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Decision-making in a Non-neutral World[1]

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Consider a decision maker A and the set B = non A of all other decision-makers in the world. The decision maker A might control a set of variables a and the set B a set of variables b. The consequences ξ_b of specific choice of a can to some extent be influenced by b. We assume that the conditional probability distribution of the consequences ξ_b, given a is a function of the form:

$$(1) \qquad P(\xi_b \leq x;\ a, \beta) = F(x,\ a, \beta) .$$

The parameter β introduced in this formula we interprete as an indicator of non-neutrality of B towards A. The value $\beta = 0$ corresponds to neutrality of B towards A. If $\beta < 0$ A is of the opinion that B will control b in such a manner that the resulting probability distribution $F(x;\ a, \beta)$ is less favorable to A than the distribution $F(x;\ a,\ 0)$. If $\beta > 0$ he expects that $F(x;\ a, \beta)$ is better than $F(x;\ a,\ 0)$. The parameter β is estimated by A and these estimates β_a can have a bias $\beta_a - \beta$, which depends on the pessimistic or optimistic outlook of A.

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To a special result $\xi_b = (x; a)$ the decision maker assigns a value $U(x; a, \alpha)$ which can vary with the procedure α used for finding the plan a chosen. By increasing α the probability that the best plan will be chosen increases, the variance in $U(x; a, \alpha)$ decreases, but the mean-value of $U(x, a, \alpha)$ has a tendency to decrease because larger values of α corresponds to increasing cost for decision-making. To a special distribution $F(x; a, \beta)$ of ξ_b we assume that A assigns a value

(2) $$U(a; \beta, \alpha) = \int_x U(x; a, \alpha)\, dF(x; a, \beta).$$

A rational decision-making procedure $\hat{\alpha}$ maximise the indicator of goodness (CCDP, Economics 2074).

(3) $$U(\alpha; \beta) = \int_a U(a; \beta, \alpha)\, dF(a; \alpha)$$

where

(4) $$dF(a; \alpha, \beta) = e^{\alpha V_A(U(a; \beta, \alpha))}\, dF(a; 0) \Big/ \int_a e^{\alpha V_A(U(a, \beta, \alpha))}\, dF(a, 0)$$

and

(5) $dF(a; 0, \beta)$ denotes the probability of a choice of a when the decision-making procedure $\alpha = 0$ is used, that is a random choice is made between the alternatives considered, without any significant efforts to find the best a. The monotonically increasing function $V_A(U)$ of U can perhaps be put equal to $U = U(a, \beta, \alpha)$. Ex analogia we can assume that the distribution function $F(x; a, \beta)$ depends on the non-neutrality β of B in a manner corresponding to (4)

(6) $$dF(x; a, \beta) = \frac{e^{\beta V_B(U(x; a, \alpha))}\, dF(x; a, 0)}{\int_x e^{\beta V_B(U(x; a, \alpha))}\, dF(x; a, 0)} \quad ;$$

giving when α is < 0 a higher weight to those results which are relatively bad to A, than in the case when B is neutral to A. In the case when $V_B(U\ (x;\ a,\alpha) = U(x;\ a,\alpha)$ we can express $U(a;\beta,\alpha)$ in the form

$$(7) \qquad U(a;\beta,\alpha) = k'(a;\beta,\alpha) = \frac{\partial k}{\partial \beta}\ (a,\beta,\alpha),$$

where $k(a,\beta,\alpha) = \log \int\limits_x \log^{-1}\beta\, U(x;\ a,\alpha)\ dF(x;\ a,\ 0)$ denotes the cumulant function to the distribution of $U(x;\ a,\alpha)$ in the case when B is neutral to A. The indicator $U(\alpha;\beta)$ of the goodness of the decision-making procedure α given β is then expressable by the formula:

$$(8) \qquad U(\alpha;\beta) = \int k'\ (a;\beta,\alpha)\ e^{\alpha V_A\ (a;\beta,\alpha))} dF(a;\ 0): \int e^{\alpha V_A(k'(a;\beta,\alpha))} dF(a,\ 0).$$

The optimality condition for choice of α can be written in the form:

$$(9) \qquad \frac{\partial U(\alpha;\beta)}{\partial \alpha} = \frac{\partial C_{\alpha\beta}}{\partial \alpha} + r\ \sigma_{k'}\ \sigma_{v'} + \alpha\, \rho\, \sigma_{k'}\ \sigma_{v'} = 0$$

where

$$(10) \qquad \frac{\partial C_{\alpha\beta}}{\partial \alpha} = - \int\limits_a \frac{\partial^2 k(a;\alpha,\beta)}{\partial \alpha\ \partial \beta}\ dF(a,\alpha) - \int \frac{\partial U(a;\beta,\alpha)}{\partial \alpha}\ dF(a,\alpha)$$

denotes the marginal costs of increasing α and r denotes the coefficient of correlation between $k' = k'\ (a;\beta,\alpha) = U(a;\beta,\alpha)$ and $v = V_A(k')$, and $\sigma_{k'}$, σ_v their standard deviations. The quantity ρ is the coefficient of correlation between k' and $v' = \frac{\partial v}{\partial \alpha}$, a coefficient that can be expected to be very small. The same can be said of the standard deviation of $\sigma_{v'}$, of v'. If we neglect the term $\alpha\rho\sigma_k\,\sigma_{v'}$, and assume that $v \approx k'$, we get the result that

(11) $$\frac{\partial c_{\alpha\beta}}{\partial\alpha} \approx \sigma^2_{k'} = \sigma^2_{k'}\,(\alpha, \beta)$$

when α is optimal.

The order of size between the values assigned to different decisions a, is influenced by β, but it seems reasonable to expect that $\frac{\partial c_{\alpha\beta}}{\partial\alpha}$ shall be approximately independent of β and also only weakly dependent on α. σ^2_k is, however, strongly influenced by both α and β. When the distribution F(a, 0) and F(x; a, 0) are unknown, it seems to be a good principle to assume that dF(a, 0) is constant $\neq 0$, for every alternative considered, and = 0, for all alternatives which will not be considered at all during the decision-making activity in question. In respect to the distribution of ξ_b given a, and $\beta = 0$ it is more difficult to make any rough estimates. The method to try to stratify ξ in such strata that A thinks they are approximately equally probable seems to be one of the few methods which can be expected to lead to acceptable results. In a case of strictly opposite interests between A and B (two persons games) where $U(x, a, \alpha) \approx U(b; a, \alpha)$ it seems to be a safe estimate to put $\beta \approx -\alpha$; that is, to assume that others are equally interested in the problem to decrease $U(\alpha, \beta)$ as A is interested to increase this function. If $\beta = -\alpha$ the problem reduces itself to a problem of finding an approximative solution to the equation:

(12) $$\overset{-\alpha}{\underset{\beta}{\uparrow}}\;\frac{\partial c_{\alpha\beta}}{\partial\alpha} \approx \sigma^2_{k'}\,(\alpha, -\alpha),$$

that is to use a stochastic procedure of choice such that the variance of

$k' = k'(a, \alpha, -\alpha)$ becomes of the same order of size as the marginal costs of efforts to increase α.

The problem of decision making in a non-neutral world can in the case where $U_A(U) \approx V_B(U) \approx U$ be condensed to a problem of finding a decision-making procedure characterized by a parameter $\alpha = \hat{\alpha}$ such that: $u(\hat{\alpha}, \beta) \approx 0$, where

$$(13) \qquad u(\alpha, \beta) = \frac{\partial}{\partial \alpha} \int_{\alpha}^{\alpha} \frac{\partial}{\partial \alpha} \log \left(\int_a \alpha \log^{-1} \frac{\partial}{\partial \beta} \log \left(\int_x \beta \log^{-1} U(x; a, \alpha') \, dF(x, a, 0) \right) dF(a, 0) \right) .$$

In the case when the interests of A and B are opposite it seems reasonable to put $\beta \approx -\alpha$.

In the extreme case when $\alpha = \infty$, $\beta = -\infty$ this principle reduces to a "Max Min" principle and in the case when $\alpha = 0$ to a purely random choice, between the alternatives considered.

If we consider the problem from the point of a neutral observer, the question of which alternatives a and b will be considered at all, that is, will have positive probabilities $dF(a, 0) > 0$, seems to be closely connected with problems of advertising, inventions, and creative processes. In decision-making problems, where similar decision-making situations are frequent, the decision maker can use information gathered in the past to get better and better estimates about the probability distribution $F(x; a, \beta)$. The decision maker will then be involved in a learning process, which results in increasing values of α and more precise estimates of the relative goodness of different decisions. The question if $dF(a, 0)$ is approximately constant for every alternative considered; in the case when very little efforts are made to find optimal decisions, is closely connected with the question how decision-making activity is influenced by the social pressure field, in the case when the decision maker does not play an active role purposively directing his parameters of action.

The question how to assign numerical values to specified pairs (x, a) is perhaps the most difficult part in rational decision-making. Also at this point in the decision-making activity the social pressure field, education, and earlier experiences can have a great influence on a decision maker who tries to find a rational decision-making procedure.

<u>NOTE:</u> It would perhaps have been more realistic to assume that the value $u_{a;\alpha\beta}$ assigned to a probability distribution is a stochastic variable, and let $U(a;\beta,\alpha)$ denote for instance the median value corresponding to the distribution

$$G_{\alpha\beta}(u;\ a) = e^{-L_{\alpha\beta}(u;\ a)} = P(u_{a;\alpha\beta} \leq u).$$

If we denote by

$$G_{\alpha\beta}(u;\ A) = e^{-L_{\alpha\beta}(u;\ A)} = P\left(\operatorname*{Max}_{a \in A} u_{a;\alpha\beta} \leq u\right)$$

$$G_{\alpha\beta}(u;\ A-a) = e^{-L_{\alpha\beta}(u;\ A-a)} = P\left(\operatorname*{Max}_{a' \in A-a} u_{a';\alpha\beta} \leq u\right)$$

then the probability that a specific plan <u>a</u> shall be assigned the highest value of all the plans A considered is

$$dF(a;\alpha,\beta) = \int_{u=-\infty}^{\infty} e^{-L_{\alpha\beta}(u;\ A-a)}\, d\, e^{-(L_{\alpha\beta}(u;\ A-a)-L_{\alpha\beta}(u,\ A)}$$

$$= \int_{u=-\infty}^{\infty} e^{-L_{\alpha\beta}(u,\ A)}\, d(L_{\alpha\beta}(u;\ A) - L_{\alpha\beta}(u;\ A-a))$$

$$= 1 - \int_{u=-\infty}^{\infty} e^{-L_{\alpha\beta}(u,\ A)}\, dL_{\alpha\beta}(u;\ A-a) \quad .$$

In the case when $u_{a,\alpha\beta}$; $a \in A$ are mutually independent we have

$L_{\alpha\beta}(u;\ a) = L_{\alpha\beta}(u;\ A) - L_{\alpha\beta}(u;\ A-a)$. If for instance

$$L_{\alpha\beta}(u,\ A) - L_{\alpha\beta}(u;\ A-a) = L_{\alpha\beta}(u;\ A)\ dF(a;\alpha,\beta)$$

we can make the assumption made earlier about $dF(a;\alpha,\beta)$ without contradictions with this more general approach. An especially simple and interesting case we get when

$$L_{\alpha\beta}(u;\ A) = \Big(\sum_{a \in A} e^{\alpha(U(a;\alpha,\beta)-u)}\Big) \log 2$$

in which case $U(a;\alpha,\beta)$ corresponds to the median of $G_{\alpha\beta}(u;\ a)$ and the variance of $u_{a;\alpha\beta}$ is proportional to $\frac{1}{\alpha^2}$. It is perhaps reasonable to assume that in many cases α^2 is proportional to the time used per alternative considered for finding the alternative, which at the point of time when the decision is made, is believed to be the best alternative.