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Commodity Futures I: Static Theory

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1. Definitions.

A <u>futures contract</u> is an agreement whereby the seller undertakes to deliver a definite quantity of a commodity, in one or more alternative grades and locations chosen at his (the seller's) option, within a definite period (which may or may not include the present) at a price fixed when the contract is made, and whereby the buyer undertakes to pay for this quantity on delivery. For instance a Chicago May wheat contract calls for the delivery at any time in May in a licensed Chicago warehouse of 5,000 bushels of No. 2 Hard, Yellow Hard or Red wheat at par, or of other grades at fixed premiums or discounts from the price agreed for these "standard grades." In this example there is no provision for delivery outside Chicago, but on a New York cotton contract, for instance, delivery can also be made in certain Southern cities, again at seller's option. The market for futures contracts is known as the <u>futures market</u>,[1] as distinct from the <u>cash market</u>, in which the quantities traded

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1 A different definition has recently been proposed by Working [10], who enumerates its economic advantages rather than its institutional characteristics.

are specified in complete detail as regards grade and location. In the cash market there are both <u>spot</u> transactions (for immediate delivery) and forward transactions (for delivery in a later period).

For each trader we can draw up a commodity balance sheet in physical terms (bushels, bales, etc.), showing all items for which the price has been fixed. For the i-th person it looks as follows

Assets		Liabilities	
Stocks owned	x_i	Sold on forward contracts	y_i^-
Bought on forward contracts	y_i^+	Sold on futures contracts	z_i^-
Bought on futures contracts	z_i^+		

With the aid of this device we can distinguish certain groups of traders. <u>Speculators</u> are those for whom $x_i + y_i^+ + z_i^+ \neq y_i^- + z_i^-$. Among them <u>cash-speculators</u> are defined by $x_i + y_i^+ \neq y_i^-$ and <u>futures-speculators</u> by $z_i^+ \neq z_i^-$. <u>Arbitrage</u> occurs if $x_i + y_i^+ = y_i^- \neq 0$; <u>Spreading</u> (also called straddling) if $z_i^+ = z_i^- \neq 0$. <u>Hedgers</u> are those for whom $x_i + y_i^+ - y_i^- = -z_i^+ + z_i^- \neq 0$; they will be called <u>long hedgers</u> if $z_i^+ > z_i^-$, and <u>short hedgers</u> if $z_i^+ < z_i^-$.

In the above formulation certain simplifications are implicit; more particularly we have ignored grade differences, delivery times, growing crops and related commodities[2]; these complications will come up later. Furthermore the above terminology is not entirely identical with that commonly used; e.g. in [8] spreading is regarded as a form of futures speculation.

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2. Related commodities, particularly products of the commodity considered, can sometimes be expressed in equivalent units of the latter. Thus a miller may sell flour forward without having wheat on hand, and this may lead him to buy wheat futures as a (long) hedge.

An incidental advantage of the balance sheet representation is that it brings out the analogy between actual stocks (x_1) and cash and between forward or futures contracts and securities (loans).

2. A general static model of traders' behavior.

Nearly all traders in a commodity market, viz. the hedgers, spreaders and speculators, stand to gain or lose by rises or falls in the various price quoted and arrange their positions accordingly. The only traders to whom this does not apply are those engaged in arbitrage as defined above[3].

It is important to recognize this almost universal reliance on favorable price movements because it is often suggested that hedgers, somehow finding themselves in possession of stocks, enter the futures market merely in order to evade the price risks they thus run. In fact, however, as Working [10] has pointed out, the decision to buy or sell in the cash market (whether spot or forward) is not usually separate from the decision to establish or lift a hedge; expectations as to price movements in both markets will influence those decisions. Hence hedging really amounts to spreading between the cash and futures markets; this is also the reason why we dissented from the practice of regarding spreading (as defined) as a form of speculation. It would be equally logical to regard hedging in the same light, but to describe most transactions as speculative is not very informative. Our terminology at any rate has the merit of avoiding this inconsistency. The matter is not merely of terminological importance, for under the Commodity Exchange Act there are certain restrictions on "speculation" (including spreading) which do not apply to hedging.

To start our survey of the various groups of traders we consider the

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3 This definition is not quite complete: it is also necessary that the delivery time on forward purchases (if any) is not later than on forward sales, since otherwise additional transactions with attendant price risks have to be undertaken.

arbitrageurs, whose behavior is simplest of all since they are not troubled by uncertainty. They only have to compare the difference between two cash prices for different delivery dates with the carrying and transportation cost. This cost, more particularly the carrying charges (which include interest), will not be the same for all traders, though the limits of variation are generally small. In any case the risklessness of these operations will ensure that forward prices cannot exceed the spot price in the same market by more than the marginal carrying cost, and that regional price differences cannot exceed freight charges plus interest during transit.

We have defined arbitrage as taking place in the cash market, but actually something analogous is possible in the futures market, viz. by making delivery of previously hedged stocks. In this case, however, there is always a theoretical possibility of making an additional profit if the grade held happens to command a higher price in the cash market than the expiring future (the cash price can only be lower in very exceptional circumstances), so that he will not deliver. Although this transaction is therefore not pure arbitrage it will again keep the difference between futures and spot price at or below the marginal carrying charge.

It will be seen that this condition provides only an upper limit to the price spreads, and stocks may well be held even if the futures price is less than the spot price.

In principle the behavior of speculators, spreaders and hedgers can also be very simply described. Each of them will, for a given state of information, want to have particular values for the entries in the balance sheet of p. 2. If we denote quotation time by t, the vector summarizing the information of the i-th trader by $k_i(t)$, the spot price by $p_x(t)$ the forward price by $p_y(t)$ and the futures price by $p_z(t)$ then this trader will have the following excess demand functions:

$$(1) \quad x_i(t) = f_{ix} \left\{ p_x(t),\ p_y(t),\ p_z(t),\ k_i(t),\ t \right\} \qquad x_i \gtreqless 0$$

$$(2) \quad y_i(t) = f_{iy} \left\{ p_x(t),\ p_y(t),\ p_z(t),\ k_i(t),\ t \right\}$$

$$(3) \quad z_i(t) = f_{iz} \left\{ p_x(t),\ p_y(t),\ p_z(t),\ k_i(t),\ t \right\}$$

The excess demand functions for futures and forward commitments can assume both negative and positive values.

The prices appearing in these functions are strictly speaking not scalar magnitudes; neither are the excess demands themselves. There are as many spot prices as there are grades and locations, and in the case of forward prices there is an additional time dimension. The latter is the only one which differentiates the futures price.

Each of these three functions contains two shift variables. The first one, $k_i(t)$, represents the state of information of the i-th trader and consists of all observed facts and figures which he considers relevant to his excess demand. This will include crop forecasts, stock and consumption data, prices in other markets, etc. It is not assumed that this information will always influence his excess demand in the same way, hence the presence of the second shift variable t. Together these two shifts, which for practical purposes need not always be distinguished, make excess demand highly unpredictable and this is intentional. We do not think there is much prospect in a theory of futures markets which lays great stress on the explanation of shifts in demand; rather we should try to find the more stable components of this system of relations and see how adjustments to these shifts are made.

It is possible to say that the shift variables represent the influence of "expectations," though this concept by itself explains nothing. In part of the literature (e.g. in [4], [5]) strict assumptions about the "expected price" are made, the purpose being to relate the latter to the futures price.

There it is assumed that all traders have the same expectations which are moreover single-valued (i.e., certain) or made equivalent to certainty by transformation. In that case it is indeed possible to work with an "expected price." However, as Hawtrey [3] has remarked, the assumption of unanimity is unrealistic to an unadmissible degree, for if individual expectations were identical there would clearly be no transactions between speculators. The reduction to certainty-equivalents tends to obscure differences in risk appreciation which cause some traders to be hedgers, others to be speculators, and still others to stay out altogether. These assumptions are moreover unnecessary: in order for a speculator to buy futures he need not have any precise idea what the price in the future will be; all that is needed is that he hopes for a price rise large enough to make the purchase worth his while. Nevertheless an important conclusion from the theory of futures markets, viz. Keynes' doctrine of "normal backwardation," involves the notion of an "expected price," and therefore an attempt will be made below to redefine the concept in a way which avoids these difficulties.

If there is to be an instantaneous equilibrium prices have to be such that

$$\sum_i x_i(t) = x(t) \tag{4}$$

$$\sum_i y_i(t) = 0 \qquad \text{i.e. } \sum_i y_i^+(t) = \sum y_i^-(t) \tag{5}$$

$$\sum_i z_i(t) = 0 \qquad \text{i.e. } \sum_i z_i^+(t) = \sum z_i^-(t) \tag{6}$$

where $x(t)$ is the total stock at time t. However the model discussed so far is too general to admit of detailed consideration or statistical verification; we will therefore proceed with a much simpler version.

Before doing so it may be helpful to say a few words about the reason why there are both forward and futures markets[4]. The main reason is the increased volume of sales obtained by standardization of the contract, which makes it of interest to a much larger group of traders, including outside speculators. The futures market thereby comes much closer to a perfect market than a cash market, with all its allowances for grade, location and delivery time, could be. In fact the futures market usually dominates the price structure quotations in the cash markets being made in terms of premiums and discounts from a futures price.

The principal drawbacks of futures trading compared to forward trading are the possibility of corners (since sellers of futures contracts will often be neither willing nor able to settle by delivery) and, more important, the impossibility of obtaining a complete hedge because futures prices represent only one or a few out of many grades and locations.

3. Speculation and hedging.

Instead of looking at the dependence of particular types of assets on their prices, as was done briefly in the preceding section, we will now look at particular groups of traders, viz. speculators, hedgers and arbitrageurs. The arbitrageurs have already been dealt with and the spreaders do not require separate consideration. For convenience we assume that the spot price p_x, which is already a highly complex variable, now also represents the whole array of forward prices, so that we need not introduce a separate p_y. We then have

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4 The importance of this distinction was stressed by Blau [1]. Though of great value in understanding actual commodity markets it is not always as clear-cut as in the theory. The higher the volume of futures contracts settled by delivery in comparison with the total volume of open contracts, the more the futures market approaches to a forward market. In the grain and cotton futures markets we are mostly thinking of here deliveries are only a small percentage of the volume of open contracts before delivery time; in some other markets such as lard on the contrary futures contracts are often used as a means of trading spot supplies.

$$u(t) = u(p_x, p_z, k_u) \tag{7}$$

for the stocks held by cash speculators and arbitrageurs

$$v_1(t) = v_1(p_x, p_z, k_{v_1}) \tag{8}$$

for the stocks held by short hedgers

$$v_2(t) = v_2(p_x, p_z, k_{v_2}) \tag{9}$$

for the commitments of long hedgers in the futures market

$$w(t) = w(p_x, p_z, k_w) \tag{10}$$

for the net holdings of futures-speculators. The k's are again shift variables[5]; they as well as the p's refer to time t.

We can discuss (7) and (10) together. p_z in (7), and p_x in (10), should also be regarded as shifts in the appropriate excess demand functions. For given values of all the shift variables it is clear that the lower the price the greater the excess demand, because of the greater chance to make a profit over time. This applies also to cash demand from consumers, which is not excluded here though consumption itself does not fit into a stock model. The relations between p_x and u, and between p_z and w, are nothing but schedules of reserve prices. Hence

$$\frac{\partial u}{\partial p_x} < 0 \tag{11}$$

$$\frac{\partial w}{\partial p_z} < 0 \tag{12}$$

Similar statements cannot be made about the effect of the other price in each equation, for there are two conflicting tendencies here. On the one hand a high value of p_z, p_x and k_u being given, will not encourage speculators to

5 "Shift" does not necessarily mean a parallel movement of the curve.

transfer their operations from the cash to the futures markets, but on the other hand it may lead some potential cash-speculators or arbitrageurs to hedge their holdings. Similarly a high cash price will not make the cash market more attractive to speculators now operating in futures, but it may induce some longs, who have bought futures in anticipation of later as yet unsold consumption, to sell forward and thus become long hedgers.

We now come to the hedgers, whose behavior is of central importance in commodity markets because they are the only traders who deal simultaneously in the cash and futures markets. They are only interested in movements of the "basis" (the difference between a cash price and the futures price). If they are short hedgers they will gain if the relevant cash price rises more (or falls less) than the futures price; the reverse applies if they are long hedgers. According to the preceding argument this will mean for the short hedgers (whose commitments are measured by their stocks)

$$\frac{\partial v_1}{\partial (p_x - p_z)} < 0 \tag{13}$$

and for the long hedgers (whose commitments also have a positive sign)

$$\frac{\partial v_2}{\partial (p_x - p_z)} > 0 \tag{14}$$

The static equilibrium conditions are, in accordance with (4) - (6)

$$u + v_1 = x \tag{15}$$

$$v_1 = v_2 + w \tag{16}$$

which can be contracted to

$$u + v_2 + w = x \tag{17}$$

showing that the total stock is ultimately held by cash-speculators (including arbitrageurs), long hedgers and futures-speculators, the short hedgers acting only as intermediaries. Of course a single equation such as (17) cannot determine both p_x and p_z and for this (8) is also needed.

We must now see whether (15) - (16) determine prices uniquely. For small changes they can be written

$$(18)\quad \left(\frac{\partial u}{\partial p_x} + \frac{\partial v_1}{\partial p_x}\right)dp_x + \left(\frac{\partial u}{\partial p_z} + \frac{\partial v_1}{\partial p_z}\right)dp_z + \frac{\partial u}{\partial k_u}dk_u + \frac{\partial v_1}{\partial k_{v_1}}dk_{v_1} = dx$$

$$(19)\quad \left(\frac{\partial v_1}{\partial p_x} - \frac{\partial v_2}{\partial p_x} - \frac{\partial w}{\partial p_x}\right)dp_x + \left(\frac{\partial v_1}{\partial p_z} - \frac{\partial v_2}{\partial p_z} - \frac{\partial w}{\partial p_z}\right)dp_z - \frac{\partial v_1}{\partial k_{v_1}}dk_{v_1} - \frac{\partial v_2}{\partial k_{v_2}}dk_{v_2}$$

$$- \frac{\partial w}{\partial k_w}dk_w = 0$$

The determinant of this system is

$$(20)\quad \begin{vmatrix} \dfrac{\partial u}{\partial p_x} + \dfrac{\partial v_1}{\partial p_x} & \dfrac{\partial u}{\partial p_z} + \dfrac{\partial v_1}{\partial p_z} \\ \dfrac{\partial v_1}{\partial p_x} - \dfrac{\partial v_2}{\partial p_x} - \dfrac{\partial w}{\partial p_x} & \dfrac{\partial v_1}{\partial p_z} - \dfrac{\partial v_2}{\partial p_z} - \dfrac{\partial w}{\partial p_z} \end{vmatrix} < 0$$

The first diagonal element is negative by (11) and (13), the second one is positive by (13), (14) and (12). We do not know the signs of all the individual terms in the off-diagonal elements, but it has been pointed out that $\frac{\partial u}{\partial p_z}$ and $\frac{\partial w}{\partial p_x}$ would be positive but for possible shifts to hedging; the latter will however be absorbed in the v_1- and v_2-terms and the whole element in the first row will therefore be positive and the other one negative, so that the determinant as a whole is negative and prices are determinate. This is of course a purely static condition.

4. Normal backwardation.

With the aid of the simplified model we can discuss one of the most interesting items in the theory of futures markets, viz. Keynes' doctrine of "normal backwardation" [6, vol. II, p. 142-47], taken over in a more explicit but less comprehensive form by Hicks [4, p. 137-9] and Kaldor [5]. "Backwardation" is said to exist if the futures price is below the spot price. "Normal" is a situation where the spot price in the future is "expected" to be equal to the spot price now. This condition of normality is made only by Hicks and it is clear from other expositions, including Keynes', that the theory really declares the futures price to be below the expected spot price in the future. In fact Keynes was chiefly interested in the case of surplus stocks, when the futures price will exceed the current spot price by the marginal carrying cost. The name "normal backwardation" is therefore ill-chosen.

In order to consider this theory in our terms we first have to reintroduce the concept of "expected price" without making the unduly restrictive assumption of unanimity. We define p_z^*, the expected price, by

$$w(p_z^*, p_x, k_w) = 0 \tag{21}$$

It is thus the futures price at which, given the spot price, the futures-speculators would be in equilibrium with each other. Since under the unanimity assumption futures-speculators would not trade with each other at all the old definition is clearly a special case of the proposed one.

Because of (12) and (16) the question whether $p_z^* > p_z$ now reduces to whether $v_1 > v_2$, i.e. whether short hedging is more important than long hedging. According to (13) and (14) this depends on the basis. There will be some values of $p_x - p_z$ for which $v_1 \lesseqgtr v_2$ and no general statement, such as Hicks has attempted, can therefore be valid. Nevertheless there are reasons for thinking that at the relatively small values of $p_x - p_z$ usually observed short hedging will predominate.

Hicks' argument [4, p. 137] is based on the decreasing flexibility of entrepreneurial decisions as their production processes advance in time[6]

6 In more sophisticated form this is expressed by Samuelson's "Le Chatelier principle" [7, p. 38]. Curiously enough this principle holds only for small changes; it is not clear how this affects the present argument.

". . . technical conditions give the entrepreneur a much freer hand about the acquisition of inputs (which are largely needed to start new processes) than about the completion of outputs (whose process of production--in the ordinary business sense--may be already begun). Thus, while there is likely to be some desire to hedge planned purchases, it tends to be less insistent than the desire to hedge planned sales. If forward markets consisted entirely of hedgers. . . . a smaller proportion of planned purchases than of planned sales would be covered by forward contracts." This is an interesting point; in our terms[7] it means that the value of $p_x - p_z$ at which producers (in the technical sense) are willing to hedge all their planned sales is higher than the value at which consumers are willing to hedge all their planned purchases.

Hicks' discussion of futures prices, as quoted and elsewhere, abounds in implicit assumptions not all of which we can uncover here. To understand the above argument one must bear in mind that Hicks (following Keynes) is apparently thinking of markets with continuous production, such as rubber and tin, and not of the seasonal markets which are more important in practice. In these markets no stocks other than pipeline stocks need be held if everything is in equilibrium, and in that "normal" case the spot price will indeed be above the futures price, since the "expected" price then coincides with the current spot price. This "normal" case is evidently of very limited interest. Keynes [6], Kaldor [5] and Dow [2] have considered more general cases in which the Hicksian argument can no longer be immediately applied.

An essential element in this line of thought is the concept of a risk premium, which hedgers hand over to speculators as a reward for taking over the risk of price fluctuations. This premium is in fact equal to the difference between the expected price and the futures price, i.e. to the backwardation in Hicks' normal case.

The principal difficulty arising here is the inapplicability of much of the argument to middlemen, who neither consume nor produce the commodity, whose

7 It should be noted that Hicks does not distinguish between futures and forward contracts and does not consider cash speculation explicitly.

positions can be adjusted to changing data at any time, and whose gain is derived entirely from changes in the basis. Since these middlemen are probably the most important group in many if not most commodity markets we have to see what reasons, if any they have for preferring short hedging to long hedging[8]. If these reasons are sufficiently general they will also shed light on the behavior of producers and consumers, so that less reliance need be placed on Hicks' remarkable but tenuous line of reasoning.

Since the gain from hedging is derived from movements of the basis, the willingness to hedge will increase with the possibility of predicting such movements. This implies in the first place that hedgers must have a much more detailed knowledge of market conditions, especially in the short run, than futures-speculators[9]. More relevant to the present problem is that the prediction of basis movements will be easier when stocks are large than when they are small; in other words the basis risk is a decreasing function of total stocks. Strictly speaking it is not only the size but even more the distribution of stocks between grades and locations which determines the basis risk, but in practice the total size may be a sufficient indicator of the stock position. Therefore (8) and (9) should be written as

$$v_1 = v_1(p_x - p_z, k_{v_1}, x) \tag{22}$$

$$v_2 = v_2(p_x - p_z, k_{v_2}, x) \tag{23}$$

8 In the most important commodity markets, viz. those for primary products, in the U. S. there is very little hedging by producers (farmers), who prefer to act as cash-speculators. Hedging by producers is more important in the futures markets for semi-manufactured products such as fats and oils.

9 It will be seen from this that we do not regard hedging as a consequence of risk aversion. The division of commitments between hedgers and speculators is based primarily on different sorts of knowledge rather than on different attitudes to risk.

with

$$\left.\begin{array}{lll} (24) \quad & \dfrac{\partial v_1}{\partial x} > 0 \\ (25) \quad & \dfrac{\partial v_2}{\partial x} > 0 \end{array}\right\} \quad \text{for constant } p_x - p_z$$

There is consequently an asymmetry between short and long hedging: large stocks by themselves encourage both short and long hedging, whereas a large basis by itself discourages short and encourages long hedging. It is to be noted that in the earlier version of the model there was already an influence of x on $p_x - p_z$, viz.

$$(26) \qquad \frac{\partial(p_x - p_z)}{\partial x} = \frac{1}{D}\left(- \frac{\partial w}{\partial p_z} - \frac{\partial w}{\partial p_x}\right) < 0$$

where D is the determinant of the matrix in (20). As argued previously $\frac{\partial w}{\partial p_x}$ is of uncertain sign and probably small; hence by (12) the basis moves inversely with stocks, assuming constant values of the k's. Looking only at (26), in conjunction with (13) and (14), we can say therefore that short hedging will be large, and long hedging small, if stocks are large. As regards short hedging this tendency is reinforced by (24), but as regards long hedging it is counteracted by (25). Now we know that as $p_z - p_x$ approaches carrying costs short hedging approaches to arbitrage and hence the demand for hedged stocks will be very large. At the same time the scope for long hedging becomes very small because the spot price cannot fall short of the futures price by more than the carrying charge and the large volume of stocks makes irregular movements of non-deliverable grades and locations unlikely. In this price range then, which must prevail in seasonal markets for a large part of the crop year, long hedging is small compared to short hedging, and the spot price must be below the expected price as defined by (21). Since the elasticity of long hedging with respect to stocks is smaller than that of short hedging the latter must be more important for a considerable range of stocks. In seasonal markets long hedging can only become dominant towards the end of the crop year, when it will in fact amount to anticipation on the next crop. This conclusion does not apply to markets with continuous production.

5. The influence of stocks on the basis.

So far we have considered the effect of changes in x for constant values of the k's. This is not realistic, for traders' expectations are no doubt influenced by the supply position. We can discuss this by making the k's functions of x; this will also be done for (8) and (9) so that we do not need (22) and (23) any longer. Moreover since the k's are arbitrary up to a monotonic transformation we can normalize them in such a way as to make their coefficients in (13) and (19) equal to unity. (24) and (25) can be translated as

$$\frac{\partial k_{v_1}}{\partial x} > 0 \tag{27}$$

$$\frac{\partial k_{v_2}}{\partial x} > 0 \tag{28}$$

It also seems reasonable to assume that the presence of large stocks will make speculators less bullish and more bearish than they would otherwise be, hence

$$\frac{\partial k_u}{\partial x} < 0 \tag{29}$$

$$\frac{\partial k_w}{\partial x} < 0 \tag{30}$$

We consider now the net effect of an increase in stocks on the basis. Solving (18)-(19) with the above convention as to the k's we obtain, after some arithmetic,

$$\frac{\partial (p_x - p_z)}{\partial x} = \frac{1}{D}\left[\left(1 - \frac{\partial k_u}{\partial x} - \frac{\partial k_{v_1}}{\partial x}\right)\left(\frac{\partial v_1}{\partial p_z} - \frac{\partial v_2}{\partial p_z} - \frac{\partial w}{\partial p_z} + \frac{\partial v_1}{\partial p_x} - \frac{\partial v_2}{\partial p_x} - \frac{\partial w}{\partial p_x}\right) - \left(-\frac{\partial k_{v_1}}{\partial x} + \frac{\partial k_{v_2}}{\partial x} + \frac{\partial k_w}{\partial x}\right)\left(\frac{\partial u}{\partial p_z} + \frac{\partial v_1}{\partial p_z} + \frac{\partial u}{\partial p_x} + \frac{\partial v_1}{\partial p_x}\right)\right] \tag{31}$$

where D has the same meaning as in (26). For convenience, and not because it is strictly necessary for the proof, we make the rather harmless assumptions that

(32) $$\frac{\partial k_{v_1}}{\partial x} = \frac{\partial k_{v_1}}{\partial x} < 1$$

since the two effects must be of the same order of magnitude in any case and the shift in short hedging resulting from a rise in stocks is hardly likely to absorb the whole rise, and that

(33) $$\frac{\partial u}{\partial p_z} = \frac{\partial w}{\partial p_x} = 0$$

for reasons discussed earlier.

(31) can then be written

(34) $$d(p_x - p_z) = \frac{1}{D} \frac{\partial u}{\partial p_x} \frac{\partial w}{\partial p_z} \left[- \frac{dx - dk_{v_1} - dk_u}{\frac{\partial u}{\partial p_x}} - \frac{dk_w}{\frac{\partial w}{\partial p_z}} \right]$$

As it stands the sign of this expression is indeterminate and we must therefore try to interpret each term separately. The first term inside the square brackets is just equal to the change in p_x necessary to have the change in stocks, less the shifts in hedgers' and speculators' demand, absorbed by the cash speculators only. Similarly the second term is the change in p_z necessary to keep w unchanged despite the shift in its curve (the shifts in long and short hedging cancel each other by (32) . The denominators in both terms are negative, and if we are thinking of an increase in stocks the first numerator is positive by (29) and (32), the second one is negative by (30). To see which term is larger in absolute value we split up the first term in two parts, viz.,

$$(35) \qquad \frac{dx-dk_{v_1}}{\partial u/\partial p_x} - \frac{dk_u}{\partial u/\partial p_x}$$

The first part is certainly negative, the second certainly positive and comparable with the second term in (34), both being indications of the change in equilibrium prices for the two groups of speculators consequent on a rise in stocks. If these two changes are equal, as they should be approximately, the first part of (34) will make the whole expression in square brackets in (34) positive. Even if this particular price change is somewhat smaller in the cash market than in the futures market, the whole expression may still be positive. In fact one would guess that the change in the cash market will be the larger of the two[10] since cash speculation is more a short term operation. In any case there are strong reasons for thinking that the bracketed expression will be positive, and that therefore

$$(36) \qquad \frac{\partial(p_x-p_z)}{\partial x} < 0$$

The only case in which (36) will not hold is to phrase it more suggestively but less exactly, if the expectations of futures-speculators are much more sensitive to changes in stocks than the expectations of cash-speculators.

The inverse relation between stocks and the basis given by (36) has been noticed by other students of futures markets, particularly by Working [9]. His interpretation as a "supply function of storage" is seemingly quite different, though actually a reconciliation with the above view may be possible. A similar remark applies to the interpretation in terms of "convenience yield" put forward by Kaldor [5].

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10 The ratio of the second term in (34) to the second part of (35) bears some resemblance to Hicks' "elasticity of expectations." Thus a sufficient but not necessary condition for (36) is that this ratio does not exceed one.

An immediate corollary of (36) is that a variation in stocks will change the spot price more than the futures price.

It also means, by (13), that short hedging will increase with stocks.

The static theory given here is still incomplete. Many interesting problems, such as the relation between different futures prices (as distinct from the relation between "the" cash and "the" futures price discussed here) await consideration. For these purposes, however, it is convenient to introduce dynamic elements.

In conclusion I wish to express my indebtedness to Lester G. Telser for many helpful suggestions, criticisms and comments.

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