Remark on Properties of Nonnegative Matrices

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[The following is a quotation from a letter dated September 24, 1951 received by Koopmans]

I have noticed a number of Cowles Commission Discussion Papers on properties of nonnegative matrices. It seems to me that many of these results follow immediately from properties of transition probability matrices in the theory of finite Markov chains. Let $A = (a_{ij})$ with $a_{ij} \geq 0$ and $\max \Sigma_j a_{ij} = 1$. Let $b = (b_i)$ with $b_i = 1 - \Sigma_j a_{ij}$. Then

$$C = \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}$$

is a transition probability matrix. Each characteristic root of A is a root of C because if

$$Ax = \lambda x$$

then

$$C \begin{pmatrix} x \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} x \\ 0 \end{pmatrix} .$$

Herstein's theorem in Cowles Commission Discussion Paper, Mathematics 406 follows from a well known result in Markov chain theory and his requirement of nonsingularity of A can be dropped. I would guess that conversion of nonnegative matrices to transition probability matrices would make it possible to take results directly from the literature instead of rediscovering them.