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Introduction to a study of Decision Making [1]

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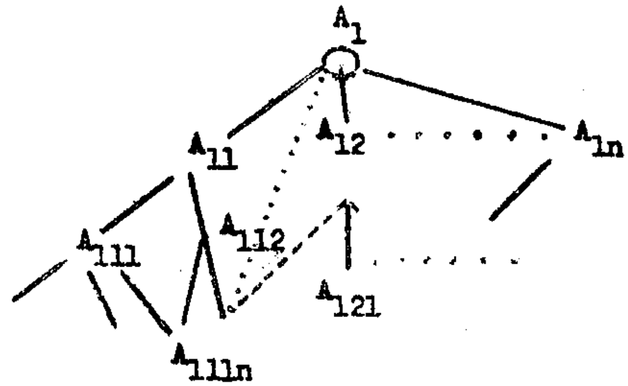
1. The Primary Concepts of Economics

1.1. The Concept—An Economic Subject

The physical or judicial persons, associations and unions of persons, which have ability and will to make economically important actions, we will call "economic subjects." The economic subjects are more or less dependent of each other. That means that the subjects are in some respects directly or indirectly subordinated to some superordinated subjects and are generally themselves superordinated to some other subjects. The economic subjects are thus, if we observe their mutual dependence, parts of more or less firmly organized societies. The organization means an expansion of the ability to act for superordinated subjects and some restrictions and regulations on the freedom to perform actions by subordinated subjects.

1.2. We will use the term "institutional frame" as a name for the distribution of power and the regulations of the interdependence between the economic subjects. The institutional frame is of course changing during time but many characteristics of this frame will generally be constant during long periods of time. We could perhaps give a graphical illustration of the institutional frame of the subjects by a diagram of "network type." [2]



1.3. The existence of an institutional frame of such a complex nature has to be taken in account if we will construct a general theory for the activities of economic subjects including devices for how to rationalize the "activity of decision making." The activity we call "decision making" is an "intellectual process" performed by the persons, which by their behavior can influence the activities of the economic subjects, resulting in some changes in these activities or in the distribution of "objects of value" between the subjects.

1.4. The concept of an economic circle that is a set of economic subjects with some common attribute.

We suppose that every economic subject A_α that exists at point of time t has been put on cards, on which many attributes characterizing the subjects are marked. We can then group the cards according to some combination of these attributes. The set of subjects corresponding to such a group of cards we will call an economic circle. Subjects belonging to an economic circle C_a will thus have one or more attributes a in common. Of specific interest is the "central directed" circle C_α which has a specific subject A_α as a directly or indirectly superordinated subject.

To an economic circle C_a corresponds a complement circle C_a^* to which all other subjects except those of C_a belong. This complement circle to C_a could be called the external economic circle. If we, to the cards of the external economic circle C^*, add a card $\overset{*}{C}$ representing the nature we will get a group of cards representing the external world $\overset{*}{C}_a = \overset{*}{C} + C_a^*$ to C_a. Economic centrally directed circles C_α containing subjects, which are not subordinated to any subject in the external circle C_α^*, may be called organizationally complete economic circles.

1.5. Characteristics of the institutional frame.

The institutional frame of an economic subject A is built up by the restrictions on the freedom of the subject to act and the expansions of its ability to perform actions which need cooperation of many subjects, made possible due to the existence of the frame. If the position of the superordinated subjects to A_α is relatively weak or if it is, notwithstanding that it, in principle is strong, in practice will be used only in few cases to restrict the activities of A_α we say that the "upward" institutional frame of A_α is liberal. The subordinated subjects to A_α can nevertheless have a nonliberal "upward" institutional frame because the power position of A_α over them may be strong. The downward institutional frame of A_α could be "socialistic."

In the "socialistic" circles the power to act has been concentrated to a very high degree to the superordinated subjects of the highest order.

The subordinated subjects have, however, in general the possibility to influence the decision of superordinated subjects by mutual cooperation, petitions, negotiations, and other actions. In the democratic circles these possibilities are a part of the institutional frame and are effectively used. In circles built according to the leader principle the possibility that the subordinated subjects can effectively influence the decisions of the leaders are relatively small. The institutional frame of an organizationally complete circle at a determinate point of time is a combination of all these types of interdependence. Because there exists continuous transitions between these types of frames it seems to be impractical, if not absolutely necessary for the study, to make any specifications about the type of institutional frame of the subjects in question. If we try to study the influence of the institutional frame on the activities of the subjects, we could perhaps start with extreme types of frame and thereafter study how the results change by a continuous transition from the one extreme frame to the other.

The institutional frame is changed by legislation, agreements, integration of new economic subjects, and their disintegration. The struggle of the subjects to get a more favorable frame will often result in changes of the frame.

2. The social pressure field influencing the subject A

The subject A_α considered is a human being or a group of human beings gathered to make decisions in the name of A_α . The subject A_α is not isolated from other subjects, but receives frequently information of many kinds from them. This information includes also recommendations, advice, and announcements about rules of action of other subjects. Thus, between A_α and other subjects belonging to the set A there exist mutual, easily changing, relationships, which are not all of such a firm nature that we can include them in the institutional frame of A_α. We have to consider them as a special kind of interdependence between the subjects A_α in the set A, influencing the decisions of A_α. We can perhaps use the term "social pressure field" to denote all opinions, advice, norm systems, ideologies, advertising, habits, etc., which influence A_α, when A_α makes decisions. These factors would not easily be expressed as a set of time functions which could be considered as input for the decision making activity in question, but are more or less permanent parts of the "machinery of decision making for the subject A_α."

All these factors in the social pressure field are such that they influence the decision making more or less unconsciouly making the decision of A_α to some degree an automatic response to the situation. It could be questioned whether we can, in such a case, speak of a purposive decision at all, rather than of a response to some observations of the subject A directed by the social pressure field. Such short-cut decisions are not results of such purposive intellectual processes of independent thinking, feeling, and judging which we have in mind when "we study the problem of decision making," but could be products of such a process done earlier by the same or some other subject. The existence of a social pressure field results thus in a pattern of behavior, which makes it possible for the subjects A_α to act without much real independent decision making.

The use of short-cut decisions can sometimes result in a more satisfactory path of action than if the subject A himself, had tried to perform a real decision making process, every time a slight need for doing so had been felt. But if all the subjects in A would have used only "short-cut" decisions we can be sure that the development of the world happenings would not have been as satisfactory as the actual historical development especially not in the Western World.

A necessary condition for really good decision making is thus that the decision maker is able to resist the social pressure field to some degree, but at the same time take it in account as a source of already worked out and tried rules of action, which perhaps, but not necessarily, include the best solution of the problem in question. Another necessary condition is to resist the temptation to avoid the trouble of careful judgment by letting uncontrolled variables of random character wholly determine the choice between alternatives not effectively excluded by the social pressure field.

3.

Some Remarks About the Set of Time Functions $x_{\to t}$ Describing the World Happenings Until Point of Time t.

The exact and precise history of the world happenings until point of time t is a set of time functions that can be classified in two fundamental classes $x_{\to t} = [x^{I}_{\to t}, x^{II}_{\to t}]$.

The first set contains time functions which, in principle, are observable at every point of time t. It is not necessary to know the world happenings during a period of length $\tau > 0$ to get an estimate of the value $x^{I}_{t_v}$ of a function $x^{I}_{\to t}$ at time points $t_v \leq t$. We say then that $x^{I}_{t_v}$ = exists, and is usually $\neq 0, = \infty$. Time functions belonging to this set we could call directly observable instant functions. x^{I}_{t} changes only slowly with t, during periods of time long enough for making an observation. (Ex. 1. The cash of an individual)

The second set of functions $x^{II}_{\to t}$ are observable only if we observe the world happenings during a period of time $\tau > 0$. If the observation period τ decreases toward zero, we usually cannot make any observation of interest. (Ex. 2. The payments for some group of goods bought by an individual). Time functions belonging to the set of interval functions can thus be expressed as functions $x^{II}(t - \tau \longrightarrow t)$ of the interval $(t - \tau \longrightarrow t)$ during which observations have been made. When $\tau \longrightarrow 0$ they usually "degenerate" to 0 or ∞. When $\tau \longrightarrow \infty$ this function, however, converges to functions of t, which in principle can be expressed numerically by a number $x_t^{II} = x^{II}(\longrightarrow t)$. The whole set of values of an interval function $x^{II}(t - \tau \longrightarrow t)$ when t and τ varies can be considered as the difference

$$x^{II}(t - \tau \longrightarrow t) = x^{II}(\longrightarrow t) - x^{II}(t - \tau)$$

between two values of the function $x_t^{II} = x^{II}(\longrightarrow t)$ for two different points of time. The whole development of the set of functions x_t^{II} from $-\infty$ to t is denoted by the symbol $x^{II}_{\longrightarrow t}$.

The interval functions $x_t^{II} = x^{II}(\longrightarrow t)$ are mathematically similar to the instant functions x_t^{I} but they cannot, without a special "information producing process," be observed at a point of time t, their values are results of observations done during the whole past $\longrightarrow t$. If the observations are done only during the period $t_0 \longrightarrow t$ we can know $x_t^{II} = x^{II}(t - \tau \longrightarrow t) + x^{II}(\longrightarrow t_0)$ only up to an additive constant for the period of observation.

The instant functions x_t^{I} can, however, also be considered as a difference between two interval functions

$$x_t^{I} = \overset{+}{x}^{(II)}(\longrightarrow t) - \overset{-}{x}^{(II)}(\longrightarrow t),$$

where $\overset{+}{x}(\longrightarrow t)$ denotes the sum of all increases of x_t^{I} and $\overset{-}{x}^{II}(\longrightarrow t)$ the sum of all decreases of x_t^{I} during the past if we assume that $x^{I}_{-\infty} = 0$. The problem of making observations and registering world happenings is solved if we can invent methods of producing functions belonging to the set $x^{I}_{\rightarrow t}$ (as an output of a production process, where the functions belonging to the set $x^{II}_{\rightarrow t}$ are used as "inputs"), functions which are such that the observation of the values of the set of functions x^{I}_{t} gives us all the information about the past $x_{\rightarrow t}$ we can use, when we try to make good decisions about how to influence the world happenings in the future $x_{t \rightarrow}$. By further processing of the information x^{I}_{t} it is possible to get results, that extends the set x^{I}_{t} to a new set $X^{I}_{t+\tau'}$ which can be more readily usable and therefore more valuable for solving decision making problems at point of time $t + \tau'$. The purpose of a process for gathering information about the past is thus to get a set of directly observable instant functions of time which, in principle, contain all such information about the past, which seems to be useful in a process of decision making influencing the future happenings. The set of all available directly observable instant functions x^{I}_{t} we call our observable data at point of time t. A look at our data gives us the input we use in the "intellectual production process," that aims to produce an output of decisions.

In the mind (memory) of the decision maker there may be other intellectual products similar to observed data, but they are perhaps better to be considered as belonging to the "storages and machinery of the decision producing factory." As soon as the decision producing factory decides to put the results of the activity of the mind in the form of observable instant functions, we get new data usable as inputs in later decision making processes.

It is perhaps necessary to point out that data always are facts in one sense of this word, but the message about the past, which we get by a process of interpretating them as pictures telling us how not directly observable interval functions have developed can be misleading and usually gives us only a rough and always only an incomplete picture of the past development. This is also true of the "pictures" of the past we have stored in our mind. Our memory is seldom correct and the picture will change during the elapse of time becoming often more and more biased and ultimately so unreliable that it will be useless as a data for decision making.

4. On possible transformations of information about the past to get in a more readily usable form.

The persons active in developing the theory and practice of statistics and of all other sciences concerned with the external world are trying to transform and translate the information of mankind about the past to a form, which is more readily usable for decision making purposes than the raw material of information contained in our direct observations about the external world. As a complimentary aim we perhaps have to take into account the purpose to transform them to a form which is more meaningful and easier to grasp than the primary observations. The results of the processing will thus often by themselves satisfy a human need. But, I think, the most important driving force for the activity, of man, which tries to process observations about past world happenings, is the thought, that the results of this activity can be helpful for the end of making better decisions. The need to get such information about the external world which may be useful for this end has also resulted in very considerable expansion of the activity of systematically observing and recording the world happenings, and sometimes also in decisions to perform experiments to get answers to questions which could not be answered without the experimenter's interference. Many decisions made are thus motivated by the hope to increase our capital of human knowledge.

The possibilities to transform the set of time functions contained in our information about the past is practically unlimited. From the given set we can, by using mathematical, statistical and other operations, get a set of time functions of which the primary given set is only a relatively small part. But if such a processing shall not be unprofitable the increase in the value of the knowledge available after the end of such a processing must be great enough to give an excess over the value of the costs of the processing process. This requirement often limits the set of profitable transformations to a relatively small set of operations, which usually means a procedure to concentrate the essence of the information contained in the primary set of observations to a picture, (map, model) or a relatively small set of meaningful characteristics, giving us such information about the primary set, which could not be grasped without further processing of the data. We will not here try to elaborate in detail the possibilities of processing the primary set of information, because such a trial would lead us outside the scope of this investigation, and would probably only cover well-explored ground of statistics and other sciences. But we wish to stress the fact that our information about the past can be processed in many ways to get it in a more useful form as a starting point for decision making. We do not necessarily have to start with the primary given set of functions describing our observations, but with a set of functions which already have been processed and perhaps also observed with the aim to increase the value of our information about the past. The human activity aiming at improving our knowledge is however controlled by decision making. To construct a theory of good decision making for the process of producing useful information is however, I think, the most difficult part in an attempt to construct a general theory of decision making, but perhaps the most urgently needed part because the process of producing information has a key position in the struggle for improving the conditions of human beings. We will later come back to this question in connection with a discussion of the possibilities of improving the social pressure field by education of the decision makers. We shall however first try to investigate the concept "decision" and construct a scheme for "rational" decision making, hoping that we thereafter could say something of interest about this matter.

5. Some Clarifying Remarks About the Decision Concept.

We have already used the concept decision many times assuming that the concept is clear enough for following the general discussion of the decision problem. This familar concept is however not so clear that it could safely be used for more precise attempts to clarify this problem, without a discussion of the set of events classified under the group name decisions.

In the following, we will try to characterize the set of events and sequences of events we think could be included in the set of decisions. This set is in fact a large set containing elements of a highly varying degree of complexity. To give a definition of the set decision, such that it would be easy to say if an event or a sequence of events is a decision or not, seems to be very difficult. For instance, a subject, who does "nothing" during a period of time, can have made a decision to be inactive during that time, but it is (perhaps) possible to do nothing, without making a decision to do nothing. It is also (perhaps) possible to do many things and perhaps to perform a complicated sequence of movements, without interference of decisions made by the active subject during this activity. One possible definition is: a decision is a <u>purposive choice</u> between more than one possibly way of continuing a sequence of movements or nonmovements. The meaning of this sentence is however not fully clear in the case in which the set of theoretically possible ways contain an infinity of different elements, which can be varied continuously inside this set. Nobody can perform an activity exactly according to one uniquely predetermined way in such a set and it is impossible to say if a slight difference between the way actually used and some other way in the set is due to a purpose choice or not. It seems to me, therefore, to be necessary to make a more or less arbitrary classification of such an infinite set into a denumerable set of ways. (Ex. Choice of tolerances, quality control limits, etc.)

One subset of ways in this set may be called a possible "journey." Different journeys may have "junction" places in common. Journeys with the same first junction place after the start of the journey we could consider as a fundamental class of "journeys." A choice of a specific fundamental class may be called an elementary decision. We call it a decision to make a move. But decision makers often, at once, decide to make a whole sequence of moves to be performed one after another without any new act of decision making. Such sequences of moves decided at the same time are usually called "plans of action." The performance of a plan is, however, usually not possible without a large set of secondary decisions necessary to secure that the way actually used, not withstanding the difficulties to follow the plan, really will belong to the set of ways determined by the plan. These secondary decisions, can be eliminated by means of such instruments, which automatically react to counterbalance departures from a good planned way in the center of the ways belonging to the plan. Those instruments are built according to the theory of servo (systems) mechanism. In most cases a human subject has to play the same role as a servomechanism for controlling a path of movement under the "load" of outside disturbances, to secure that a plan of action will be actually performed. We may perhaps call decisions made during the activity of fulfilling a plan of action for servomechanical decisions. The usefulness of fine classifications of the set of the theoretically possible ways of action is dependent on the precision attainable when we try to follow a planned way by help of servomechanical decisions.

The class of all decisions is however not fully covered by moves, plans, and servomechanical decisions. We can also use a random process to determine the choice we make between different possible moves. We could call these sequential process decisions. The real decision is in this case, however, the choice between different possible stochastical processes, and classifications of their developments, and not the decisions done automatically after one development of the stochastical process has gone so far that it is clear to what class it belongs. These real decisions belong to the set of choices of a plan from a set of possible plans. To this class of decisions belong also a decision to use a so-called mixed strategy in the theory of games, where the player plays a sequence of games using different "rules of action" each time but where the choice of the rule of action is determined by a decision to use a specific sequential process as an "automatic decision maker." A rule of action could perhaps be considered as a new class of decision, but they will then be an extended class of servomechanical and sequential process decision, where the moves of the decision maker are predetermined by the observations he has been able to make about the development of a set of functions of a more or less stochastical nature. The real decisions are, in this case, the choice of the rule of action, the decision to cease to use it or to change it to another rule of action. It is thus possible to decrease the number of cases in which real decisions have to be made by the use of servomechanical decisions and by decisions to use rules of action, thereby increasing the degree of automatization of the behavior of the subject. He will then perhaps become more effective from some point of view but at the same time less human. But there is, I think, a minimum degree of automatization which is necessary if the human mind is not to be almost wholly occupied by decision making processes of small importance.

The decision to use a special rule of action is in itself of so much more importance than the choice of individual moves which the rule makes automatic, that it can be worth while to perform a much more careful decision making process to get a good rule of action, than in the case where we decide to consider each move as a problem to be solved. But I think, there will also be a risk in choosing a very high degree of automatization, because then it can happen that the human subject will degenerate to an automaton, and thereby lose his most precious gift of nature, his ability to make purposive choices between different plans of action.

Interesting subsets of the set of all decisions are also such classes of decisions which are consequences of each other. For instance, the subject of highest order in a centrally directed economic circle may decide to give an order to subordinated subjects, an order which sets a goal for them and perhaps also points out a class of allowed ways to get there. The order receivers are able to choose between many ways included as allowed ways in the given order for fulfilling the goal. They can then decide to give one but often a whole set of orders, to their subordinates, which determine some subsets of ways allowed for in the first order, and so on until we come down to orders, which are performed using only simple servomechanical decisions during the fulfillment of the orders. In many cases we perhaps can distinguish the same structure in the process of decision-making also in the case, when the ultimate performer of the order is the same subject as the order giver.

Good order giving constitutes a science in itself, where we try to accomplish a goal by means of a chain of orders, which ultimately produce a set of very simple orders of the type: Start some specific activity, stop an activity, increase the speed of the activity, decrease the speed, change the direction of the activity, accept or refuse a proposal, for instance, to buy or sell an object of value; and last but not least important, record and/or rapport the accomplishments done to some one.

The above discussed "decision —>order cascades" could be defined in the following way: A decision —>order cascade is a set of successive attempts to determine a usable subset of ways to a goal by a chain of decision —> orders —> decision —> orders —> ... In the ultimate subset arrived at, the way actually used by the subject, usually an organization, is determined by a random process controlled by servomechanical decisions. The mathematical model for a decision-order cascade is a chain of set-theoretic multiplications of sets of ways to define a desired subset of ways in a large set of possible ways to a goal.

A cascade of research decisions is in some sense an inversion of the above cascade. We define a research — decision-cascade as a set of decisions done by starting, performing and stopping a research program with the end to seek a set of ways leading to some subset of a set of goals by help of one or a whole set of organized subjects simultaneously and/or successively following all ways in a successively expanding sample of ways taken from a set of ways that maybe includes ways leading to some subset of the set of goals considered. The mathematical model of research decisions is a successive set-theoretic summation of sets of ways until the sum includes some elements which can be proved to belong to a set of interesting ways. The elements in the sum are found by a sequential sampling procedure from a set of ways that maybe contains elements of interest.

It is likely that many more interesting classes of decisions exist in the actual world, but I think this discussion is already sufficient for reaching the goals the auth r had in mind, when he decided to start a research-activity to find a set of usable concepts for an attempt to construct a theory of decision making.

6. Controlled Stochastical Processes

By a controlled stochastical process we mean a set of developments of a vector $x^{(\xi)} = (x_{1,}^{(\xi)} \ldots x_k^{(\xi)})$; until $(\rightarrow t)$ and after $(t \rightarrow)$ the point of time t, if this set $x^{(\xi)}$ is measurable by means of a conditional probability

$$(1) \quad P(x_{t\rightarrow}^{(\xi)} \subset H_{t\rightarrow} \mid x^{(\xi)} \subset H_{\rightarrow t} \;\&\; H_{t\rightarrow}^{(a)}) = P(H_{t\rightarrow} \mid H_{\rightarrow t} \;\&\; H_{t\rightarrow}^{(a)})$$

$$= P(H_{t\rightarrow} \mid H_{\rightarrow t \rightarrow}^{(a)})$$

The probability, that the development $x^{(\xi)}_{t\longrightarrow}$ of the vector $x^{(\xi)}$ after the point of time t will belong to a set $H_{t\longrightarrow}$ of logically possible developments is thus dependent on the course the developments have taken until t and on the development of some variables a which are assumed to be controlled by a subject. The variables $a = (a_1, a_2, \ldots, a_m)$ could be called the parameters of action of the decision maker. The parameters of action a are also assumed to be coordinates of the vector $x^{(\xi)}$. From a practical point of view such controlled stochastical processes are of special interest if the probabilities (1) P can be approximated by a Taylor expansion $P^{(n)}$ of the numbers $x^{(\xi)}_{t-v\tau/\sqrt{n}}$, $v = 1,2, \ldots, n$ and $a_{t+v\tau/\sqrt{n}}$, $v = 1,2, \ldots, n$ which converges rapidly to P when $n \longrightarrow \infty$. In this case it is possible to get approximately the correct results describing the development of the process if we know the development of the process (and thereby also the controlled variables) during a past period $(t_a - T \longrightarrow t_b + T'')$ of finite length, and have a definite plan of action $H^{(a)}_{t\longrightarrow}$ for the future. Usually we are most interested in the development of such a process in the near future $t \longrightarrow t + T'$; $T' < T$. It is then expedient to restrict the study of the process to such sets $H_{t\longrightarrow}$ of possible developments which put no restrictions on the development after the point of time $t + T'$. A property of conditional probabilities that sometimes perhaps can be helpful when we try to estimate $P(H_{t\Rightarrow} \mid H_{\Rightarrow t} \;\&\; H^{(a)}_{t\Rightarrow})$ and make a good choice of $H^{(a)}_{t\Rightarrow t+\tau}$ is the formula:

$$P(H_{t\Rightarrow} \mid H^{(a)}_{\Rightarrow t\Rightarrow}) = P(H_{t+\tau\Rightarrow} \mid H^{(a)}_{\Rightarrow t+\tau\Rightarrow}) \cdot P(H_{t\Rightarrow t+\tau} \mid H^{(a)}_{\Rightarrow t+\tau})$$

obtained from the multiplication rule of probabilities. The conditions $x^{(\xi)}_{\Rightarrow t\Rightarrow} \subset H_{t\Rightarrow t+\tau}$ and $a_{\Rightarrow t\Rightarrow} \subset H^{(a)}_{t\Rightarrow t+\tau}$ restrict the development of $x^{(\xi)}_{\Rightarrow t\Rightarrow}$ respective $a_{\Rightarrow t\Rightarrow}$ only during the time interval $t \Rightarrow t + \tau$.

In some cases it will be of interest to study how the process develops in the limit, when $t \to \infty$, but in most cases the subject controlling the variables a has no interest in the process of the distant future (because the subject will not then be alive). It may thus be good to have a more precise description of the process in the near future, than in the more distant future. We might then use such a system for classifying the developments $x^{(\xi)}$ that every development, is given a diccrete code number h attached to a well defined subset of possible future developments of $x^{(\xi)}$. Such subsets could be defined by a set of tables containing only integers.

$$h = \left\{ \left\{ h_{t_v \rho}; \begin{array}{l} \rho = 1, \dots, k \\ v = -T + 1, -1, 1, \dots, T \end{array} \right\} \right\}.$$

If we say that $x^{(\xi)}$ belongs to the subset $H_t^{(h)}$ this can be interpreted to mean, for instance, that:

$$\log x_{t_v, \rho} = \log x_{t, \rho}\left(1 + \frac{t_v - t}{T}\right) + \frac{t - t_v}{T} \log x_{t-T, \rho} + \beta \cdot (t_v - t) \operatorname{tg} \frac{\pi}{19} (h_{t_v, \rho} + \theta_{t_v \rho})$$

where $\theta_{t_v \rho}$ is bounded to the interval $-\frac{1}{2} < \theta_{t_v \rho} - \frac{1}{2}$ and $h_{t_v \rho}$ is a whole integer from the set $(p, \pm 1, \pm 2, \pm 9)$, for all v in the set $(v = 1, 2, \dots, T - 1)$. The values of $x_{t, \rho}$ and $x_{t-T, \rho}$ are assumed to be known with sufficient accuracy. For $t_t = t$ and $t_v = t - T$ the number $h_{t_v \rho}$ is thus always $= 0$.

By using this system of discrete codes for classifying the past developments we have in all $19^{k(T-1)}$ different subsets of developments, which subsets taken together contain all possible past developments of $x^{(\xi)}$. All possible subsets of future development $H^{(h)}_{t \rightarrow t+T'}$ is defined by the corresponding tables of h for $v = +1, 2, \ldots, T'$. We have in all $19^{T'k}$ different subsets of future developments of $x^{(\xi)}$.

The stochastic process $x^{(\xi)}$ can by such classifications, during the period of interest $t - T \rightarrow t+T'$ be described by a "simpler" discrete process ${}_{t-t}h^{(\xi)}_{\rightarrow t+T'}$, where ${}_{t-T}h^{(\xi)}_{\rightarrow t+T'}$ denotes a matrix of whole integers. The process $h^{(\xi)}$ has a finite number $N_{T+T'} = 19^{(T+T'-1)k}$ of different possible realizations, which if we like could be identified by an integer λ from the set $1, \ldots, N_{T+T'}$. For a system of time functions x_t, known from $t - T$ to $t + T'$, the corresponding integer λ is λ_t. The numbers in the matrix ${}_{t-T}h^{(\xi)}_{\rightarrow t+T'}$ are assumed to be only partly known at the point of time t_c in the interval $t_b \leq t_c < t_b + T'$, namely, the numbers relating to the submatrix ${}_{t-T}h^{(\xi)}_{\rightarrow t}$. By going backwards in the past from t_b, we may have a whole interval $t_b \geq t \geq t_a$ for which the numbers λ_t can be calculated. If we classify λ_t and say that

$$\lambda_t \subset L_{t_c}(a) \text{ if } h^{(\xi)}_{t-T \rightarrow t} = h^{(\xi)}_{t_c-T \rightarrow t_c} \text{ and the rows in } {}_{t_c-T}h^{(\xi)}_{\rightarrow t_c+T'}$$

corresponding to the action parameters are fixed hypothetically it will be possible to state the relative time

$$p_{t_a \rightarrow t_b}(\lambda_t = \lambda \,|\, \lambda_t \subset L_{t_c}(a)) = \int_{t_a}^{t_b} \Phi((\lambda_t \subset \lambda)\, \Phi(\lambda_t \subset L_{t_c}(a))dt : \int_{t_a}^{t_b} \Phi(\lambda_t \subset L_{t_c}(a))dt$$

(where $\Phi(\;) = \binom{1}{0}$ depending on if λ_t belongs to the subset mentioned in or not) the realization $\lambda_t = \lambda$ has occured during those time intervals when $\lambda_t \subset L_{t_c}(a)$, and $t_b \geq t > t_a$.

If $p_{t_a \to t_b}(\lambda_{t_c} = \lambda \mid \lambda_{t_c} \subset L_{t_c}(a))$ converges in probability sufficiently rapidly [3] towards a number $p(\lambda_{t_c} \subset \lambda \mid \lambda_{t_c} \subset L_{t_c}(a))$ when $(t_b - t_a) \to \infty$, it is possible to use the relative time $p_{t_a \to t_b}(x_{t_c} = \lambda \mid \lambda_{t_c} \subset L_{t_c}(a))$ as an estimate of the conditional probability that the stochastical process $x^{(\xi)}_{t_c \to}$ will belong to the subset corresponding to the number λ if the past history and the planned action is such that we know that $\lambda_{t_c} \subset L_{t_c}(a)$.

The decision problem the subject has to solve is to find some <u>rule</u> of action $\hat{a}_{t_c, \rho} = \hat{a}\rho(h_{t_c - T \to t_c})$; $\rho = 1, \ldots, k$, that is a function of the subject's information about the past, which is such that the controlled stochastical process, which is controlled by this rule results in a distribution

$$p(\lambda_{t_c} = \lambda \mid \lambda_{t_c} \subset L_{t_c}(\hat{a}(h_{t-T \to t_c})$$

which the subject considers better than the distribution corresponding to every other possible rule of action. Because the purpose of estimating the probabilities of different developments in the last instance is to determine an indicator of goodness for different rules of action it is sufficient to estimate the probability distribution of some indicator of goodness U_λ given the rule of action considered. The different developments λ can therefore conveniently be classified into broader classes according to the partition of U_λ in a sequence of intervals, thereby decreasing the number of probabilities that has to be estimated from observations.

It might be of interest to state that already if, the probability, that the subject will choose a definite rule of action for the point of time t_{c1} following the actual point t_c, is a given function of

h the controlled stochastical process has a probability measure.

$t_c - T \rightarrow t_c$

If such a "traditional strategy" has been followed during the period of time $t_a - T \rightarrow t_b + T'$ it is possible to estimate the probability of different developments λ, if the subject continues to act in the traditional way, by studying the relative time of $\lambda_t = \lambda$ during the period $t_a \rightarrow t_b$. But before the problem of choosing an optimal rule of action (an optimal strategy) can be said to have been solved in a satisfactory manner, the subject has to be sure that no other strategy than the traditional strategy gives a probability distribution of λ, which is preferrable to the probability distribution corresponding to the traditional strategy. I think, most subjects could invent new strategies which are better than the traditional one. The only exception seems to me to be the paradoxical case, when the traditional strategy itself includes a rule defining the optimal amount of activity, the subject shall devote to the problem of improving the traditional rules of action the subject tries to follow.

FOOTNOTES:

1/ Research undertaken by the Cowles Commission for research in Economics under contract Nonr-358(01), NR 047-006 with the Office of Naval Research.

2/ Graph theory as a Mathematical Model in Social Sciences by Frank Harvey, Robert Z. Norman, Ann Arbor, 1953.

3/ For stochastical processes of a more general character, than those which are called stationary processes, the best estimate to the probability $p(\lambda \mid \lambda \subset L_{t_c}(a))$ that can be got by varying $t_b - t_a$ may be got for a period $t_b - t_a$ of final length. It may then be in principle, impossible to get an estimate with a variance smaller than ε^2.