COWLES COMMISSION DISCUSSION PAPER: STATISTICS NO. 365

A Suggested Application of the Limited Information Maximum Likelihood Method of Estimation

T. W. Anderson

[The following is a quotation from a letter dated September 20, 1951, received by Koopmans]

While reading Cowles Commission Discussion Paper, Statistics 363, I thought again about the procedure of using the single equation limited information maximum likelihood method successively on each equation of a system. Usually one treats each equation independently using only the restrictions on that equation. I wonder if one can not improve on this procedure by making some use of the computations on a given equation in the estimation of later equations. For example, when one estimates the parameters in the first equation, he obtains at the same time an estimate of a particular part of the Π matrix of a certain rank. In treating the second equation, one could use the estimates of Π_{ij} resulting from the computations on the first equation instead of the p_{ij}. If one goes on to other equations, one could successively obtain adjusted estimates of elements of the Π matrix.

Whether or not a general procedure can be developed, there are examples in which one can trivially modify this limited information maximum likelihood method to make some use of restrictions on other equations. For example, let the system be

$$\begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} y_t + \begin{bmatrix} 0 & 0 & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & 0 \end{bmatrix} z_t = u_t .$$

Then the reduced form is

$$y_t = - \begin{bmatrix} -\beta\gamma_{21} & -\beta\gamma_{22} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & 0 \end{bmatrix} z_t + v_t .$$

One can use the limited information maximum likelihood method to estimate β appearing in the first equation. Simultaneously, one obtains an estimate of

$$\begin{bmatrix} -\beta\gamma_{21} & -\beta\gamma_{22} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

which is of rank one. This leads to estimates of γ_{21} and γ_{22}. These estimates may be better than the limited information maximum likelihood method estimates of the coefficients of the second equation which are the regressions of y_{2t} on z_{1t} and z_{2t}.