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Optimal Routings of Empty Boxcars, 1940-1950*

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Summary: This paper reports an attempt to extend the earlier work of Kirk Fox [1] on quarterly optimal routings of empty boxcars for the year 1949. A simpler, though admittedly less accurate, computational method of estimating surplus and deficit boxcars for each region of the United States was developed, and the ease of this method permitted us to find optimal routings for each year in the period 1940-1950, under the assumption that the flows of full freight would remain constant, indefinitely, at the level to which they attained in any particular year for which the optimal routing was computed. The *raison d'etre* of the study was to see whether there was sufficient stability in the optimal routings, in a period when great strain was put upon the transportation system to warrant further investigation with the ultimate purpose of using such optimal routings as obtained to improve the allocation of resources in the railroad industry. It should be emphasized

* I am greatly indebted to C. B. McGuire, under whose direction the early portions of this study were carried out, and to Martin Beckmann and Kirk Fox for very helpful and stimulating conversations and correspondence. A great deal of credit is due to the computing staff, particularly to Bill Foster without whose interest and application this paper could hardly have been completed. I am, however, solely responsible for any errors which occur.

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that the actual routings are, in themselves, of little importance due to the inaccuracies in estimating surplus and deficit boxcars, to the relatively crude approximation of the rail network of the United States by a twelve point network, and to the static nature of the optimal routings themselves. It is mainly stability or lack of stability which concerns us here.

A. Method:

Data on the number of tons of revenue freight originated and terminated in carloads (not by carloads) were available for each of six large commodity groups, by state, for the years 1940-1950, in Statistics of Railways in the United States, (Washington: 1942, ..., 1952). Only Class I line-haul railroads reported. In order to estimate the empty boxcar surplus or deficit for each state and Canada in any given year, it was necessary to transform this available data. To facilitate this we had, for each year 1940-1950, average tons per car for each of the six large commodity groups, from Statistics of Railways in the United States, 1950. (See Table 1).

Tons per Car of Carload Revenue Freight Originated, β^M,

by commodity group, 1940-1950.1/

Year	Products of Agriculture	Animals and Products	Products of Forests	Manufactures and Miscellaneous	Forwarder traffic
1940	27.6	12.9	53.8	31.6	12.1
1941	28.4	13.4	54.2	31.7	12.7
1942	30.7	14.6	54.4	33.0	17.4
1943	34.1	15.4	54.8	34.5	19.1
1944	32.9	15.5	55.2	34.5	18.7
1945	33.4	15.0	55.5	34.3	18.2
1946	32.6	14.2	55.6	34.5	17.8
1947	34.7	14.3	55.9	34.1	17.5
1948	34.5	14.5	56.3	34.3	17.2
1949	34.4	14.2	56.9	34.0	15.9
1950	33.8	14.3	57.3	33.8	14.3

Table 1

1/ From Statistics of Railways in the United States, 1950, U. S. Government Printing Office (1952), p. 41.

In addition, we had "Carload Waybill Analyses" for the years 1949-1951, which gave us the proportion of boxcars to all cars used in shipping commodities within a given large commodity group. (See Table 2.)

Boxcars/ Total Cars, by Commodity Group, 1949-1951 2/

Year	Products of Agriculture	Animals and Products	Products of Forests	Manufactures and Miscellaneous	Forwarder traffic
1949	0.750	0.081	0.064	0.578	0.962
1950	0.725	0.094	0.062	0.585	0.964
1951	0.739	0.083	0.055	0.558	0.964

Table 2

These analyses were based upon a 1 percent sample of all audited waybills of Class I railroads representing their carload terminations . Data on the various proportions of boxcars to all cars were unavailable for the years 1940-1948.

Since only Class I line-haul railroads reported, there was a discrepancy between tons originated and tons terminated for the whole U. S., **within** each commodity group. If one had summed originations and terminations for any commodity group over each state and Canada, one should have found that total originations equaled total terminations within each commodity group. The actual discrepancy between the two was undoubtedly caused by "leakage" of freight tonnage ~~on~~ to the Class II and Class III railroads (i.e., railroads which do not gross over $1,000,000 a year). Since no data were available for the true originations and terminations, it was decided to adjust the figures initially so that tons originated would equal tons terminated. The initial adjustment was carried out as follows: Let $\bar{O}_i^M$ and $\bar{T}_i^M$ equal the number of tons in

2/ From "Carload Waybill Statistics", statements Nos. 5058, 5159, 5256, File 40-C-8, I.C.C., (Washington: 1949, 1950. 1951).

commodity group M reported originated and terminated in state i, respectively. Then for each M

$$\sum_i \bar{O}_i^M - \sum_i \bar{T}_i^M = C^M ,$$

where C^M equals the discrepancy between the two. To adjust $\bar{O}_i^M$ and $\bar{T}_i^M$, to find O_i^M and T_i^M, respectively, we divide C^M evenly and apportion each half among the $\bar{O}_i^M$ and $\bar{T}_i^M$ according to their percentage of $\sum_i \bar{O}_i^M$ and $\sum_i \bar{T}_i^M$. That is,

$$O_i^M = \bar{O}_i^M - \frac{C^M}{2} \cdot \frac{\bar{O}_i^M}{\sum_i \bar{O}_i^M} = \bar{O}_i^M \frac{\sum_i \bar{O}_i^M + \sum_i \bar{T}_i^M}{2 \sum_i \bar{O}_i^M} ;$$

$$T_i^M = \bar{T}_i^M + \frac{C^M}{2} \frac{\bar{T}_i^M}{\sum_i \bar{T}_i^M} = \bar{T}_i^M \frac{\sum_i \bar{O}_i^M + \sum_i \bar{T}_i^M}{2 \sum_i \bar{T}_i^M} .$$

Clearly, $\sum_i O_i^M = \sum_i T_i^M$.

The transformation of the adjusted tons originated and tons terminated was carried out as follows: Let α^M equal the proportion of boxcars to all cars used in shipping commodities in group M for a given year. Let β^M equal the reciprocal of the average number of tons per car for commodities in group M and for a given year. Then

$$\sum_M \alpha^M \beta^M (T_i^M - O_i^M)$$

gives us an estimate for surplus or deficit boxcars attributable to each state i. Data for the β^M were available for each year (Table 1); but estimates for the α^M, based on a 1 percent sample of all **waybills** for terminated freight, were only available for 1949 - 1951, at the time when this study was made. If one merely glances at Table 1, one observes fairly definite trends in the β^M. Is there reason to suppose similar trends in the α^M? There are differences between the α^M for different years and a given M, but if one applies a Chi-square test one finds that all the differences are insignificant at a 5 percent critical level. In other words, it is 95 percent probable that the samples for different years, with proportions α^M of boxcars to all cars for each M, were all drawn from populations with the same proportions of boxcars to all cars for each M. Since the differences between the α^M were insignificant for the years 1949-1951, the provisional assumption that they were also insignificant for the years 1940-1948 was made. On the basis of available evidence, it was felt that this assumption was both necessary and not unreasonable. The transformation constants, $\alpha^M \beta^M$, actually used are shown in Table 3.

Transformation Constants

	M	$\alpha^M \beta^M$
1.	Products of Agriculture	0.021790
2.	Animals and Products	0.005700
3.	Products of Mines	0.001125
4.	Products of Forests	0.015489
5.	Manufactures and Misc.	0.019190
6.	Forwarder Traffic	0.060427

Table 3

A rather serious error was, however, committed when we used the $\alpha^M \cdot \beta^M$ shown in Table 3. As indicated above, the $\alpha^M \beta^M$ were derived from data for data for the whole U. S. For any region i and any commodity group M the use of a U. S. $\alpha^M \beta^M$ introduces an error due to the actual shipments of commodities within class M originated and terminated in i. To be strictly accurate we should have used the type of transformation used by Fox [1] to transform tons into boxcars for each state i.

Fox's method of estimating surpluses and deficits was as follows: Fox had data on tons originated and tons terminated for each of 500 different commodities in each state and Canada for the four quarters of the year 1949 (from Freight Commodity Statistics first, second, third and fourth quarters 1949); he had Association of American Railroads, File: 448-1C, June 9, 1950, which showed as an average for the whole country and for the entire year how many tons of each of the 500 commodities were carried per car; and he had "Carload Waybill Analyses, 1949", Statement 5058, as we did, which showed as an average for the entire year for a 1 percent sample what percent of each of 500 commodities was distributed to each type of car. Fox first aggregated the states i into twelve regions, R, and Canada. He then subtracted tons originated from tons terminated for each region R and for each of the 500 commodities, j. Fox was able to arrive at $\alpha^j \ \beta^j$'s for the U. S. and the entire year from the last two sources of data mentioned above. He was then able to transform surplus and deficit tons for each R and each j into surplus and deficit boxcars for each quarter. Fox then aggregated over commodities j to find surplus and deficit boxcars for each region R. In a word, Fox's estimate for each R was

$$\sum_j \alpha^j \beta^j \sum_{i \in R} (\bar{T}_i^j - \bar{O}_i^j).$$

Fox did not, as we did, initially adjust his figures on tons; hence, it was necessary for him to adjust the final result. In doing this final adjustment he also adjusted out the Canadian deficit. Fox's adjustment was carried out by assuming that the percentage error by which the surpluses were too small, x, was equal to the percentage error by which the deficits were too large, x, yielding the following relation:

$$(1 + x) \Sigma \text{(surpluses)} + (1 - x) \Sigma \text{(deficits)} = 0$$

whose solution gave the correction factors 1 + x and 1 - x to be multiplied by each of the surpluses and deficits respectively.

By taking surplus and deficit figures for the relatively more homogeneous j rather than the aggregate M, Fox could use the U. S. $\alpha^j \ \beta^j$ to transform to boxcars with relatively more accuracy than our $\alpha^M \ \beta^M$. The actual commodity shipments into and out of a state or region within each commodity group M determine a constant $\alpha_i^M \ \beta_i^M$ (or $\alpha_R^M \ \beta_R^M$) appropriate for each i or R. To get an idea of how much we erred in using our $\alpha^M \ \beta^M$ we compared our result, without adjustment, for surplus boxcars in New England due to the shipment of Agricultural Commodities (one of our M) in the second quarter 1949. Due to a minor but unfortunate error in the computation of Fox's surplus figures, comparison necessitated the recomputation of his comparable result. To have been as accurate as possible, we should have found a surplus of boxcars in New England due to the shipment of Agricultural Commodities in the second quarter, 1949 of 20,019; we actually estimated 13,045, using our computation procedure. If we had been interested in arriving at "true" optimum routings, this kind of error would have seriously invalidated our results. Since, however, we were not interested in the actual optimum routings, but only in their stability or instability, we expected, or rather hoped, that our error would be immaterial to our purpose.

It might be useful at this point to make some suggestions for more accurate estimates of surplus and deficit boxcars still retaining most of the computational ease of our procedure. Let α_i^M equal the proportion of boxcars to all cars used in shipping M for a given i. Let β_i^M equal the reciprocal of the average tons per car of M for a given i. It might be possible to estimate these for a given year as follows:

$$\beta_i^M = \frac{\sum_{j \in M} O_i^j \alpha^j \beta^j}{\sum_{j \in M} O_i^j \alpha^j} \qquad \text{and}$$

$$\alpha_i^M = \frac{\sum_{j \in M} O_i^j \alpha^j}{\sum_{j \in M} O_i^j} = \frac{\sum_{j \in M} O_i^j \alpha^j}{O_i^M} ,$$

for originations. $\hat{\alpha}_i^M$ and $\hat{\beta}_i^M$ for terminations could be estimated in an exactly similar way. These estimates would be absolutely correct for the year chosen for estimation; however, they would be incorrect for any other year. Once having done this for one year, however, we could project each β_i^M back along the U. S. trend to arrive at estimates (admittedly incorrect) of the β_i^M for other years. Since we suspect that the α^M do not differ significantly from year to year, we could also make the assumption that the α_i^M did not differ significantly. In this way we might arrive at much better estimates of surpluses and deficits than we actually did, but without the estreme computational difficulty of Fox's method. In a sense, though, by using this method we would be avoiding the possibility of an interesting result. There are considerable differences in the tonnage figures from year to year, and weighting our estimates with one particular year's tonnage figures would give an entirely different result than if we had weighted them with another.

Once estimates for surplus and deficit boxcars for each M and i had been made, it was simple matter to arrive at optimum routings. Boxcars were first summed over M to arrive at total surplus or deficit boxcars for each i. Due to shortness of time, it was decided to use Fox's rail network for the U. S. and his aggregation of i into R (except that Canada's deficit was supplied from region II instead of being adjusted out). However, both the surplus - deficit figures by state and a 42 point rail network are available in my files should it be possible at some future date to use electronic computing facilities to do optimal routings for so large a network. Fox's aggregation of i into R is shown in Table 4 which is followed by a map summarizing it. The rail network used, or the railroad distance between the various representative points, is shown in Table 5. It was assumed for the purposes of this study that distance was an adequate index of the total cost of moving an empty boxcar from one point to another. Consequently, the sense in which the routing plans of this study are optimal is that for a given program of loaded movements the total distance moved by all empty boxcars is made as small as possible, under the assumption that the given program of loaded movements is expected to remain the same for an indefinite period. We thus attempt, given a set of surplus and deficit points in our network, to minimize the number of car- miles used in getting empty boxcars from the surplus points to the deficit points. The theoretical model for this work was developed by T. C. Koopmans [2,3], and the computational technique was that of Dantzig [4]. That the routings given in section B are indeed optimal may be checked using the necessary and sufficient conditions of Koopmans [3], i.e., that each routing plan has associated with it a set of potentials, p_i, such that

Table 4 Regions and Representative Points

Number	State	Representative Point
I	Maine New Hampshire Vermont Massachusetts Rhode Island Connecticut	Boston, Massachusetts
II	New York New Jersey	New York, New York
III	Ohio Pennsylvania Michigan West Virginia	Cleveland, Ohio
IV	Maryland Delaware Virginia District of Columbia Florida Georgia North Carolina South Carolina	Columbia, South Carolina
V	Indiana Illinois Wisconsin	Chicago, Illinois
VI	Alabama Arkansas Kentucky Mississippi Tennessee Louisiana	Jackson, Mississippi
VII	Minnesota North Dakota South Dakota	Minneapolis, Minnesota
VIII	Colorado Iowa Kansas Nebraska Missouri	Kansas City, Missouri
IX	New Mexico Texas Oklahoma	Fort Worth, Texas

Table 4 continued

Regions and Representative Points

Number	State	Representative Point
X	Wyoming Utah Idaho Montana	Ogden, Utah
XI	Arizona Nevada California	Monterey, California
XII	Oregon Washington	Portland, Oregon

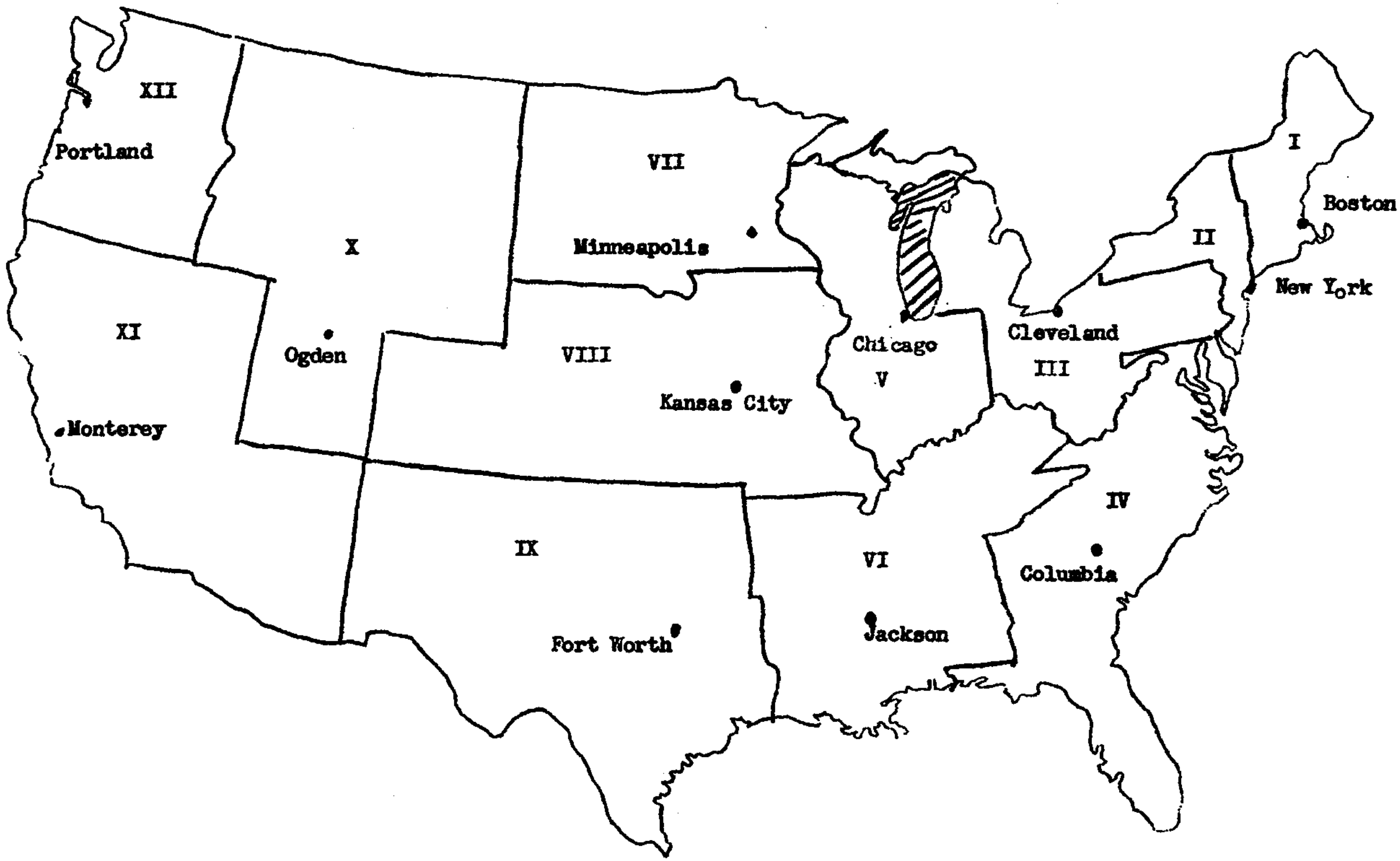

XII
Portland
X
VII
Minneapolis
I
Boston
II
New York
XI
Ogden
VIII
Chicago
V
Cleveland
III
Kansas City
Monterey
IV
IX
VI
Columbia
Fort Worth
Jackson

Table 5 Railroad Distances[3/]

	Boston, Massachusetts	New York, New York	Columbia, South Carolina	Cleveland, Ohio	Jackson, Mississippi	Chicago, Illinois	Minneapolis, Minnesota	Kansas City, Missouri	Fort Worth, Texas	Ogden, Utah	Portland, Oregon	Monterey, California
Boston		232	935	681	1470	1033	1441	1513	1937	2521	3256	3432
New York	232		703	579	1239	909	1317	1365	1773	2397	3132	3308
Columbia	935	703		863	669	886	1294	1163	1106	2265	3050	2916
Cleveland	681	579	863		992	357	765	786	1256	1845	2580	2756
Jackson	1470	1239	669	992		747	1123	726	437	1780	2615	2247
Chicago	1033	909	886	357	747		408	480	1004	1488	2223	2399
Minneapolis	1441	1317	1294	765	1123	408		501	1007	1381	1815	2292
Knasas City	1513	1365	1163	786	726	480	501		536	1169	1954	2080
Fort Worth	1937	1773	1106	1256	437	1004	1007	536		1343	2178	1810
Ogden	2521	2397	2265	1845	1780	1488	1381	1169	1343		851	911
Portland	3256	3132	3050	2580	2615	2223	1815	1954	2178	851		900
Monterey	3432	3308	2916	2756	2247	2399	2292	2080	1810	911	900	

3/ Adapted from the table of the same name in Fox [1] by alteration of distances which did not satisfy the triangular inequality.

(1) $p_i \leq p_j + s_{ij}$ for all (i, j),

(2) $p_i = p_j + s_{ij}$, if $x_{ij} > 0$,

where the s_{ij} are the distances between points in the network and the x_{ij} are the flows between points. The results for each year are given in the form of a flow matrix $\| x_{ij} \|$ where i is a deficit point and j a surplus point. A list of the potentials associated with the optimal routing are given for each point, as well as the number of car-miles associated with the routing. A map of the routing follows the flow matrix for each year.

B. Results:

Optimal Routing, 1940

Surplus Areas

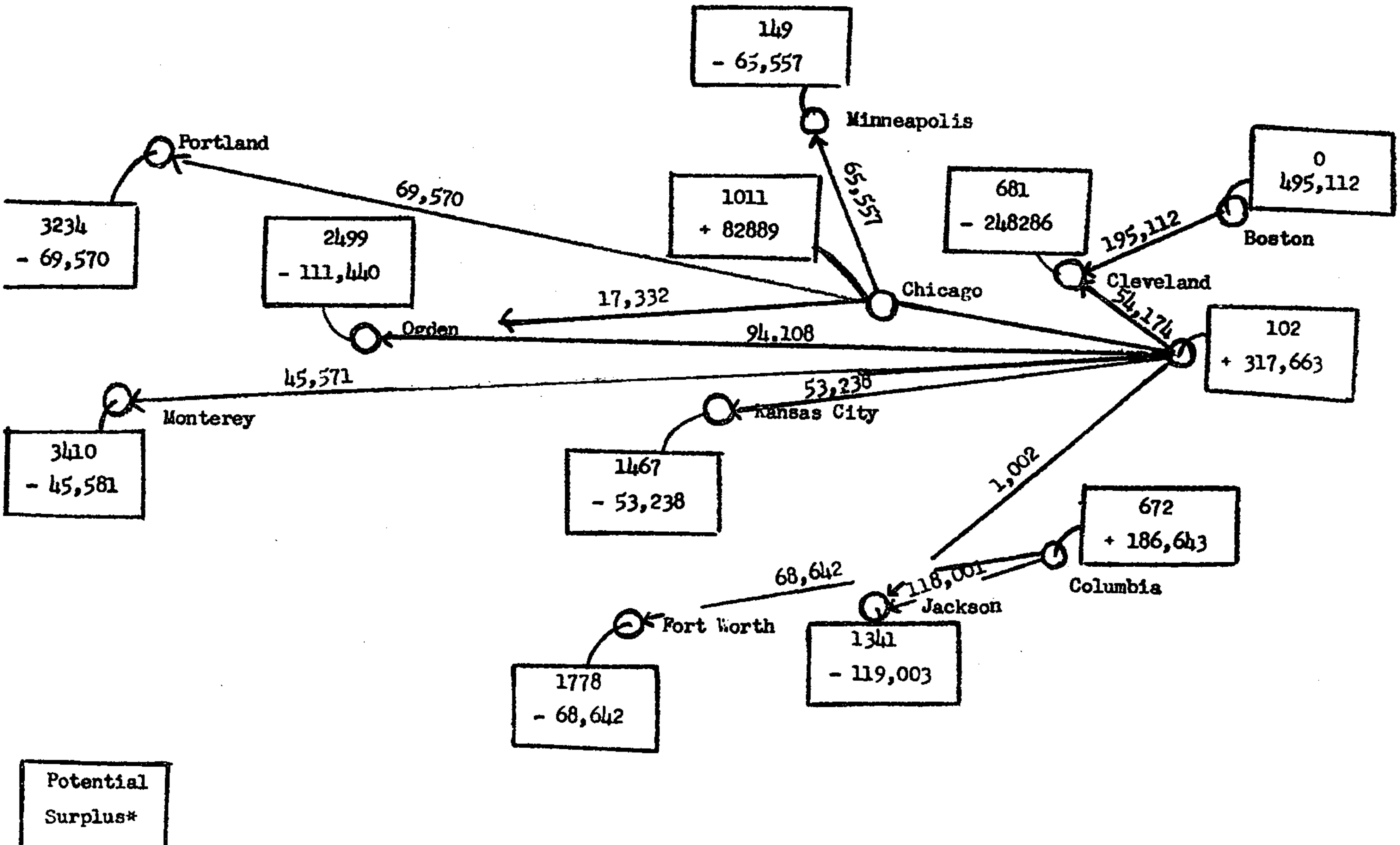
Deficit Areas	I	II	IV	V	Σ
III	195,112	54,174	0	0	249,286
VI	0	1,002	118,001	0	119,003
VII	0	0	0*	65,557	65,557
VIII	0	53,238	0	0	53,238
IX	0	0	68,642	0	68,642
X	0	94,108	0	17,332	111,440
XI	0	45,571	0	0*	45,571
XII	0	69,570	0	0*	69,570
Σ	195,112	317,663	186,643	82,889	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	672	1011	1341	1419	1467	1778	2499	3410	3234

Car-miles = 1,039,766,343

Optimal Routing, 1940



* a deficit is a negative surplus

Optimal Routing, 1941

Surplus Areas

Deficit Areas

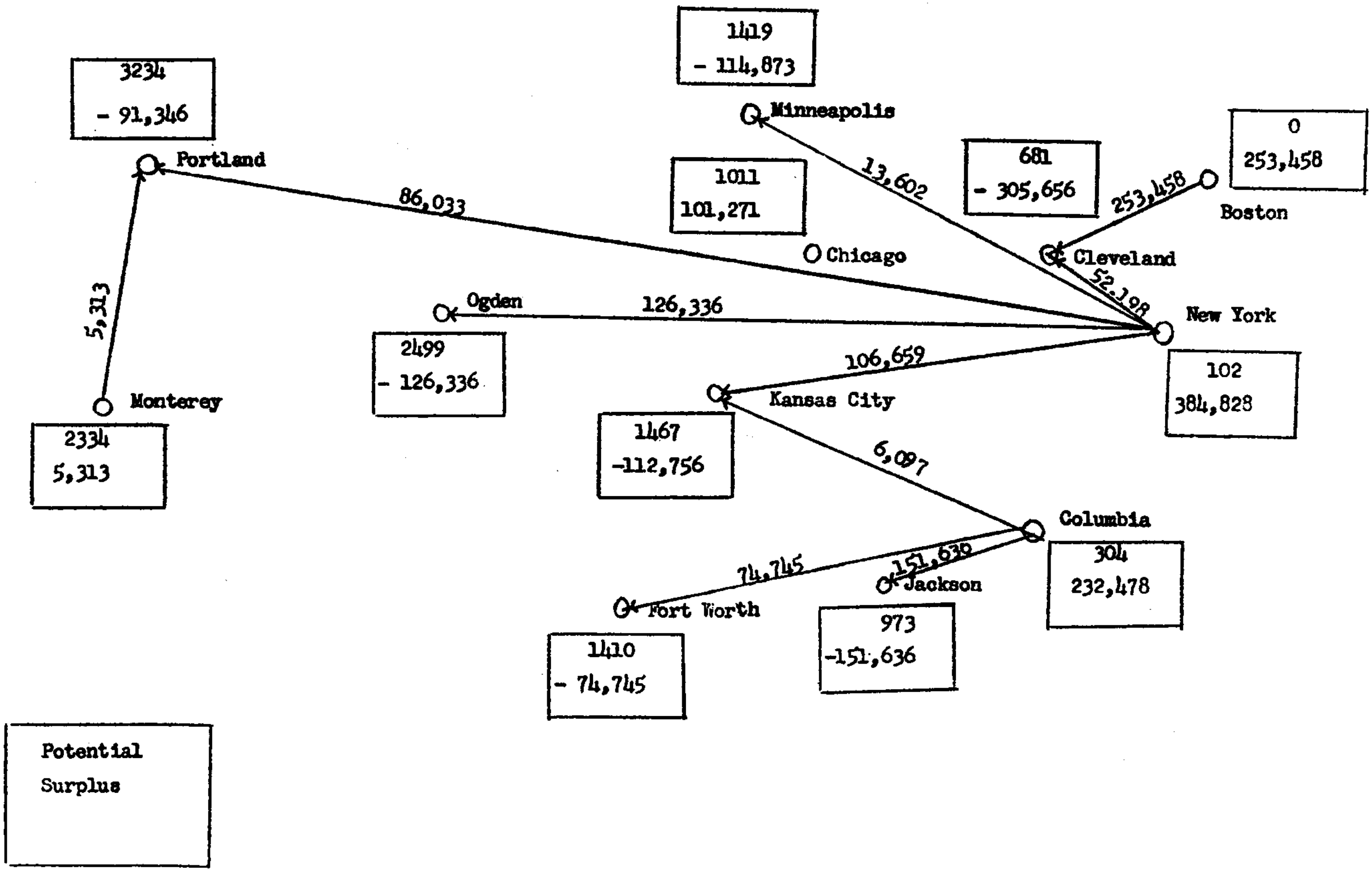
	I	II	IV	V	XI	Σ
III	253,458	52,198	0	0	0	305,656
VI	0	0	151,636	0	0	151,636
VII	0	13,602	0	101,271	0	114,873
VIII	0	106,659	6,097	0	0	112,756
IX	0	0	74,745	0	0	74,745
X	0	126,336	0	0*	0	126,336
XII	0	86,033	0	0*	5,313	91,346
Σ	253,458	384,828	232,478	101,271	5,313	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

Car-miles = 1,175,917,190

Optimal Routing, 1941



Optimal Routing, 1942

Surplus Areas

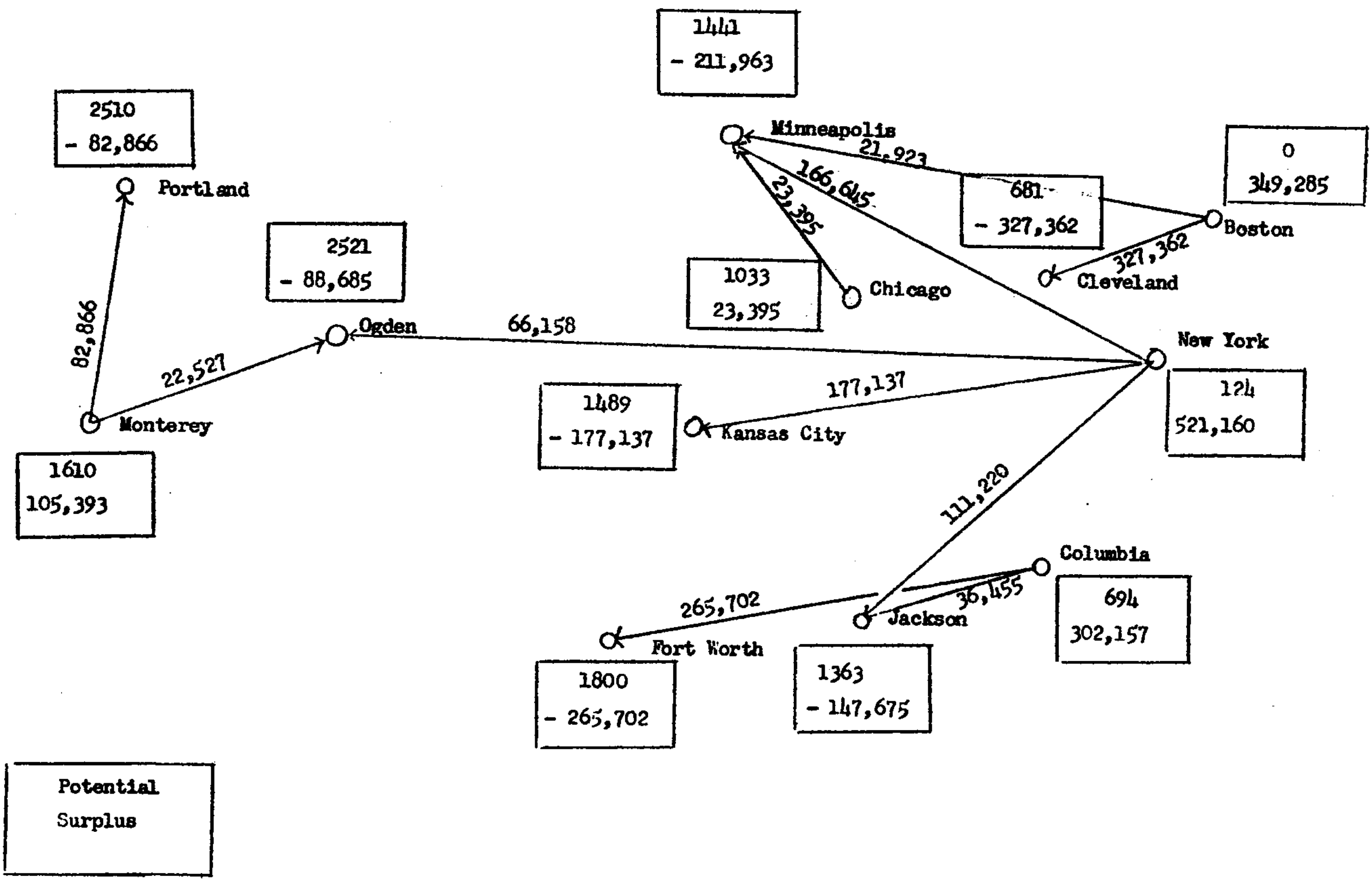
Deficit Areas	I	II	IV	V	XI	Σ
III	327,362	0	0	0	0	327,362
VI	0	111,220	36,455	0	0	147,675
VII	21,923	166,645	0	23,395	0	211,963
VIII	0	177,137	0	0	0	177,137
IX	0	0	265,702	0	0	265,702
X	0*	66,158	0	0*	22,527	88,685
XII	0	0	0	0	82,866	82,866
Σ	349,285	521,160	302,157	23,395	105,393	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	124	681	694	1033	1363	1441	1489	1800	2521	1610	2510

Car-miles = 1,435,071,805

Optimal Routing, 1942



Optimal Routing, 1943

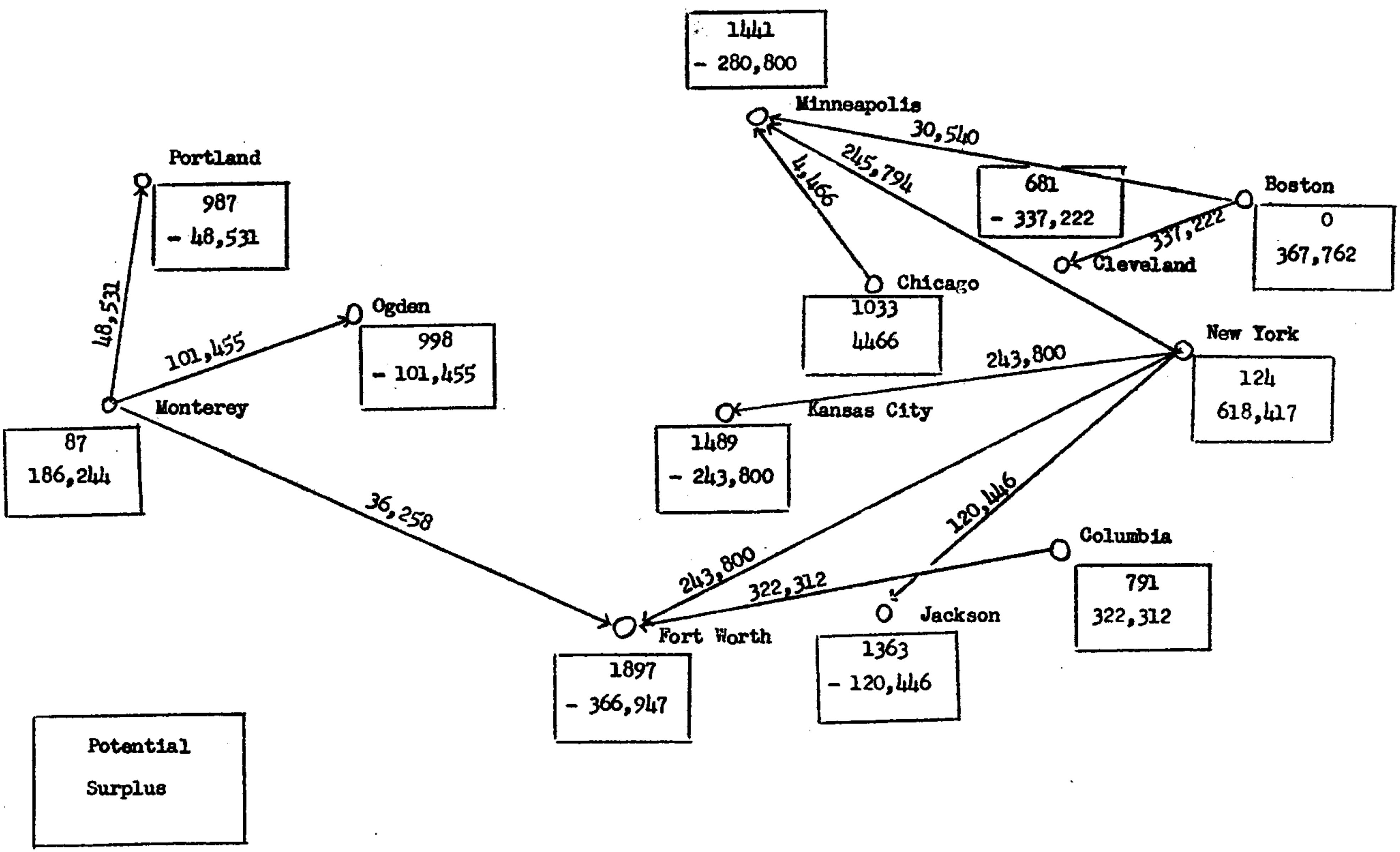
Surplus Areas

Deficit Areas	I	II	IV	V	XI	Σ
III	337,222	0	0	0	0	337,222
VI	0	120,446	0	0	0	120,446
VII	30,540	245,794	0	4,466	0	280,800
VIII	0	243,800	0	0	0	243,800
IX	0	8,377	322,312	0	36,258	366,947
X	0	0	0	0	101,455	101,455
XII	0	0	0	0	48,531	48,531
Σ	367,762	618,417	322,312	4,466	186,244	

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	124	681	791	1033	1363	1441	1489	1897	998	87	987

Car-miles = 1,654,268,620

Optimal Routing, 1943



Optimal Routing, 1944

Surplus Areas

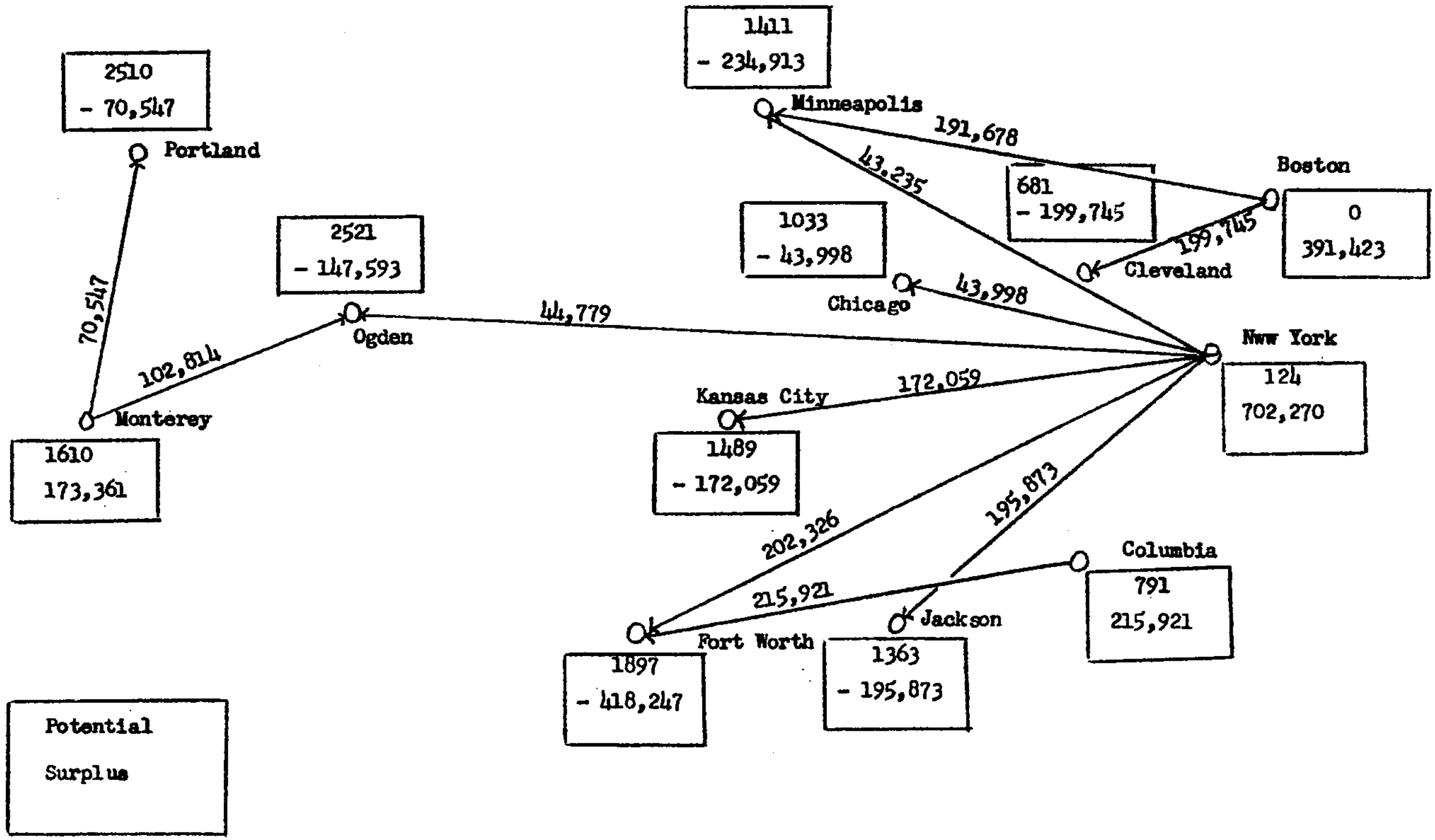
Deficit Areas	I	II	IV	XI	Σ
III	199,745	0	0	0	199,745
V	0*	43,998	0	0	43,998
VI	0	195,873	0	0	195,873
VII	191,678	43,235	0	0	234,913
VIII	0	172,059	0	0	172,059
IX	0	202,326	0	0	418,247
X	0*	44,779	215,921	102,814	147,593
XII	0	0	0	70,547	70,547
Σ	391,423	702,270	215,921	173,361	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	124	681	791	1033	1363	1441	1489	1897	2521	1610	2510

Car-miles = 1,848,739,943

Optimal Routing, 1944



Optimal Routing, 1945

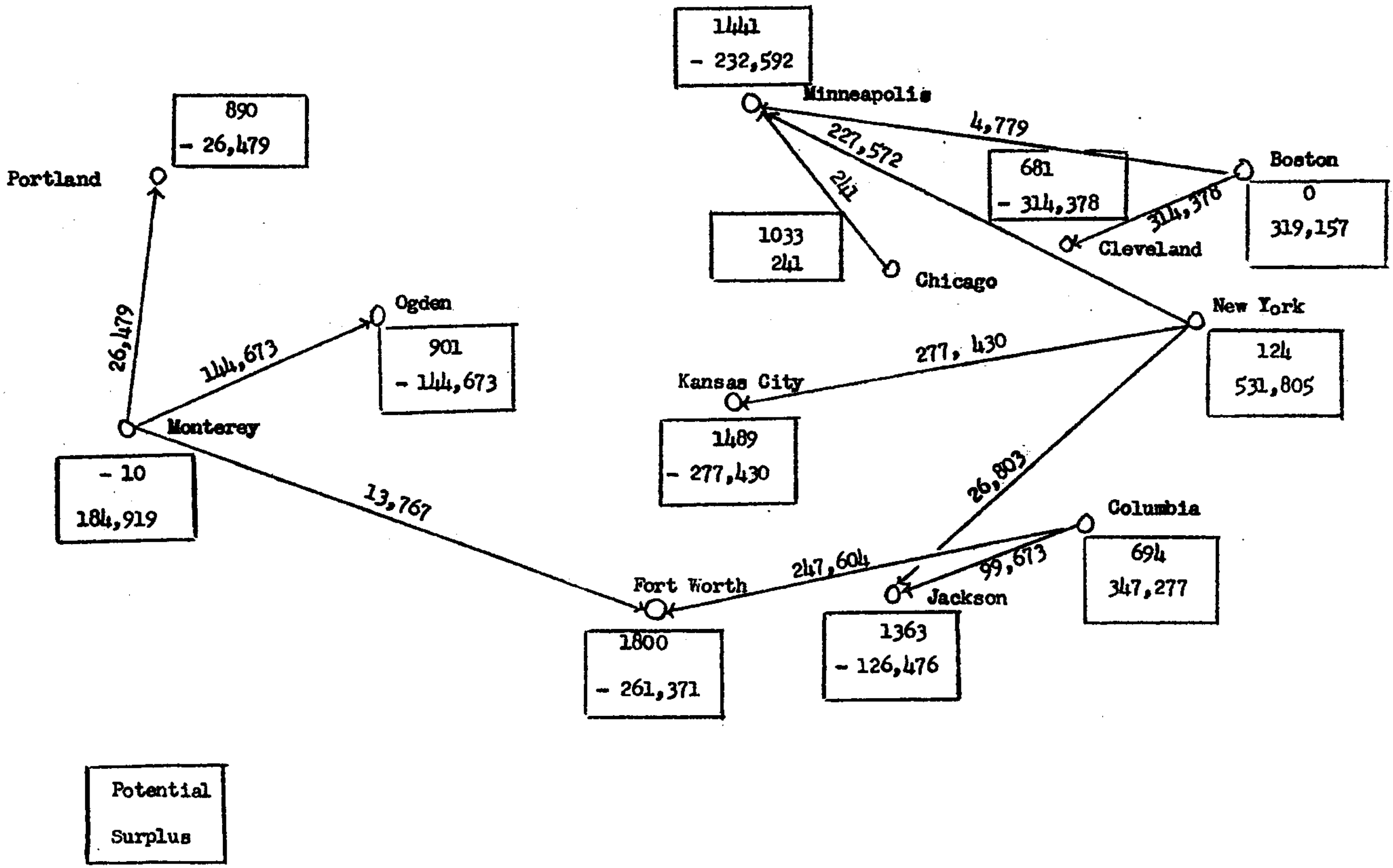
Surplus Areas

Deficit Areas	I	II	IV	V	XI	Σ
III	314,378	0	0	0	0	314,378
VI	0	26,803	99,673	0	0	126,476
VII	4,779	227,572	0	241	0	232,592
VIII	0	277,430	0	0	0	277,430
IX	0	0	247,604	0	13,767	261,371
X	0	0	0	0	144,673	144,673
XII	0	0	0	0	26,479	26,479
Σ	319,157	531,805	347,277	241	184,919	

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	124	681	694	1033	1363	1441	1489	1800	901	-10	890

Car-miles = 1,453,767,210

Optimal Routing, 1945



Optimal Routing, 1946

Surplus Areas

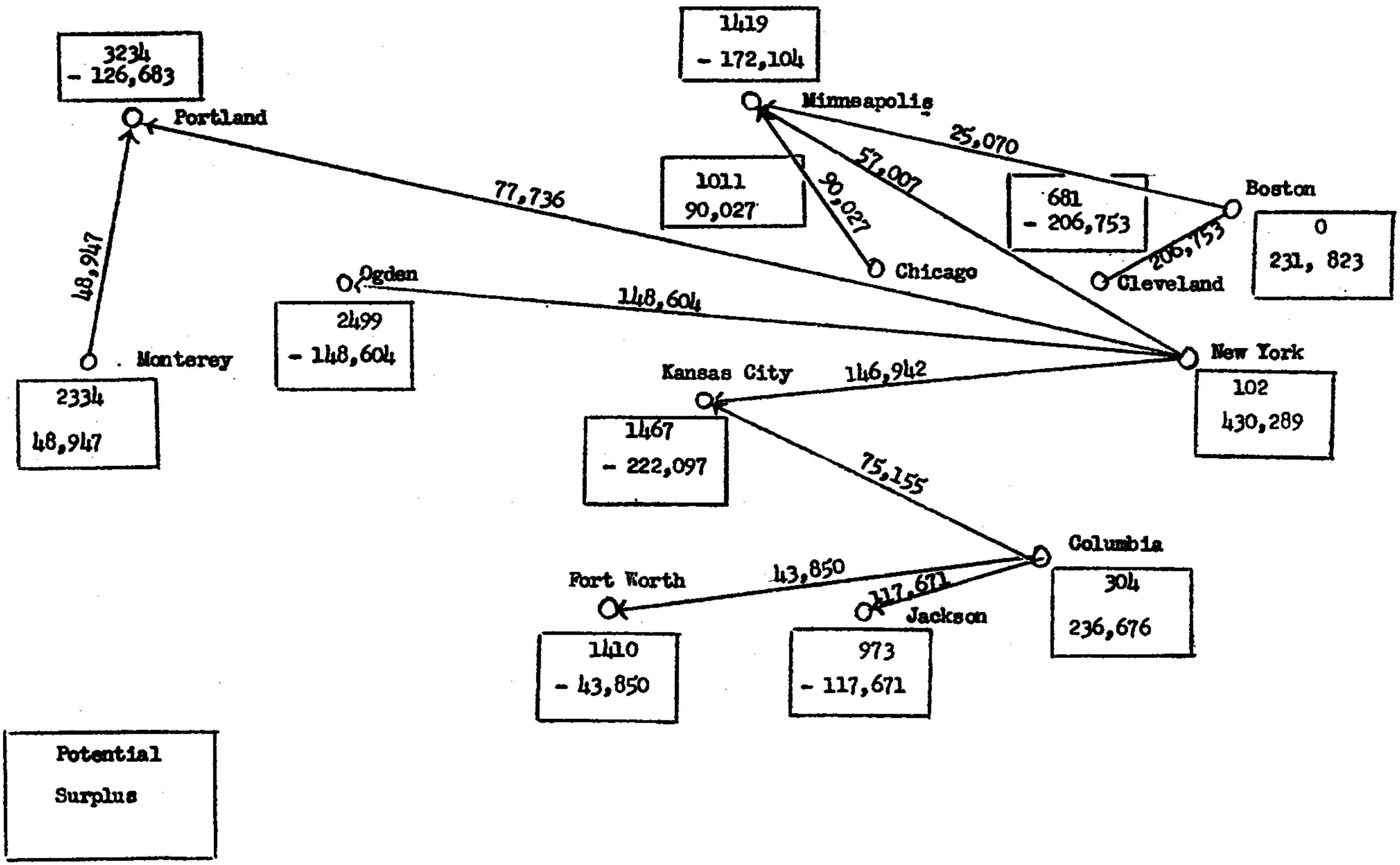
Deficit Areas	I	II	IV	V	XI	Σ
III	206,753	0	0	0	0	206,753
VI	0	0	117,671	0	0	117,671
VII	25,070	57,007	0	90,027	0	172,104
VIII	0	146,942	75,155	0	0	222,097
IX	0	0	43,850	0	0	43,850
X	0	148,604	0	0*	0	148,604
XII	0	77,736	0	0*	48,947	126,683
Σ	231,823	430,289	236,676	90,027	48,947	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

Car-miles = 1,347,108,692

Optimal Routing, 1946



Optimal Routing, 1947

Surplus Areas

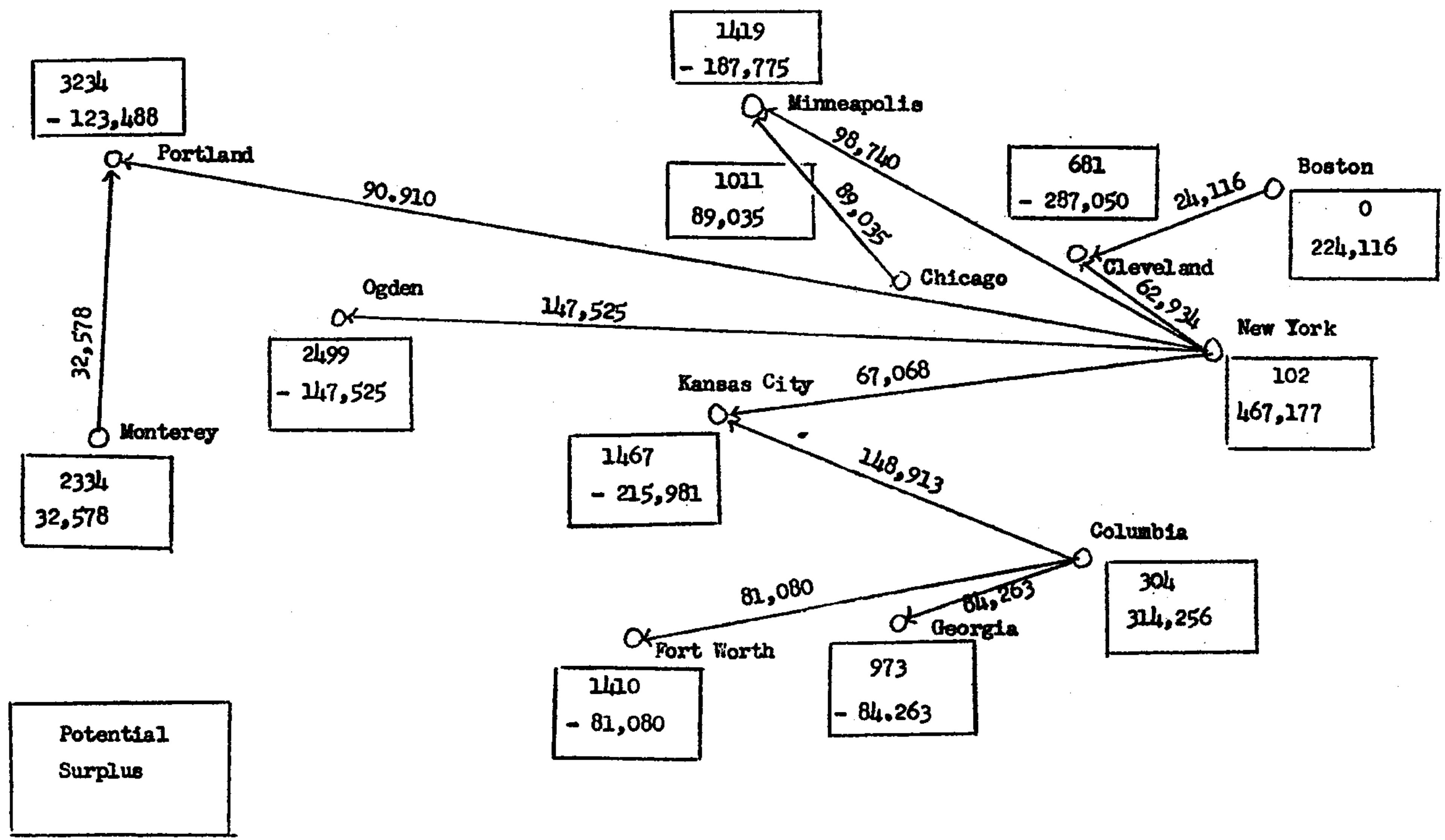
Deficit Areas	I	II	IV	V	XI	Σ
III	224,116	62,934	0	0	0	287,050
VI	0	0	84,263	0	0	84,263
VII	0	98,740	0	89,035	0	187,775
VIII	0	67,068	148,913	0	0	215,981
IX	0	0	81,080	0	0	81,080
X	0	147,525	0	0*	0	147,525
XII	0	90,910	0	0*	32,578	123,488
Σ	224,116	467,177	314,256	89,035	32,578	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

Car-miles = 1,433,876,453

Optimal Routing, 1947



Optimal Routing, 1948

Surplus Areas

Deficit Areas

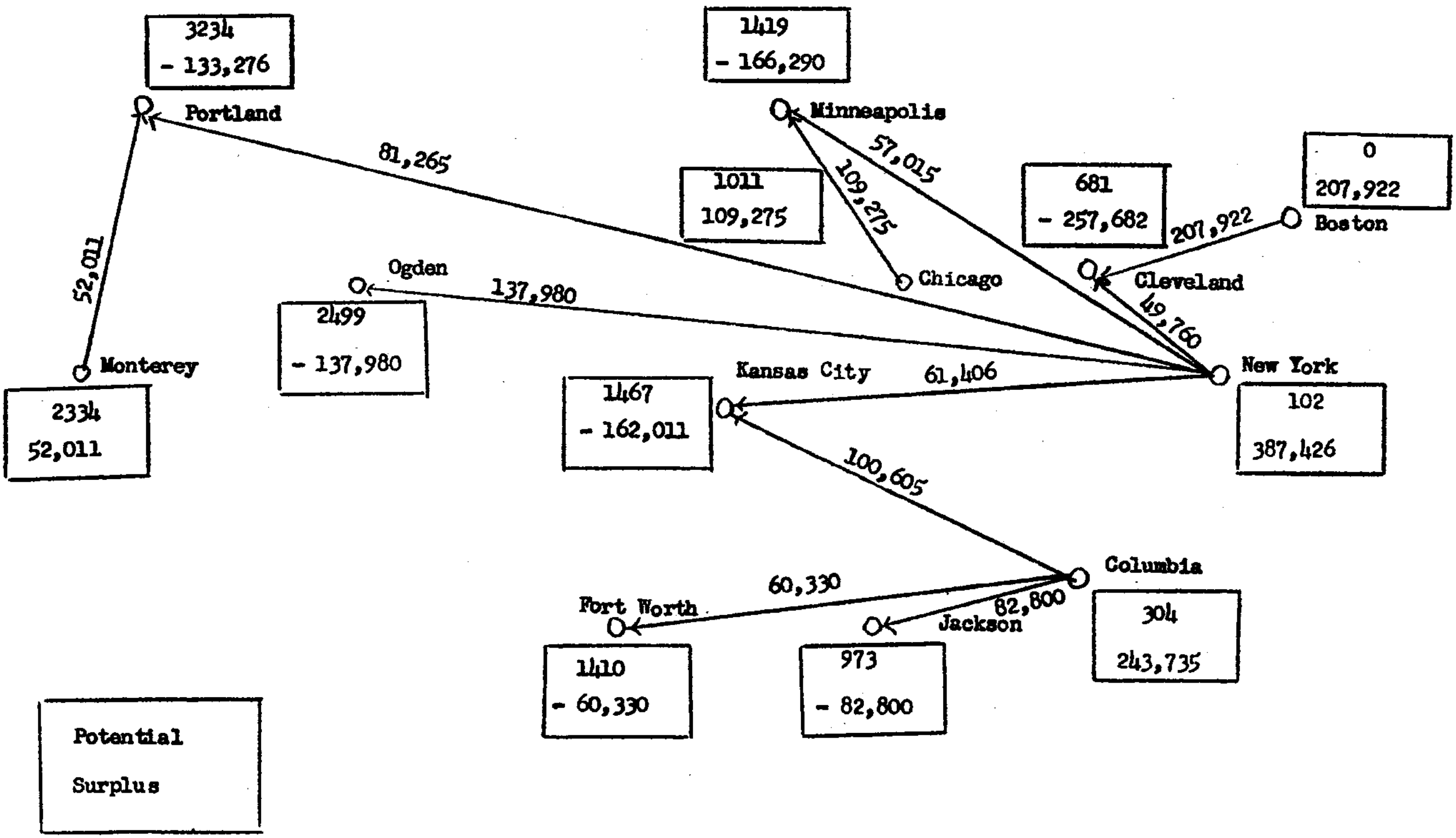
	I	II	IV	V	XI	Σ
III	207,922	49,760	0	0	0	257,682
VI	0	0	82,800	0	0	82,800
VII	0	57,015	0	109,275	0	166,290
VIII	0	61,406	100,605	0	0	162,011
IX	0	0	60,330	0	0	60,330
X	0	137,980	0	0*	0	137,980
XII	0	81,265	0	0*	52,011	133,276
Σ	207,922	387,426	243,735	109,275	52,011	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

Car-miles = 1,245,091,212

Optimal Routing, 1948



Optimal Routing, 1949

Surplus Areas

Deficit Areas

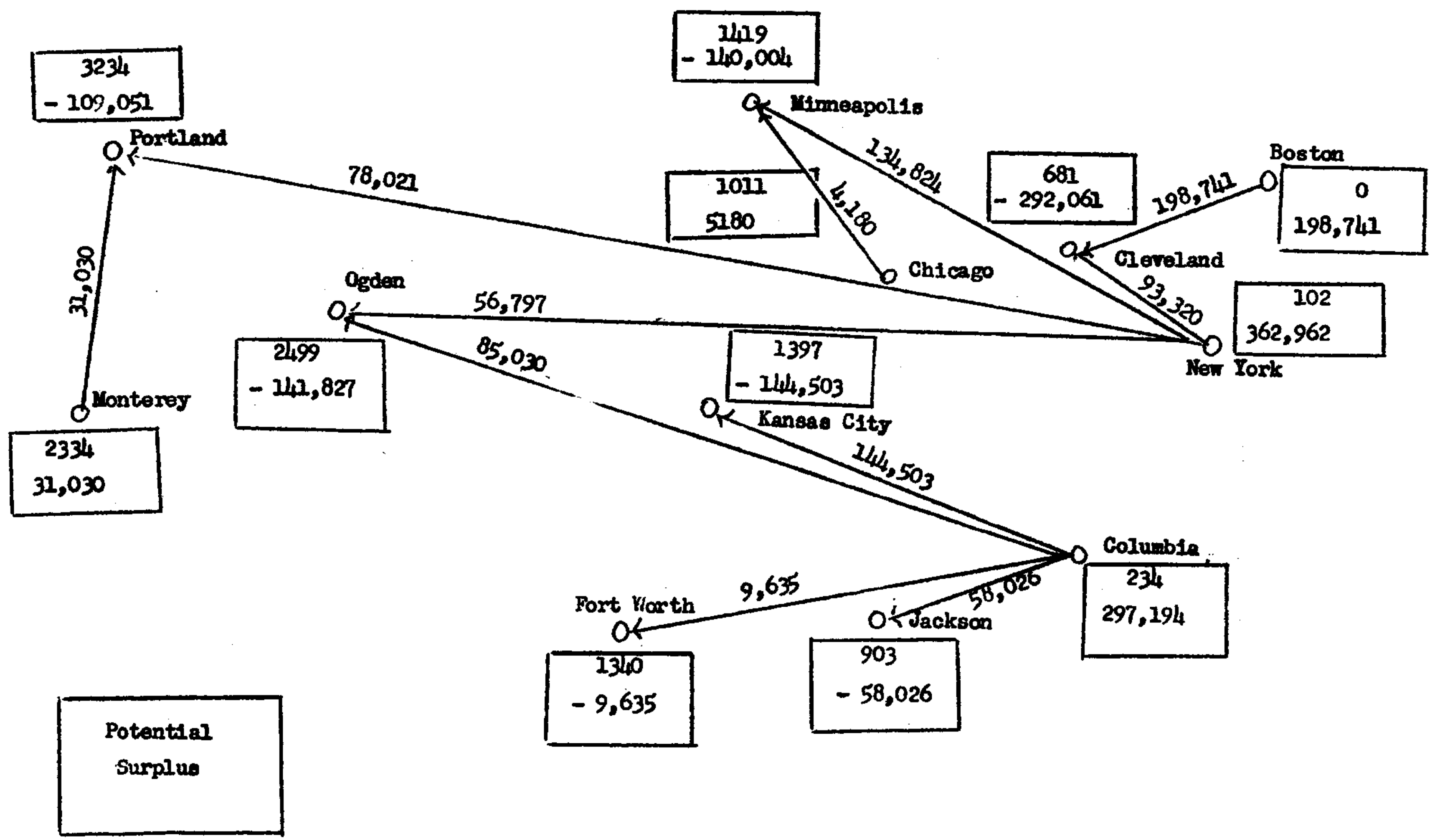
	I	II	IV	V	XI	Σ
III	198,741	93,320	0	0	0	292,061
VI	0	0	58,026	0	0	58,026
VII	0	134,824	0	5,180	0	140,004
VIII	0	0	144,503	0	0	144,503
IX	0	0	9,635	0	0	9,635
X	0	56,797	85,030	0*	0	141,827
XII	0	78,021	0	0*	31,030	109,051
Σ	198,741	362,962	297,194	5,180	31,030	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	234	1011	903	1419	1397	1340	2499	2334	3234

Car-miles = 1,187,608,373

Optimal Routing, 1949



Optimal Routing, 1950

Surplus Areas

Deficit Areas		I	II	IV	V	XI	Σ
	III	205,260	97,808	0	0	0	303,068
	VI	0	0	106,436	0	0	106,436
	VII	0	0*	0	87,176	0	87,176
	VIII	0	6,848	130,946	0	0	137,794
	IX	0	0	15,215	0	0	15,215
	X	0	144,257	0	5,669	0	149,926
	XII	0	68,807	0	0*	45,910	114,717
	Σ	205,260	317,720	252,597	92,845	45,910	

* Neutral Circuit

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Potential	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

Car-miles = 1,092,693,917

Optimal Routing, 1950

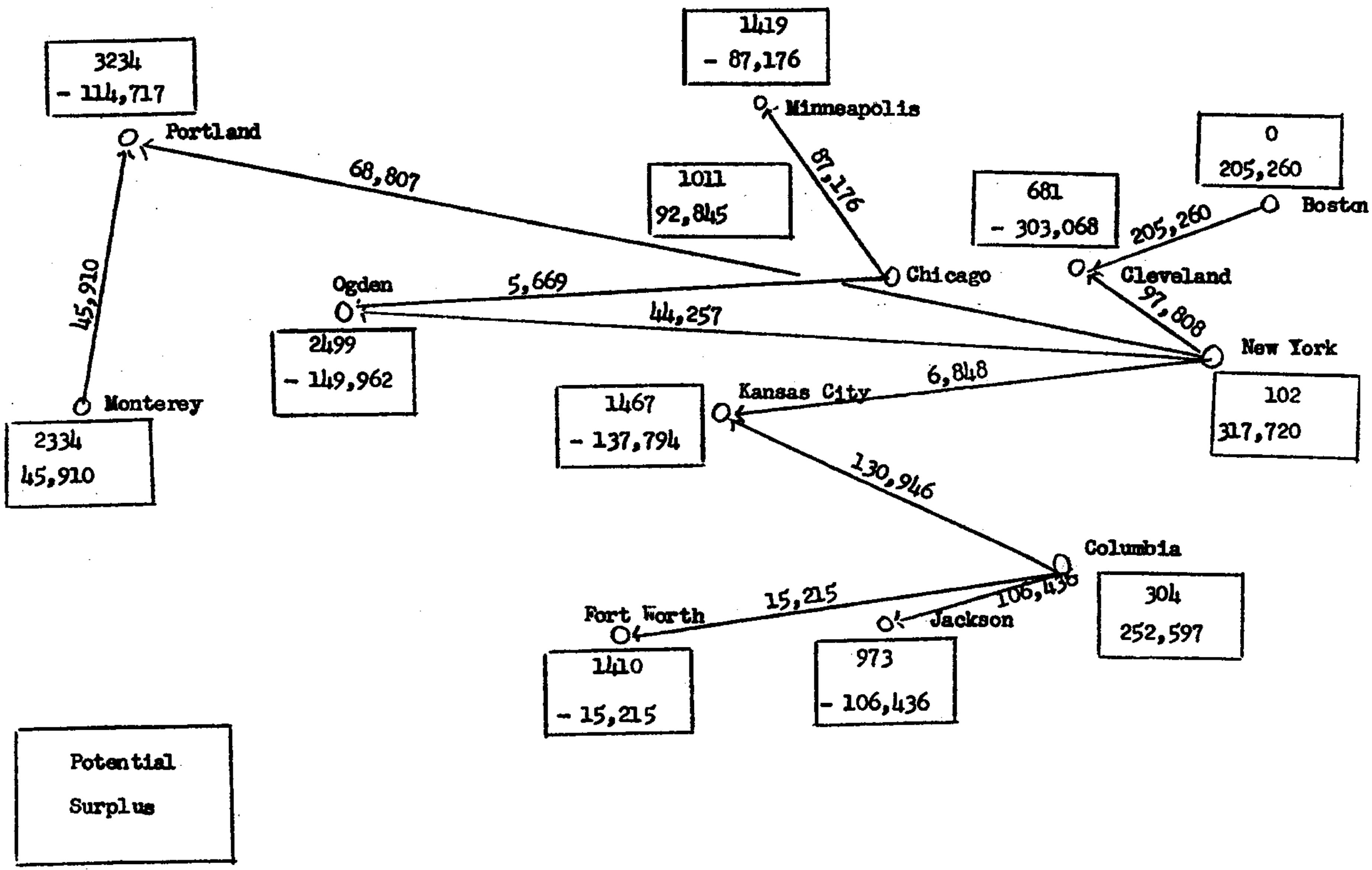


Table 6 Regional Potentials, 1940 - 1950

Regions

Year	Boston I	New York II	Cleveland III	Columbia IV	Chicago V	Jackson VI	Minneapolis VII	Kansas City VIII	Fort Worth IX	Ogden X	Monterey XI	Portland XII
1940	0	102	681	672	1011	1341	1419	1467	1778	2499	3410	3234
1941	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234
1942	0	124	681	694	1033	1363	1441	1489	1800	2521	1610	2510
1943	0	124	681	791	1033	1363	1441	1489	1897	998	87	987
1944	0	124	681	791	1033	1363	1441	1489	1897	2521	1610	2510
1945	0	124	681	694	1033	1363	1441	1489	1800	901	-10	890
1946	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234
1947	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234
1948	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234
1949	0	102	681	234	1011	903	1419	1397	1340	2499	2334	3234
1950	0	102	681	304	1011	973	1419	1467	1410	2499	2334	3234

C. Conclusions

Except for the two years 1940 and 1944 the pattern of surpluses and deficits shows amazing stability over the entire period 1940-1950 in spite of a fairly definite upward trend in the surplus and deficit figures themselves. It should be mentioned too, that this same stability is true, but to a lesser degree, in the state-wise pattern of surpluses and deficits. In spite of the relative stability of the pattern, the relative magnitudes of the surplus-deficit figures show considerable fluctuation during the war period but amazingly little there after. This is reflected in the relative instability of the potentials for t six year period, 1940-1945 as compared with their extraordinary stability in the five year period, 1946-1950.(see Table 6) The exact duplication of the potentials in all the years after the war except for 1949 indicates identical optimal routings, taking account of neutral circuits. If Fox had chosen a year during the war, he might not have found the quarterly stability he did find.

The results indicate, I believe, that further investigation is justified. The evidence presented here, however inaccurate the routings may be and in spite of the static character of the analysis, seems to indicate that a prediction of the optimal routing for the year ahead, based on the potentials of the previous year's optimal routing, might be used to considerably improve the allocation of resources in the railroad industry, provided the routing and estimation procedures were made a good bit more accurate. There is no way in which we can say, however, by how much allocation would have been improved during the post-war period by the use of such projective techniques; for we do not know nor do we have any way of finding out, how actual shipments of empties were made. The Congressional Hearings of 1946 on the car shortage in the west indicate, however, that serious mis-allocation was taking place.

The case for further investigation with a view to the use of optimal routings during mobilization is less clear. During the period 1940-1945 our routings show the kind of instability likely to be increased by more accurate methods of routing and estimation. It is safe to say that, should the government attempt to impose optimal routings during war, the simple yearly projection would not suffice. Since quarterly data are not available for any years prior to 1946 we have no real way of finding out whether quarterly stability exists even under the distorting influence of war. It seems a good bet that quarterly stability does not exist, in which case the government would have to predict surpluses and deficits for the period during which an optimal routing would be enforced. Again further investigation is warranted, although this time along a different and less superficial tack.

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