COWLES COMMISSION DISCUSSION PAPER: STATISTICS NO. 351

Computation at the Cowles Commission

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The subject described by the title may be discussed under three headings: what we have done, what we are doing and what we may do in the future. Of these the first is at present of interest as providing some idea of what can be expected to be done; the second is of some interest, but as it happens not of major importance; and the third is largely unknown. Nevertheless some purpose may be served by drawing the attention of members of the Commission to this side of the Commission's activities.

1. Almost all the computational work done in the past few years seems to have been on estimation of coefficients of simultaneous-equations economic models. The estimation procedure may be least-squares (Borts' railway model), limited information (models such as Allen's for the linseed-oil market, Klein's and Christ's twelve-equation models of the U. S. economy and others) or maximum likelihood (with Klein's three-equation model the only extensive example so far worked out). Computational techniques for the last two methods have been discussed with examples in an article by Chernoff and Divinsky to appear in Cowles Commission Monograph 14. This article discusses computations for the Klein three-equation model in some detail, but, perhaps, only an inspection of the several large volumes of work sheets in the files gives an adequate idea of

the very great labor involved in maximum-likelihood estimation. Limited-information estimation is much cheaper and better understood, and for the case of linear restrictions on the coefficients has been reduced to a routine.

2. Computation jobs presently in progress include limited-information estimation, integration of a pair of simultaneous differential equations and estimations by various methods in a certain artificial linear-equation model. The first of these (No. 349) is a two-equation model for the Burley Tobacco market in the U. S. for the years 1935-1949. Estimation of the coefficients has been completed for the case in which no restrictions are placed on the coefficients. Some attention is being given to possible methods of computation to allow for two linear restrictions on the coefficients. Methods given in [Rubin] may be applicable.

The second job (No. 348) is connected with the problem of maximizing a function, subject to linear constraints. This can be shown to be equivalent to a problem in game theory which in turn is related to the asymptotic solution of certain differential equations. Work has been done in this field at The RAND Corporation by Brown and von Neumann, and also by Arrow and Hurwicz. Hurwicz suggested that it might be useful to study the approach to equilibrium of solutions of the system

$$\frac{dx}{dt} = \frac{1}{2\sqrt{x}} - 2ux$$

$$\frac{du}{dt} = x^2 - 1$$

This system has the equilibrium point $(1, 1/4)$. We are trying out various methods for constructing approximate solutions of the system, subject to the initial conditions $x(0) = 0.8$, $u(0) = 0.2$.

The third job is being done in connection with Gurland's work, as discussed in Cowles Commission Discussion Paper No. 349. The model in this case is given by the single equation

$$y_t + \alpha + \beta z_t = u_t, \qquad t=1,2,\ldots, T$$

where u_t is generated by the relation

$$u_t = \rho\ u_{t-1} + \varepsilon_t\ , \qquad t=1,2,\ldots, T,$$

ε_t is taken from a table of random unit normal deviates and u_0 is a random normal deviate $=0$. The values of z_t are fixed (random numbers such that $-\frac{1}{2} \leq z_t < \frac{1}{2}$), the u_t are taken from [Kendall], and the values of y_t are calculated for a set of values of ρ, α, β. In the present work, we have chosen $\rho = 0.5$, $\alpha = +2$ and $\beta = 5$. We use $T = 15$ (20 samples) and $T = 25$ (20 samples).

Estimates of ρ, α, β are to be found for each sample by the maximum-likelihood method and also by the method described in [Gurland]. It is hoped that the results will give some information as to the relative accuracy of the two methods, and also as to their relative cost in computation time. We are also fitting a least-squares regression $y_t = a + b\ z_t$ to the same data, to obtain at slight cost a further comparison of methods.

3. What I have to say about future computation projects consists mainly of a report of a meeting to discuss the subject held early in October, attended by Gurland, Hildreth, Hurwicz, Slater and Templeton. It was agreed that more planning of future computation requirements was required in order to avoid extreme fluctuations in the supply of work for the computing staff. Several suggestions were made as to projects that might be kept on hand to assure a smooth flow of work to the computation section.

In addition to jobs which have since been started on, the following were considered:

1. (Hildreth) Since we have only one example of a system in which estimation has been carried out both by the limited information method and by maximum likelihood, it seems useful to carry through maximum likelihood on another system, chosen partly for its intrinsic interest, in which limited information estimates are available.

2. (Hildreth) A simple artificial model might be set up to obtain some empirical information as to what might be expected of limited information estimation when specification bias exists.

3. (Hurwicz, Slater) Several problems in linear programming lend themselves to a computational approach. These computations would be undertaken to give information on the specific problems considered, to provide examples illustrative of more general problems, and to educate the computing group in the techniques applicable to these problems (e.g. numerical integration of differential equations).

Some attention was given to the possibility of using larger and more powerful computing facilities than those provided by the Cowles Commission's desk calculators. Some of the computation projects discussed above can certainly be carried out more efficiently on electric punch card equipment or by an electro-analog computer. No doubt some projects that could reasonably be suggested (e.g. maximum likelihood estimation of coefficients in a fifteen-equation system) might call for the use of one of the large-scale digital computers recently completed (or "soon" to be completed). In justification of our plans to proceed with these problems using our present facilities, we may argue that the work presently at hand is not extensive enough to justify the expense in money and time of having outside computing done, and that results of the present computations may indicate whether or not work on a larger scale is likely to be productive.

REFERENCES

[Curland] Cowles Commission Discussion Paper 349

[Hildreth] Cowles Commission Discussion Paper 333

[Kendall] Tables of Autoregressive Series, Biometrika, Volume 36, 267-289, 1949, especially p. 282.

[Brown and von Neumann] Solutions of Games by Differential Equations, The RAND Corporation, P- 142, 1950.

[Rubin] Cowles Commission Discussion Paper 329