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A Class of Criteria for Decision-Making under Ignorance

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1. Let

(1.1) $w = \psi(d, e)$

where

(1.2) $w \in W$

$d \in D$

$e \in E$

and

(1.3) w = scalar.

It is desired to maximize w with d, but not e, subject to the decision-maker's control.

[The use of probability "mixtures" (= "mixed strategies") over D, other than those already contained in D, is not permitted. I.e., an element of D may happen to be a mixture of some other elements of D, but it is not permitted to add new elements to D by "mixing" those already in it. The same is true for E.]

2. <u>Definition:</u> an element $\hat{d}^{\varrho}_{DE\psi}$, or more briefly $\hat{d}^{\varrho}$, in D is said to be ϱ-optimal in D for E and ψ if[1]

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1. This approach is closely related to suggestions made by Hildreth in several staff meetings.

(2.1) $\varsigma(W_{\hat{d}_\varsigma}) \geq \varsigma(W_d)$ for all $d \in D$

where $W_{d_o} \subseteq W$ is the "image" carried by ψ from E to W for a given d_o.[2] (ς is a single-valued set function.) $v(d) = \varsigma(W_d)$ is said to be the "value" of d.

3. Denote by M_d and m_d respectively the supremum and infimum of W_d. Now assume that

(3.1) W and all W_d are bounded intervals, and

(3.2) $\varsigma(W_d)$ does not depend on whether both, one, or neither of the relations $M_d \in W_d$, $m_d \in W_d$ hold.

It follows that

(3.3) $v(d) = \sigma(M_d, m_d)$

with σ single-valued.

Let $\mathcal{S}$ denote the class of all σ and denote by $\mathcal{S}^L$ the subset of $\mathcal{S}$ defined by

(3.4) $\sigma(M_d, m_d) = \alpha M_d + (1 - \alpha)m_d, \qquad 0 \leq \alpha \leq 1.$

An element of $\mathcal{S}^L$ corresponding to a given α will be denoted by $s(\alpha)$.

4. It is easily seen that the maxmin principle is obtained as a special case of $s(\alpha)$-optimality, viz. for s(0). Similarly the maxmax principle[3] is that obtained by using s(1).

The two principles are respectively open to the charges of excessive (systematic) conservatism and recklessness.

Such charges would not, however, seem to be justified with regard to the principle of $s(\frac{1}{2})$-optimality, i.e., with regard to the principle of maximizing the average

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2. I.e., W_{d_o} is the set of values (in W) of $w = \psi(d_o, e)$ obtained by letting e assume all possible values in E.

3. Suggested by F. Modigliani, Papers and Proceedings, American Economic Review, Vol. 39, No. 3, May, 1949, p. 205 ("maximaxing profits").

(4.1) $$\mu_d = \frac{1}{2} M_d + \frac{1}{2} m_d.$$

5. The result (3.3) is based on the definition (2.1) and on the assumptions (3.1), (3.2).

The restriction implied in the definition (2.1) is very strong, perhaps unjustifiably so, as can be seen from the fact that it does not imply the nonoptimality of an "inadmissible" d [i.e., of a d' such that there exists d'' with $\psi(d', e) \leq \psi(d'', e)$ for all $e \in E$ and $\psi(d', e_o) < \psi(d'', e_o)$ for some e_o].

Assumption (3.1), on the other hand, is introduced only in order to obtain a principle of sufficient generality to cover the "continuous" case. For example, it eliminates from competition principles that are valid for discrete cases only.

There seems little likelihood of controversy with regard to (3.2).

6. Chernoff has pointed out to the writer that examples could be made up to make the choice of $s(\frac{1}{2})$ appear unreasonable. Such examples would be so designed that while $\mu_{d'}$ is only slightly higher than $\mu_{d''}$ we would have $\psi(d'', e) - \psi(d', e) > 0$ and large for "most" $e \in E$.

Thus, presumably, the choice of $s(\frac{1}{2})$, while not systematically pessimistic or optimistic, is unduly "extremist" in that it attaches too much importance to the extremal values of $\psi(d, e)$.

It seems to the writer that this type of objection is based on the implicit assumption that there is a "natural measure" in E. If it is desired to abstract from considerations that are not invariant under transformations in E, examples of the type described are not valid as counterarguments.

7. Let it be made clear that $s(\frac{1}{2})$ is not being proposed as an "ideal" principle for decision-making under ignorance. It definitely has undesirable properties, although it may in some cases be superior to the maxmin [i.e., $s(0)$] criterion.