the matrix are zero (the diagonal elements are assumed to be non-zero).

There are six off-diagonal elements. If all the variables, x, y, and z occur at different points in time, we may use the Hypothesis of the Direction of Influence to show that three of the off-diagonal elements are zero. Equation (4.4) and the two other corresponding equations may now enable us to test which of the remaining three off-diagonal elements are zero. In summary, this is the logic we follow in order to determine, by introducing a new variable, z, whether the correlation between two variables, x and y, is true or spurious.

Before proceeding with the algebra, it may be helpful to look a little more closely at the matrix of coefficients in(III), disregarding the numerical values of the coefficients, but considering only whether they are non-vanishing (X), or vanishing (0). An example of such a matrix would be:

$$\left\| \begin{matrix} X & 0 & 0 \\ X & X & X \\ 0 & 0 & X \end{matrix} \right\|$$

In this case x and z both influence y, but not each other, and y influences neither x nor z. Moreover, a change in v_2—v_1 and v_3 being constant—will change y, but not x or z; a change in v_1 will change x and y, but not z; a change in v_3 will change z and y, but not x. Hence the causal ordering may be depicted thus:



In this case the correlation between x and y is true, and not spurious.

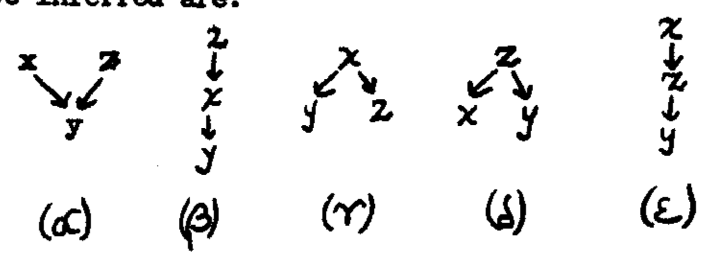
Since, there are six off-diagonal elements in the matrix, there are $2^6 = 64$ possible configurations of X's and 0's. If the variables occur at different times, however, each ordering of the variable requires three 0's in specified cells, and hence for each time ordering there are only $2^3 = 8$ possible distinct configurations. If (to make a definite assumption) x procedes y in time, then there are three possible time orderings (z,x,y; x,z,y; x,y,z), and consequently $3.8=24$ possible configurations. These 24 configurations are not all distinct. For example, the one depicted above is consistent with either the ordering (z,x,y) or the ordering (x,z,y).

Still assuming that x precedes y in time we will be interested, in particular, in the following configurations:

$$\left\|\begin{matrix} X & 0 & 0 \\ X & X & X \\ 0 & 0 & X \end{matrix}\right\| \quad (\alpha) \qquad \left\|\begin{matrix} X & 0 & X \\ X & X & 0 \\ 0 & 0 & X \end{matrix}\right\| \quad (\beta) \qquad \left\|\begin{matrix} X & 0 & 0 \\ X & X & 0 \\ X & 0 & X \end{matrix}\right\| \quad (\gamma)$$

$$\left\|\begin{matrix} X & 0 & X \\ 0 & X & X \\ 0 & 0 & X \end{matrix}\right\| \quad (\delta) \qquad \left\|\begin{matrix} X & 0 & 0 \\ 0 & X & X \\ X & 0 & X \end{matrix}\right\| \quad (\varepsilon)$$

In Case α, either x may precede z or z, x. In Cases β and δ, z precedes x; in Cases γ and ε, x precedes z. The causal orderings that may be inferred are:



The two cases we were confronted with in our earlier examples of Section 1 were δ and ε, respectively. Hence, δ is the case of spurious correlation due to z ; ε the case of true correlation with z as an intervening variable.

5. The Correlation Coefficients

It is time now to supply the details of the procedure. We consider the case where z precedes x, and x precedes y. Then $b_{12}=b_{31}=b_{32}=0$. The system (III) reduces to:

$$(5.1) \quad b_{11}x + \qquad + b_{13}z = v_1$$

$$(5.2) \quad b_{21}x + b_{22}y + b_{23}z = v_2$$

$$(5.3) \quad \qquad\qquad b_{33}z = v_3$$

Assume further that equations (4.3) are satisfied:

$$E\,(v_1 v_2) = E\,(v_1 v_3) = E\,(v_2 v_3) = 0.$$

Multiplying equations (5.1)-(5.3) by pairs and taking expected values, we get:

$$(5.4)\quad b_{11}[b_{21}E(x^2) + b_{22}E(xy) + b_{23}E(xz)] + b_{13}[b_{21}E(xz) + b_{22}E(yz) + b_{23}E(z^2)] = 0$$

$$(5.5)\quad b_{11}E(xz) + b_{13}E(z^2) = 0$$

$$(5.6)\quad b_{21}E(xz) + b_{22}E(yz) + b_{23}E(z^2) = 0$$

Because of (5.6) the terms in the second bracket of (5.4) vanish giving:

$$(5.7)\quad b_{21}E(x^2) + b_{22}E(xy) + b_{23}E(xz) = 0$$

Solving for $E(xz)$, $E(yz)$ and $E(xy)$ we find:

$$(5.8)\quad E(xz) = -(b_{13}/b_{11})\,E\,(z^2)$$

$$(5.9)\quad E(yz) = \frac{b_{13}b_{21} - b_{23}b_{11}}{b_{11}b_{22}}\,E(z^2)$$

$$(5.10)\quad E(xy) = \frac{b_{13}b_{23}}{b_{11}b_{22}}\,E(z^2) - \frac{b_{21}b_{11}}{b_{11}b_{22}}\,E(x^2)$$

<u>Case α</u>: Now in Case α of the previous section, we have $b_{13} = 0$.

Hence

$$(5.11a)\quad E(xz) = 0 \qquad (5.11b)\quad E(yz) = -\frac{b_{23}}{b_{22}}\,E\,(z^2)$$

$$(5.11c)\quad E(xy) = -\frac{b_{21}}{b_{22}}\,E(x^2)$$

<u>Case β</u>: In this case, $b_{23} = 0$. Hence,

$$(5.12a)\quad E(xz) = -(b_{13}/b_{11})\,E(z^2) \qquad (5.12b)\quad E(yz) = \frac{b_{13}b_{21}}{b_{11}b_{12}}\,E(z^2)$$

$$(5.12c)\quad E(xy) = -\frac{b_{21}}{b_{22}}\,E(x^2)$$

from which it also follows that:

$$(5.13)\quad E(xy) = E(x^2)\,\frac{E(yz)}{E(xz)}$$

Case δ : In this case, $b_{21} = 0$. Hence

(5.14a) $E(xz) = -(b_{13}/b_{11})E(z^2)$ (5.14b) $E(yz) = -\frac{b_{23}}{b_{22}} E(z^2)$

(5.14c) $E(xy) = \frac{b_{13}b_{23}}{b_{11}b_{22}} E(z^2)$

and we deduce also that:

(5.15) $E(xy) = \frac{E(xz)\,E(yz)}{E(z^2)}$

We have now proved that $b_{13} = 0$ implies (5.11a); that $b_{23} = 0$ implies (5.13) and that $b_{21} = 0$ implies (5.15). The converse also holds.

To prove that (5.11a) implies $b_{13} = 0$ we need only to set the right-hand side of (5.8) equal to zero.

To prove that (5.13) implies that $b_{23} = 0$ we substitute in (5.13) the values of the cross-products from (5.8)-(5.10). After some simplification, we obtain:

(5.16) $b_{23}(b_{11}^2 E(x^2) - b_{13}^2 E(z^2)) = 0$

Now, since, from (5.11)

(5.17) $b_{11}^2 E(x^2) - E(v_1^2) - 2b_{13}E(zv_1) \neq b_{13}^2 E(z^2)$

and since, by multiplying (5.3) by v_1, we can show that $E(zv_1) = 0$, the second factor of (5.16) can vanish only in case $E(v_1^2) = 0$. Excluding this degenerate case, we conclude that $b_{23} = 0$.

To prove that (5.15) implies that $b_{21} = 0$ we proceed in a similar manner, obtaining:

(5.18) $b_{21}(b_{11}^2 E(x^2) - b_{13}^2 E(z^2) = 0$,

from which we conclude that $b_{21} = 0$.

We can summarize the results as follows:

1) If $E(xz) = 0$, $E(yz) \neq 0$, $E(xy) \neq 0$, we have Case α

2) If none of the cross-products are zero, and

$$E(xy) = E(x^2)\frac{E(yz)}{E(xz)}$$

we have Case β.

3) If none of the cross-products are zero, and

$$E(xy) = \frac{E(xz)E(yz)}{E(z^2)}$$

we have Case δ.

We can combine these conditions to find the conditions that two or more of the coefficients b_{13}, b_{23}, b_{21} vanish:

4) If $b_{13} = b_{23} = 0$, we find that:

$E(xz) = 0$, $E(yz) = 0$. Call this Case $\alpha\beta$.

5) If $b_{13} = b_{21} = 0$, we find that:

$E(xz) = 0$, $E(xy) = 0$. Call this Case $\alpha\delta$.

6) If $b_{23} = b_{21} = 0$, we find that:

$E(yz) = 0$, $E(xy) = 0$. Call this Case $\beta\delta$.

7) If $b_{13} = b_{23} = b_{21} = 0$, then

$E(xz) = E(yz) = E(xy) = 0$. Call this Case $\alpha\beta\delta$.

8) If none of the conditions (1)-(7) are satisfied, then all three coefficents b_{13}, b_{23}, b_{21} are non-zero. Thus, by observing which of the conditions, (1) through (8) are satisfied by the expected values of the cross products, we can determine what the causal ordering is of the variables.[6/]

6/ Of course, the expected values are not, strictly speaking, observables except in a probability sense. However, we do not wish to go into sampling questions here, and simply assume that we have good estimates of the expected values.

We may add a remark on partial correlations. The numerator of the partial correlation coefficient, $N(r_{xy.z})$, is:

$$(5.19)\quad N(r_{xy.z}) = \frac{E(xy)}{\sqrt{E(x^2)\,E(y^2)}} - \frac{E(xz)\,E(yz)}{E(z^2)\sqrt{E(x^2)\,E(y^2)}} .$$

We see that the condition for Case δ is precisely that $r_{xy.z}$ vanish, while none of the cross products, r_{xy}, r_{xz}, r_{yz} vanish. From this, we see that the first illustrative example of Section 1 falls in Case δ, as asserted in Section 4.

We can now summarize the procedure for interpreting, by the introduction of an additionalvariable z, the correlation between two variables, x and y.

1. We postulate that the system is describable by the equations (III) with $E(v_1v_2) = E(v_1v_3) = E(v_2v_3) = 0$.

2. We postulate an ordering of the three variables in time, and determine from this that three of the off-diagonal elements in the matrix of III are zero.

3. We compute from the observables the expected values E(xy), E(x z), and E(y z), and deduce from these which of the remaining off-diagonal coefficients (if any) are zero. The conditions when z precedes x and x precedes y have been spelled out explicitly, and analogous conditions hold for the other time orderings of the variables.

4. The arrangement of zero and non-zero elements in the matrix determines the causal ordering of the variables, and enables us to distinguish true from spurious correlations.

Finally, in the course of our development we have shown in what sense the vanishing of the partial correlation coefficient provides evidence that a correlation is spurious.

6. The Case of Experimentation.

In sectons (3) - (5) we have treated v_1, v_2 and v_3 as random variables. The causal ordering among x, y and z can also be determined in the case where v_1, v_2, and v_3 are controlled by an experimenter, but where the time precedence of x, y and z is not known (but where they are assumed not to be simultaneous, so that $b_{ij} \neq 0$ implies $b_{ji} = 0$; and $b_{ij} \neq 0$, $b_{jk} \neq 0$ implies $b_{ki} = 0$).

Under the given assumptions at least three of the off-diagonal b's in (iii) must vanish, and the equations and variables can be reordered so that all the non-vanishing coefficients lie on or below the diagonal. If (with this ordering) v_2 or v_3 are varied at least the variable determined by the first equation will remain constant (since it depends only on v_1). Similarly, if v_3 is varied, the variables determined by the first and second equations will remain constant.

In this way we discover which variables are determined by which equations. Further, if varying v_i causes a particular variable other than the i^{th} to change in value, this variable must be causally dependent on the i^{th}.

Suppose, for example, that variation in v_1 brings about a change in x and y, variation in v_2 a change in y and variation in v_3 a change in x, y and z. Then we know that y is causally dependent upon x and z, and x upon z. But this is precisely the Case β treated previously under the assumption that the v's were stochastic variables.

7. Conclusion

In this paper I have tried to clarify the logical processes and assumptions that are involved in the usual procedures for testing whether a correlation between two variables is true or spurious. These procedures begin by imbedding the relation between the two variables in a larger three-variable system that is assumed to be self-contained, except for stochastic disturbances or parameters controlled by an experimenter.

Since the coefficients in the three-variable system will not in general be identifiable, and since the determination of the causal ordering involves considerations of identifiability, the test for spuriousness of the correlation requires additional assumptions to be made. These assumptions are usually of two kinds. The first, ordinarily made explicit, are assumptions as to the time order of the variables. These assumptions, combined with what we have called the "hypothesis of the direction of influence," reduce the number of degrees of freedom of the system of coefficients by implying that three specified coefficients are zero.

The second type of assumption, more often implicit than explicit, is that the random disturbances associated with the three-variable system are uncorrelated. This assumption gives us a sufficient number of additional restrictions to secure the identifiability of the remaining coefficients, and hence to determine the causal ordering of the variables.

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