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SYSTEMS OF LINEAR STOCHASTIC EQUATIONS

Abstract

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A method is given for estimating the coefficients of a system of equations which is a subsystem of a complete system of linear equations linear in the variables, provided that those equations are identified by linear restrictions in the complete system. Under the assumptions that all variables in the system are known, and that the disturbances are jointly normally distributed with mean zero and constant covariance matrix, point estimates are obtained by maximizing the likelihood function. This leads to the condition that

$$\frac{|A_{u_I x} M_{xx} A'_{u_I x}|}{|A_{u_I y} W_{yy} A'_{u_I y}|}$$

be minimized, where $A_{u_I x}$ is the matrix of coefficients in the subsystem to be estimated, M_{xx} is the moment matrix, and W_{yy} is the matrix of moments of the residuals of the jointly dependent variables from their regression on the predetermined variables. This is seen to reduce to the single-equation estimates obtained by Anderson and Rubin [2]. It is also seen to reduce to the maximum likelihood estimates ([1]) if the subsystem is complete.

It is shown that this method can, if certain conditions analogous to the identification restrictions are satisfied, be carried over to

general linear systems.

Under conditions analogous to those in [2, Section 6.1], it is shown that the estimates in the case of linear systems linear in the variables are consistent; and under conditions similar to those in [2, Section 6.2], the estimates in the case of a general linear system are consistent.

Under more restrictive assumptions, the estimates of the parameters in a system of linear stochastic difference equations are shown to be asymptotically normally distributed; if the disturbances in the complete system have a constant normal distribution, efficiency comparisons are made.

[1] Statistical Inference in Dynamic Economic Systems, to be published as Cowles Commission Monograph No. 10.

[2] T. W. Anderson and H. Rubin, "Estimation of the parameters of a single stochastic difference equation in a complete system," to be published.