COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 2091

NOTE: Cowles Commission Discussion Papers are preliminary materials circulated privately to stimulate private discussion and are not ready for critical comment or appraisal in publications. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has had access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

COMMODITY FUTURES III: SOME EMPIRICAL RESULTS ON HEDGERS' BEHAVIOR

H. S. Bouthakker and L. G. Telser

December 15, 1953

1. Introduction.

In a previous paper (CCDP Economics 2089) a static model for commodity markets with futures trading was outlined. We recall here that hedgers were defined to be those traders whose commitments in the cash market (whether spot or forward) are exactly offset by commitments in the futures markets. According to their futures position they were divided in short and long hedgers; short hedgers own stocks (or hold forward purchases) and have sold futures; long hedgers have sold forward and bought futures.

It was further argued that, as pointed out by Working [8], hedgers like most other traders hope to gain by favorable price movements. For hedgers this means that they hope the "basis" (the difference between a cash price and a futures price) will change in their favor during the period in which the hedge is held. The demand functions for short and long hedging were written...
(1) \[ v_1 = v_1 (p_x - p_z, k_1) \]

where \( v_1 \) represents the stocks held by short hedgers, \( p_x \) the spot price, \( p_z \) the futures price (both prices being really multi-dimensional) and \( k_1 \) a shift variable; and

(2) \[ v_2 = v_2 (p_x - p_z, k_2) \]

where \( v_2 \) represents the futures held by long hedgers and \( k_2 \) another shift variable (with a slight change of notation from the earlier paper).

About these functions the following statements were made

(3) \[ \frac{\partial v_1}{\partial (p_x - p_z)} < 0 \]

(4) \[ \frac{\partial v_2}{\partial (p_x - p_z)} > 0 \]

(5) \[ \frac{\partial k_1}{\partial x} < 0 \]

(6) \[ \frac{\partial k_2}{\partial x} < 0 \]

where \( x \) is the total stock.

In the present paper some calculations on the relation between hedging and the basis \( (p_x - p_z) \) will be reported for corn and cotton. At the end there is also an addendum to an earlier paper by one of the authors [1].

2. Sources of data.

Some of the data problems that arise in the study of hedging have already been considered in [5]. The figures for hedging, commitments derived from [5], refer only to reporting traders, i.e. those with commitments of more than 200,000 bushels of grain, or 5,000 bales of cotton, in any one future. Smaller hedgers are included in the non-reporting traders. It is
quite conceivable that some hedgers are sometimes above and sometimes below the reporting limit, and this would evidently make the figures for "large" hedgers less representative. Unfortunately we have no means of checking up on this and have therefore taken the published hedging commitments as they stand. A further comment on this source of possible errors is made below.

The calculation of the basis \((p_x - p_z)\) also presents many difficulties, since neither \(p_x\) nor \(p_z\) are scalars. For \(p_x\) the most convenient solution, though certainly not a very satisfactory one, is to take one price, viz. the price for a contract grade on the futures considered, as representative of the whole pattern of cash prices. It then remains to determine what contract grade to use in a particular period; this has to be done from an investigation of the schedules of premiums and discounts for deliveries of various grades. For the pre-war years in corn No. 2 Yellow was used, and for the post-war years No. 3 Yellow after deduction of the statutory discount of 2 cents per bushel; this discount favored the delivery of No. 2 during the thirties (when prices were low) and of No. 3 after the war. Prior to August 1940 the New York cotton contract was based on 7/8 inch Middling, thereafter on 15/16 inch Middling. From the cash price series, which represents the latter staple length, the average discount for 7/8 inch was therefore deducted for the first three crop years (this discount is not available by months, only by crop years). The cash price used was the monthly average for Houston, Texas, this being the usual delivery point on both New York and New Orleans contracts. The only other important delivery point is Galveston, which is very close to Houston and therefore has nearly the same price.

Since hedging commitments are distributed over various futures a weighted average of the values of \(p_x - p_z\) over all futures was computed. The weights were intended to reflect both the remaining life of the future (so as to get
all figures on a monthly basis) and the proportion of hedging commitments in each. The actual formula used is given below as (7). The distribution of hedging commitments was available only for corn in [6]; for cotton the distribution of total commitments by futures from [5] was used instead. Chicago quotations were taken for corn, in which there is virtually no other futures market. Cotton futures prices refer to New York only, but since the New Orleans contract is almost the same no inaccuracy arises. The multiplicity and heterogeneity of wheat futures markets was the reason why no calculations for that commodity were undertaken.

A further adjustment was made to allow for carrying charges, consisting of storage costs \( c \) and interest \( r \). This was desirable because \( p_x - p_z \) cannot be less than the marginal carrying charge and it is convenient to have all values of the same sign. Interest rates for commercial loans were obtained from the *Federal Reserve Bulletin* and increased by 1 per cent to allow for miscellaneous expenses. Storage costs for cotton are available annually in [7]; the figures used were those for Texas. For corn no reliable data have been published, but an inspection of futures prices led to the adoption of a rate of 1 cent per bushel per month throughout. Both in cotton and in corn it turned out that the total carrying charge thus calculated was not always large enough to make \( p_x + c + r - p_z > 0 \), though there were only a few instances to the contrary. In cotton they all occurred during the crop years 1940-41 and 1941-42 and may have been caused by port congestion due to submarine warfare in addition to large stocks. In corn there were two such cases, viz. in November of 1948 and 1949, when total grain stocks (wheat, corn and oats together) were unusually large and storage capacity consequently hard to find. For these observations ad hoc corrections to the storage rates were made.
The formula by which the "basis index" \( s_i \) was calculated is finally

\[
(7) \quad s_i = \frac{1}{P_{11}} \left( \frac{1}{\sum_j w_{1j}} \sum_j w_{1j} \frac{P_{i1} - P_{1j}}{t_{1j}} + c_i + r_i P_{11} \right)
\]

where (with some changes in notation) \( P_{11} \) is the spot price in the \( i \)-th period, \( w_{1j} \) the weight of the \( j \)-th future in the \( i \)-th period (based on hedging commitments for corn and on total commitments for cotton, as explained above), \( c_i \) the storage cost in the \( i \)-th period, \( r_i \) the interest rate, \( P_{1j} \) the price of the \( j \)-th future in the \( i \)-th period, and \( t_{1j} \) the remaining life of the \( j \)-th future.

\( s_i \) as defined by (7) is a pure number, which might be interpreted as a somewhat simplified version of Keynes's "own rate of interest" [4, Ch. 17]. The presence of \( P_{11} \) in the denominator acts as a sort of deflator, for the own rate of interest is expressed in units of the commodity considered.

This particular way of deflating is not always appropriate, for traders are not interested in accumulating (say) cotton but in accumulating income. In the absence of another deflator the use of \( P_{11} \) may have something to comment itself, but for calculations over shorter periods, in which \( P_{11} \) fluctuated much more than prices in general, it may be as well not to deflate at all and multiply \( s_i \) by \( P_{11} \). This "absolute" basis index has sometimes been used instead of \( s_i \).

3. The basic model.

In evaluating the following model it is necessary to bear in mind that it is designed specifically for short-term (particularly monthly) variations and that there is no intention of estimating all the equations presented.

For the rest it is based on the theoretical argument of [2] with the additional assumption (55) of that paper. We get therefore (going back to the
previous notation)

(8) \[ u = u(p_x, k_u) \]

(9) \[ v = v(p_z, k_v) \]

(10) \[ v_1 = v_1(p_x - p_z, k_1) \]

(11) \[ v_2 = v_2(p_x - p_z, k_2) \]

(12) \[ u + v_1 = x \]

(13) \[ v_1 = v_2 + v \]

where \( u \) is the volume of stocks held by cash-speculators, \( v \) the net commitments of futures-speculators, \( v_1 \) the stocks held by short hedgers, \( v_2 \) the futures held by long hedgers, \( x \) total stocks and the \( k \)'s shift variables. To discuss the identifiability properties of the system we write it in a linear form, since this is the worst case from that point of view: if the equations we are interested in are identified in the linear case they will certainly be so in the non-linear case. We obtain

(8a) \[ u - b_{11} p_x = c_{11} k_u \]

(9a) \[ v - b_{12} p_z = c_{22} k_v \]

(10a) \[ v_1 - b_{31} p_x + b_{32} p_z = c_{33} k_1 \]

(11a) \[ v_2 - b_{41} p_x + b_{42} p_z = c_{44} k_2 \]

(12a) \[ u + v_1 = x \]

(13a) \[ - v + v_1 - v_2 = 0 \]

with moreover

(14) \[ b_{31} = b_{32} \]

(15) \[ b_{41} = b_{42} \]

We regard the \( k \)'s and \( x \) as predetermined variables, which however are not independent of each other. It is clear that (10a) the equation we are primarily interested in, cannot be obtained as a linear combination of the
other equations which satisfy the identities (12a) and (13a) and the restrictions (14) and (15). Hence in the linear case (10) is identified, and a fortiori it will be so in the non-linear case. We will see that in practice (10) has to be given a non-linear form; moreover the static model (3) - (13) has to be made dynamic by introducing some lags. The non-linearity causes some estimation problems, since maximum likelihood methods for that case have not yet been worked out and we have to be satisfied with least squares.

The lags are introduced both to make the system more realistic and to adapt it to the available data. The principal variable in this context is the total stock $x_t$ which is the result of past production and production, both of which depend, among other things, on past prices. We can lump production and consumption together as the net change in stocks $\Delta x_t$, where $t$ refers in this case to the period beginning at time $t$. Hence

$$x_t = x_{t-1} + \Delta x_{t-1} (p_{x,t-1}, k_{\Delta x,t-1})$$

It will be recalled from [2] that in a stock model the net change in stocks during a period may be regarded as part of the holdings of cash-speculators at the beginning of that period, so that the model (8) - (13) needs no major revision. We need only add a "t" to all the subscripts and adjoin equation (16). The time path of the endogenous variables $u, v, v_1, v_2$ and $x$ is then determined by the shift variables $k_u, k_v, k_1, k_2$ and $k_{\Delta x}$ and by certain initial values which are not relevant here.

Little has been said so far about the nature of these $k$'s, except as regards the partial dependence of the first four on $x$. They will be further specified when we come to estimating particular equations.
4. The short hedging equation.

We now proceed to the estimation of equation (10), with the particular purpose of testing the hypothesis (3) concerning the effect of changes in the basis. For \( p_x - p_z \) we used the basis index \( s \) as described in Section 2. It remains to specify \( k_1 \). In [2] it was observed that \( k_1 \) depends on \( x \); however since \( p_x - p_z \) also depends on \( x \) (cf. Section 5 below) it does not seem advisable to introduce \( x \) as a separate variable. The estimated relation between \( v_1 \) and \( s \) is therefore strictly speaking not the one for which (3) was derived, but it will be seen from (27) and (36) of [2] that the same inequality continues to hold if the hedging-basis relation is reinterpreted to allow for changes in \( x \).

Another variable which may affect \( k_1 \) is the volume of short hedging in the preceding period, since the periods are only one month and hedging is probably rather sluggish, partly because it is costly to change one's commitments. For some commodities the government support price may affect hedging decisions, viz. in those where it puts an effective floor under the market price. This is the case in cotton, where prices have never fallen noticeably below the support level, though in corn they have done so repeatedly. If the support price is a lower bound and market prices are close to it there is evidently little prospect of profit in hedging, since futures prices can only move upwards and thus cause a loss. In cotton the ratio between the market price and the support price has therefore been introduced explicitly. For the rest \( k_1 \) has been regarded as random.

Scatter diagrams of \( s \) and \( v_1 \) were made for corn and cotton with all the available observations (about 108 months for corn and about 180 months for cotton). In the case of corn a fairly striking correlation between \( s \) and \( v_1 \) was evident, which turned out to be markedly curvilinear. In cotton
the scatter diagram was much less clear, due mainly to the additional influence of the support price. In both commodities the influence of previous volumes of hedging was discernible.

There is still a great deal to be done before this extensive material will be completely analyzed, but the following results indicate that it supports our theory to a considerable degree and that the estimated relations are sufficiently exact to be of some use for predictive purposes. In cotton we did a least-squares regression for the six crop years beginning in 1946 to 1951; four months had to be left out because the market was closed or not all data were available, so there were 68 observations in all. We obtained

\[ \log y_t = -0.231 \log s_t + 0.521 \log g_t + 0.804 \log y_{t-1} + 0.511 \]

\[ (\pm 0.077) \quad (\pm 0.251) \quad (\pm 0.059) \]

where the figures below the line are standard errors; the multiple correlation coefficient was 0.952. Here \( y_t \) is the volume of short hedging by reporting traders (those with 5,000 bales or more in any one future) at the end of each month, \( s_t \) the basis index, as defined by (7), for the same month, and \( g_t \) the ratio of the spot price at Houston to the support price (it was assumed that the support price for any crop year becomes effective in the preceding July).

The coefficients in (17) are all significant at the 5 per cent level (those of \( s_t \) and \( y_{t-1} \) also at much more exacting levels) and have the right signs. In particular (3) is shown to hold. The coefficient of \( \log y_{t-1} \) is less than unity so that \( y_t \), viewed as a time series, will be stable if \( s_t \) is bounded.

In estimating (7) it was assumed that the disturbances were serially interdependent; to make them so was in fact one of the reasons for introducing
\( y_{t-1} \). No test of this assumption has yet been made, and it is also not yet clear to what extent (17) expresses the seasonality in \( y_t \). Subject to these qualifications the fit of (17) is very satisfactory.

A similar result was obtained for corn. In this case the support price was not taken into account since the market price has often been below the support level and there is consequently no reason why it should influence hedging. The equation for the period November 1946 - September 1952 is

\[
(18) \quad \log y_t = -0.287 \log s_t + 0.591 \log y_{t-1} + 0.637
\]

\( t = 0.039 \) \( t = 0.059 \)

with \( R = 0.929 \). The coefficients are much the same as those in (17); there is less influence of lagged hedging but slightly more of the basis index.

It will be recalled from Section 2 that in corn \( s_t \) could be computed more accurately than in cotton.

Some scatter diagrams were also made for long hedging \( v_2 \) against the basis index, but no regression analyses have yet been undertaken. The scatters do not look very promising, though this does not mean much by itself. It is conceivable that long hedging is dependent on forward (as distinct from futures) prices, such as those for cotton "call" contracts or for cotton goods. No information on such prices seems to have been published.

5. **Stock-spread relations.**

In an earlier paper [1] some calculations concerning the effect of stocks on the spread between old crop and new crop futures were reported. The theory developed there at first sight seems rather different from the later one presented in [2], though actually a synthesis may be possible. The regressions in [1] may be regarded as tests of the theoretical result (36) in [2].

For the sake of completeness we give here an equation for cotton replacing the one on p. 7 of [1]. The present one covers the period 1933-53; ear-
lier years were disregarded because in 1930-31 there were major changes in the New York cotton contract and in 1930-32 the Federal Farm Board was active in a way about which no accurate statistics are available. Moreover the July-October spread was corrected for carrying charges in the manner described in Section 2 above. The form of the equation was also changed, and we derived

\begin{equation}
\log s^* = -2.10 \log x_1 - .26 \log x_2 + 2.68
\end{equation}

\begin{align*}
(\pm .22) & \quad (\pm .05) \quad (R = .924)
\end{align*}

where \( s^* \) is the adjusted July-October spread during April, in per cent; \( x_1 \) is the total stock in the U. S. excluding mill stocks and CCC stocks on April 1; \( x_2 \) is the stock owned by or pledged to the CCC on April 1.

Although \( s^* \) in (19) is not identical with \( s \) in (17) the two are, very roughly speaking, proportional and their coefficients in a logarithmic equation therefore comparable. Substituting (19) into (17) we see therefore that the elasticity of \( v_1 \) with respect to \( x_1 \) is rather less than one (viz. about .5), which means that the proportion of hedged stocks declines as free market stocks decrease. This interesting conclusion will be checked further.
REFERENCES


