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The Christ and Canadian Econometric Models—Some Comparisons in Approach[1/]

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I. Introduction

1. This paper will deal with a comparison of how the methods used in two econometric macro models may have affected the differences in results. The conclusion one would draw from Christ's paper and the discussion of it is that macro-econometric models are not particularly useful for forecasting purposes with the present level of economic theory, data, and theoretical and practical problems of estimation.[2/] However, on the basis of the five years work done in this field by those working in Ottawa I would say that the results have been fairly good and have been improving as greater experience and a larger number of observations are obtained.[3/] This paper

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1. This paper was prepared while the writer was on a Canadian Social Science Research Fellowship. It draws heavily on the experience gained while working for T. M. Brown in the early stages of the econometric work initiated by L. R. Klein in the Department of Trade and Commerce, Ottawa. Many of the points discussed here have been clarified by discussions with Klein and Brown and in helpful discussions and correspondence with Carl Christ.

2. Carl Christ, "A Test of an Econometric Model for the United States, 1921-1947," in Conference Business Cycles, (New York: N.B.E.R., 1951), pp. 35-114, reprinted as Cowles Commission Papers, New Series, No. 49.

3. T. M. Brown, "Econometric Research and Forecasting," Paper presented at the 1951 meeting in Boston and "A Version of the Consumption Function," Abstract in Econometrica, 20 (January 1952), pp. 83-84. The complete latter paper will appear in the July issue of Econometrica.

will suggest some explanations for the differences in results, and make some proposals for further empirical work in macro model building.

2. In order to limit the possible problems that might be discussed to a manageable size, some important topics will not be introduced here. The a priori economic specification and identification of the two models will not be dealt with, and the problem of serial correlation in time series data will be avoided. Furthermore, detailed results of the two studies will not be analyzed, partly because the writer is not familiar with the details of the more recent Canadian work and no full published study of the project is available.

3. To an important extent, the differences in emphasis and methods in the two projects arise from slightly different objectives, and no fair appraisal of this sort can be made without some understanding of the purposes of the studies. The objectives in the Canadian work have been twofold: (1) to improve (and in certain cases to prepare) basic time series data for the economy, both for purposes of general descriptive and factual studies in the government and for the econometric work proper, and (2) to develop statistical estimates of a system of equations for use in short-run forecasting work and (to a lesser extent) for policy purposes. Christ states a much more limited objective in one section of his comments, saying "I regard as the most valuable part of my paper the exposition and illustration of procedures for prediction and testing, rather than the particular model tested or the particular results arrived at, and that is the basis upon which I would like my work to be judged."[4/]

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4. Christ, op. cit., p. 123. See, however, statements that econometric models should be useful in prediction and business cycle research on pages 37 and 43, which suggest that his ex ante objectives were more ambitious than this. His more modest statement quoted above would not appear to go beyond Marshall's study.

4. Three points will be dealt with in the following discussion: (1) the general quality of the data used in statistical estimation in the two projects; (2) the problem of how to estimate the parameters in a system of equations in which significant non-linearities are present and how this problem is met in each study; and (3) the problem of how to use the model for forecasting and analysis of the effects of routine policy changes when the model is heavily over-identified. After discussing these topics in turn, some suggestions will be made for further research in this field.

II. Quality of the Basic Data

1. The general problem of time series data for use in the two models can best be illustrated by the preparation of constant dollar estimates of gross national expenditure. In Canada, the whole project was assisted by a general interest in a historical series on the volume of output, both in the government and from the general public. A value series on gross national product had been prepared by the Dominion Bureau of Statistics, and it was then necessary to separate the price and quantity movements out of this series. Initially, the "deflating" project was started by those engaged on the econometric work, and it would be my opinion that one-third or more of the time of two research workers and two clerks has been on this project over the last five years in this section alone. The whole period since 1926 has been completely reworked four times, and a series of interdepartmental panels and committee meetings has discussed the problem of index numbers and basic data before the material was published.5/ Furthermore, independent estimates of employment and hours worked were made to check the reasonableness of the data for the whole

5. The material was published early in 1952 in National Accounts, Income and Expenditure, 1926-1950, (Ottawa: D.B.S., 1952), pp. 28-29.

period, and annual estimates of the volume of output built up on an industrial basis for the last fifteen years have been prepared to check the deflated estimates.

Such an allocation of resources to preparing basic data was not available to Christ, as a single research person in the field. Since Klein had first prepared the basic data for the pre-war period, the Department of Commerce had published a whole new value series for gross national product with important changes in concept and significant statistical revisions for the period 1929 to date. Christ "linked" the series before and after 1929 using a regression between the two Department of Commerce Series in an attempt to get comparable data for the whole time period.[6] Price index data was then used to obtain a physical measure of output.

2. Christ's method of handling the data problem was very unfortunate. In discussing the paper Klein states, "The most serious deficiency in Christ's work is in the data he used for 1946-47 to revise my model and bring it up-to-date."[7] He points out that the series of real aggregate output drops more than 10 per cent from 1946 to 1947, which is quite different than the movement exhibited by other related series. This drop compares with a fractional increase in the most recent official estimates.[8] No satisfactory explanation of this has been made.

A further criticism can be made of the method of linking the new Department of Commerce series to the earlier data. The conceptual and statistical

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6. For the extent of the differences see National Income Supplement to Survey of Current Business, July 1947 (Washington: U. S. Government Printing Office, 1947), passim.

7. Christ, op. cit., p. 115.

8. National Income, 1951 Edition, (Washington: U. S. Government Printing Office, 1951), p. 145.

revisions introduced in 1947 were significant, affecting particularly military income, corporate profits and consumer expenditure and thus all of the main aggregates. A series of published charts show that the levels of all the aggregates are increased, and that the relative increase is greater for the more recent years.9/ A correlation between the old and new Commerce series will introduce a bias in the data even though the correlation is high. An an illustration of the bias introduced (perhaps only partially due to this factor), Christ shows an increase in real output from 1939 to 1948 of 38 per cent, while the latest Commerce series shows an increase of 57 per cent for the same period.10/ A difference of this extent is extremely serious.

Klein makes some observations on how an individual research worker might have tackled the data problem that Christ was faced with. One further suggestion to anyone else trying a similar study is to spend some time with the Department of Commerce and those working at the National Bureau on data problems. It is just too much to expect one person to be able to cope with all the conceptual and statistical problems in this field, and full discussion of the basic data with experienced people would be invaluable.

3. Some comments seem appropriate here as to just why the basic data should get a large amount of attention. For one thing, one of the specifications of the model is that "effects of errors in measuring variables are here assumed to be small relative to the disturbances (this kind of model is called a shock model, as distinct from an error model)."11/ The methods used in estimation give "good" results only if this assumption is met

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9. National Income Supplement to Survey of Current Business, July 1947, pp. 13, 15, 16.

10. L. R. Klein, Economic Fluctuations in the United States, 1921-1941, (New York: John Wiley & Sons, 1950), p. 135, Christ, op. cit., p. 90 and National Income, 1951 Edition, p. 146.

11. Christ, op. cit., p. 36.

(approximately) in the model being estimated. It is my judgment that the basic data is not of a high enough calibre to fulfill this assumption.

A further observation is required when the most serious errors in the data occur in the post-war period when the observations lie a substantial distance from the mean of all the observations. In estimating systems of equations of this size from time series data, the number of degrees of freedom are small and each observation can have an important effect on the estimates of the parameters. This is particularly important for extreme values of the variables, and this is precisely where the data is the least satisfactory. In Appendix C, Christ shows that the results are significantly affected by revisions in the 1946-1947 data. Similar significant changes in results from statistical revisions for one or two years have been obtained in the Canadian work.

When the time required to prepare the basic data is compared with the time and resources allocated to the theoretical problems of specification, identification, and estimation and the expenditure on computation, it would seem that Christ did not allocate his time in an optimum manner. This is unfortunate, but it is to be hoped that those working in the field profit by this experience.

III. Methods of Estimation

1. Most of the Canadian work has relied on least-squares estimates of the structural equations. One four equation linear model has been estimated using limited-information, maximum-likelihood methods. However, most of the time has been engaged in working with eleven and thirteen equation models generally similar to Klein's Model III or Christ's revision of it.

Although Christ used least-squares estimates of the structural equations for screening purposes, he finally made limited-information maximum-likelihood estimates of all the parameters.

2. The crucial question in this type of model is what type of estimation procedure should be used when the model consists of a non-linear system of equations. These non-linearities come about from the occurrence of certain endogenous variables as separate variables in some equations and combined in certain specified ways in other equations. Christ's model contains seven such non-linear functions, such as w/p, and $(p\,x - \mathcal{E})/q$. Klein's model also contains such variables,[12] as do all the larger Canadian models. A simultaneous solution of all the equations in Klein's model gives a cubic equation in p (the price level), Christ gets a quintic in p, and most of the Canadian models give a cubic in Y (private disposable income). However, the limited-information maximum-likelihood method of estimation has been developed for a system of linear equations. In the face of uncertainty about the properties of alternative methods of estimation for such non-linear systems for small samples, different procedures have been adopted in the two research projects. Some suggestions as to the underlying rationale in the two cases will now be made.

3. Christ followed Klein in estimating the parameters by treating the system of equations as if they were linear and used the limited-information methods developed initially for the linear case. Chernoff and Rubin argue that limited-information estimates retain the property of consistency asymptotically when used in non-linear models.[13] However, there may be some loss in efficiency of the estimates from disregarding these non-linearities in going from the structural to the reduced form equations and back again. The sampling variability of many parameters may be substantially increased from this procedure.

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12. L. R. Klein, op. cit., pp. 105-106.

13. "Asymptotic Properties of Limited Information Estimates under Generalized Conditions," Cowles Commission Discussion Paper Statistics No. 342. A revised and extended version of this paper will appear as Chapter VII in Monograph 14, to be published.

Almost from the first those engaged in the Canadian econometric work felt that these non-linearities were an important part of the a priori specifications of the model. Any method of estimation which disregarded these non-linearities was regarded as unsatisfactory, particularly when the costs of computation were greater. Least-squares estimates of the structural equations permitted the retention of the non-linearities during estimation, even though it was known that the methods used were biased asymptotically and did not fulfill the conditions for the Markoff Theorem on least squares. Secondary considerations were that least-squares estimates would be helpful in "screening" the number of alternative specifications possible, and also that the lower costs of computation and the easier comprehension of the simpler methods would be useful in the early training stages.

4. It is not clear at present just how serious the loss in efficiency may be from using Christ's procedure of using a method of estimation for linear systems of equations in a non-linear model. The following should be regarded as intuitive "hunches" on this problem. Christ obtained several unexpected results which he was not able to explain. The limited information estimates of the investment function (1.0) and the equation determining the private wage bill (4.1) differ significantly from the least-squares estimates of the same structural equations or Klein's estimates for the same equations.[14] It may be significant that both of these equations are estimated from the same sub-set of exogenous and predetermined variables and that one equation contains the endogenous variable $(p\,x - \mathcal{E})/q$ and the other contains $p\,x - \mathcal{E}$.[15] It is possible that the neglect of the non-linearities present may explain this difference in the result.

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14. Christ, op. cit., p. 85-86. See also the large standard errors for the parameters in these equations in Table 2, pp. 72-73.

15. Loc. cit., pp. 51, 66, and 103.

One further problem that Christ encountered was that the simultaneous solution of all equations (after inserting all exogenous and lagged endogenous variables for a forecast for 1948) gave a ridiculous value for the price level. Neither Klein nor Christ have been able to explain this result. Consider the following possible enterpretation for this. In estimation, all the non-linearities have been disregarded and regression equations of linear approximations to the correct reduced forms have been made. Estimates of the parameters of the structural equations are then obtained and then a forecast has been made using these estimates. It is possible that forecasts in the neighbourhood of the mean of the observations would give reasonable results, but in extrapolating beyond the range of previous observations the problems associated with disregarding the non-linearities are so great that the model breaks down and gives ridiculous results. It is interesting that in this same situation, the Canadian models have never given a wierd result in a score of forecasts done with a variety of non-linear models used.

There is another possible explanation for better results using least-squares methods than for those obtained from using some type of maximum-likelihood estimate. This is that the least-squares method does not require information on the rest of the system, and if the specification on the rest of the system is incorrect, the results may be improved. Hurwicz has suggested that the gains from dropping the incorrect assumptions may exceed the losses associated with the bias present in least-squares estimates.16/ The suggestions above on the problem of estimating a non-linear system using linear methods may be merely an illustration of Hurwicz's suggestion that specification bias may be an explanation of the results.

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16. Loc. cit., p. 129 for Tinbergen's comments, footnote 21, p. 48 for Hurwicz's suggestion, and pp. 80-81 for Christ's comments on the differences between the limited-information and least-squares estimates.

5. Neither Christ's approach of estimating the system as if it was linear, or the Canadian approach of estimating the structural equations by least-squares is completely satisfactory. Neither method has both consistent and efficient properties even in large sample cases, and little is known about the properties of these estimates in small samples. In order to get estimates approaching the optimal properties more closely, several alternatives are open. One way is to drop the equations containing the non-linear combinations of endogenous variables and obtain a smaller, but linear model. This is the procedure Brown used in his four equation just-identified model. However, the endogenous variables dropped from the bigger models included employment, the price level, imports and investment, all of which are important and interesting variables to have in the model.

A more satisfactory method would be to have a method of estimation which preserved the non-linearities in the system, but which had more of the desirable statistical properties. One solution applicable in certain cases has been proposed by Klein in informal discussions. A necessary condition for its use is that the β matrix of the endogenous variables in the system of equations be triangular — that is for one equation the structural and reduced form of the equation be identical; that another equation contain this endogenous variable and one other one, etc. For a model of this sort, the calculated values of the first variable are inserted in place of the observed value in the second equation and parameters of the second equation are then obtained, and this procedure is continued until estimates have been made of all parameters. It is my understanding that this method has the properties of full information maximum-likelihood estimates and is applicable whether the system of equations is non-linear

or not. The only application of this method that has come to my attention is by Kisselgoff in a four equation model.[17]

IV. Forecasting Procedure

1. Christ uses the reduced forms of the equations in all the naive model tests or predictions published in his paper. All the exogenous and lagged endogenous variables are inserted in each reduced forms equation and an estimate for each endogenous variable was obtained for the year 1948. It is on the results of this step in the procedure that Christ and Friedman draw their conclusions on the project as a whole. However, the quality of the results is dependent on all of the assumptions and steps made up to that point.

All of the Canadian forecasts have been true forecasts of the endogenous variables in the model for the calendar year. Estimates of the lagged endogenous variables are always available (preliminary and subject to revision in most instances, of course) but estimates of the exogenous variables in the model must be made by non-econometric methods. Forecasts of government expenditure and exports, for example, are made by direct surveys and a considerable amount of assessing of information from diverse sources and careful judgment is required. The values of these exogenous and lagged endogenous variables are inserted in the structural equations and the whole system is solved by a process of substitution until one equation in one variable is obtained — a cubic equation in disposable income. After finding the appropriate root of this equation, the values of all the other endogenous variables are obtained and the whole procedure checked for mechanical errors. The results of this solution cannot usually be finally

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17. Avram Kisselgoff, Factors Affecting the Demand for Consumer Installment Sales Credit, Technical Paper 7, (New York: National Bureau of Economic Research, January 1952), pp. 26-28. Houthakker has brought an article of Bentzel and Wold to my attention which deals with this problem. See Skandinavisk Aktuarietidskrift, 1946, pp. 95-114.

checked for perhaps a year when the final ex post estimates are available, and are conditional, of course, on the correctness of the forecasts of the exogenous variables.

2. Christ appears to have decided on this procedure initially because his prediction problem corresponded to a situation of unchanged structure when it was not regarded as necessary to estimate the structural equations. Hurwicz first made it clear that for linear models prediction from the reduced forms equations was adequate for conditions of unchanged structure, but to analyze the effect of changes in structure it was necessary to use the structural equations.[18] However, it does not appear to be widely appreciated that the reduced forms equations give the identical results as would be obtained from using the structural equations if and only if the model was linear and just identified.[19] In other cases, the use of the structural equations permits the influence of all the predetermined variables to be felt on all the endogenous variables, which would have no influence or influence only a subset of the endogenous variables if the forecast was made with the reduced forms equations from the limited information method. (This loss in efficiency when using limited information reduced forms equations in forecasting is related to the loss in efficiency in estimation from neglecting some information.)

This loss in efficiency is particularly severe in Christ's model for two reasons. One is the non-linearities which are neglected in going to the reduced forms equations as previously discussed. The second reason is that this model is heavily overidentified (even without using the non-linearities to assist in identification), containing fourteen exogenous

18. L. Hurwicz, "Prediction and Least Squares," in Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph 10, (New York: John Wiley and Sons, 1950).

19. This point was clarified in discussions with Christ in the Spring of 1950. See Christ, op. cit., p. 44.

variables and seventeen lagged endogenous variables. All of the information on the non-linearities and most of the information on the exogenous variables is disregarded in Christ's application of the limited information maximum-likelihood method. However, it seems that this information is too valuable to be disregarded in both estimation and prediction.

In summary, the procedure of using the structural equations for prediction (even with an unchanged structure) would always seem to be as good as the use of the reduced forms equations. Only for linear systems which are just-identified (Hurwicz's special case), would identical results be achieved. In this case the use of the reduced forms may be preferred as it avoids the cost of estimating the structural parameters from the reduced forms, but this cost is quite small relative to all the other work necessary to obtain the reduced forms estimates themselves.

However, certain cases may arise where some of the endogenous variables only are to be forecast, or where certain equations are to be tested by extrapolation to a period outside of the range of observations. In these cases, the costs avoided may more than offset the loss in efficiency. The purpose of the study should always be kept explicitly in mind when deciding between alternative approaches. I think that a difference in objective existed between Christ and Klein on this point that Klein does not appear to have recognised.

3. One further observation appropriate at this point relates to the particular exogenous variables used by Christ in the estimation of the reduced forms. In discussing Christ's paper Friedman states, "Christ suggests one qualification to the conclusion that, on the basis of existing evidence, the particular econometric model he tested is worthless. He writes, 'Even if such a naive model does predict about as well as our econometric model, our model may still be preferable because it may be able to predict

consequences of alternative policy measures and of other exogenous changes, while the naive model cannot.' But this argument is at best misleading, at worst invalid. The naive model can make such predictions too: one can simply assert that a proposed change in policy or in an exogenous variable will have no effect. If this kind of prediction worked as well as the econometric predictions for a change from one year to the next, might it not work as well for policy changes also?"[20] It will now be argued that for a large group of the simplest possible policy problems, Christ's reduced forms equations give exactly the result that a change in many of the exogenous variables will have no effect. For nonstructural (or "routine") policy changes of the exogenous variables, it should be possible to predict the effects from the reduced forms of the model.[21] However, a comparison of the exogenous variables in Christ's model with the variables used in estimating the reduced forms shows that only two (t and $\mathcal{E}_R$) are included in the latter group. A large group of important policy variables (government expenditures of all types and government revenues of all types) are not included in any of the reduced forms equations for the endogenous variables (including such demand equations as consumption, investment, inventories and labor). Thus changes in these variables can have no effect — the identical result of the naive model!

This particular result is not inherent in the method of reduced forms, but is rather a result of the particular group of predetermined variables used by Christ in estimation. At present, the basis of selection of predetermined variables to use in estimation is arbitrary but perhaps some more satisfactory basis of selection can be developed. Furthermore, by using the structural equations the effects of changes in such non-structural policy variables can be determined with Christ's model.

20. Christ, op. cit., p. 111.

21. J. Marschak, "Statistical Inference in Economics: An Introduction," in Cowles Commission Monograph No. 10, Statistical Inference in Dynamic Economic Models, (New York: John Wiley and Sons, 1950), pp. 11-12.

V. Suggestions for Future Work

Suggestions on future work in econometric model building were made by Christ, Friedman and Klein at the Conference on Business Cycles and further suggestions have been made by Orcutt[22] and many others. The following suggestions relate only to those arising out of this paper.

1. One large question raised is how serious the loss of efficiency is if limited information methods are applied to systems in which non-linear combinations of endogenous variables play an important part. In small sample estimates of such models, is it possible that the loss in efficiency is more serious than the gain through the use of estimation procedures which are unbiased asymptotically?

2. Can methods of estimation be developed for models of the type described here which yield both consistent and efficient methods without serious increases in cost?

3. It is probable that the scope for practical applications of structural equations has been underestimated in recent years. Only for just-identified systems of linear equations are the forecasts from the reduced forms equations as efficient as those obtained from using the structural equations.

4. In all problems of estimation, the quality of the basic data is quite important. In shock models no amount of later processing of data can correct for unsatisfactory initial data, and this is just as true for the more advanced methods of estimation as it is if least-squares estimates of the structural equations are made.

5. Judging by the differences in results between Christ's model and the Canadian work, it is quite possible that Friedman's conclusions on the

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22. Guy H. Orcutt, "Towards Partial Redirection of Econometrics" to be published in the Review of Economics and Statistics.

possibilities of further empirical work with macro models are too pessimistic. Although the writer does not underestimate the difficulties in estimating macro models which can be helpful to policy makers, it is to be hoped that Christ's results have not discouraged further empirical work in this field at the Cowles Commission and elsewhere. Although much of the recent literature on methodology has consolidated much ground and clarified many points, it should not be forgotten that the objective of this work has been to design tools to be used in practical problems of estimation. Furthermore, empirical work can frequently suggest lines of theoretical research which may be useful. In my judgment a reworking of the annual American data and the estimation of a complete macro model could now be done with the possibility of getting considerably better results than were obtained even several years ago.