COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 2063

Ethical Criteria for Group Choice: A Preliminary Formulation

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December 15, 1952

The tentative nature of this paper should be stressed even more than usual. It contains some speculation about how the application of ethical values to problems of group choice might be systematically studied with the aid of formal theoretical models. Questions related to the selection of ethical values or criteria are not discussed.

My interest in engaging in speculation of this type arises from my feeling that the ethical criterion of maximum satisfaction of individual preferences which underlies most recent discussion of welfare economics is incomplete. I believe it would be useful to have a theoretical framework which invited the consideration of various criteria and could be used to analyze the implications of various criteria. I do not wish to propose a specific set of ethical values, but to explore some possibilities for incorporating various value considerations into formal analysis. In the selection of some ethical values for use in illustrations, my limited acquaintance with important discussions of philosophers has forced me to use vaguely defined concepts from ordinary conversations, newspaper editorials, and political oratory rather than more refined concepts of

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1. I have benefited from brief discussions of some aspects of this problem with Abraham Kaplan and Roy Radner

philosophic discussions. Some aspects of the formal framework suggested below may be appropriate for the study of various groups -- clubs, classes in a school, etc. However in the presentation and illustrations of ethical criteria used we shall assume that we are concerned with large semi-autonomous groups such as nations, commonwealths or colonies.

Consider a group of n individuals. Let a_i $i = 1,2, \ldots, n$ represent the actions of the i th individual over a given interval of time. Actions are defined rather broadly to include consumption of commodities, production services rendered during the period, assets owned and liabilities owed at the end of the period, and social activities of various sorts that are important to the welfare of the individual. The concept is consciously left somewhat vague in this paper because of uncertainties about the non-economic factors that may be appropriate for inclusion and the possibility that new considerations relevant to a more precise definition of actions may arise when more thorough attempts at application are made. In the present discussion actions do not include participation in politics. This is briefly explained below.

The group is regarded as an entity separate from the individual members and is assumed to have agents through which it may act. We may think of group actions as such things as collection of taxes, gathering and distribution of information, enforcement of laws and regulations, and military and diplomatic activity. Actions of a particular individual are assumed to be a function of actions of other individuals, actions of the group, and actions of an entity that we shall call the environment. The environment includes natural forces and groups foreign to the group under consideration. Actions of the environment may include such natural phenomena as weather, earthquakes and erosion as well as offers of trade and diplomatic and military gestures of foreign groups. In many contexts it might be desirable to recognize several entities in the environment since natural forces and other groups might respond quite differently to actions of the group being studied and its individual members. However in this preliminary formulation it is convenient to treat them as a single entity.

In a democracy everyone is to some extent an agent as actions of the group or agents include all essentially political activity. For this reason political activity was excluded from actions of individuals.

Let $a = (a, a_2 \ldots a_n)$ be an array of the actions of the n individuals and let $a_{(i)}$ be obtained from a by deleting the actions of the i th individual. Let w represent actions of the environment and d represent actions of the group. We then have —

(1) $a_i = \alpha_i (a_{(i)}, w, d)$ for $i = 1, 2, \ldots, n$

α_i is a function or mapping from a space of actions of n + 1 entities to a space of actions of a single entity. α_i will be called the i-th individual's strategy. In this paper only pure strategies are considered.

It is assumed that the n equations given by (1) have a unique solution determining, for given individual strategies, the actions of all of the individuals as functions of the actions of the environment and the group.

(2) $a = \varphi(w,d)$

where the mapping φ depends upon $\alpha = (\alpha_1, \alpha_2 \ldots \alpha_n)$.

Actions of the environment are regarded as a function of actions of individuals and actions of the group. Similarly, actions of the group are regarded as functions of actions of individuals and the environment.

(3) $w = \omega(a,d)$

(4) $d = \delta(a,w)$

ω and δ are the respective strategies of the environment and the group. It is assumed that (2), (3), (4) uniquely determine individual, environmental and group actions for given strategies. Let $x = (a\ w\ d)$, x will be called the social history of the group. The relation between the strategies and the history will be indicated by

(5) $x = \xi(\alpha, \omega, \delta)$.

ξ is a mapping from the space of possible strategies to the space of possible actions. The existence of ξ implies the existence of mappings connecting specifications of strategies with elements of subspaces of the action space. Two of these that will be of use later are indicated by

(6) $a = \xi_a(\alpha, \omega, \delta)$

(7) $a_i = \xi_{a_i}(\alpha, \omega, \delta)$.

A_i represents the set of strategies available to individual i when technical possibilities and the personal limitations of the individual are taken into account. $A = A_1 \times A_2 \times \ldots \times A_n$ is the Cartesian product of the A_i. It denotes the combinations of strategies available to all of the individuals. The strategies available to the environment when technical and other (possibly personal and legal) restrictions are taken into account are denoted by Ω and Δ is the set of strategies available to the group.

Our purpose is to consider how ethical values may offer guidance in reaching group decisions. In the framework outlined above group decisions reduce to the selection of a strategy for the group, one of the δ's, from the available set Δ. To consider this choice we should like to consider the consequences of alternative selections or, more accurately, we should like to consider information that the agents of the group may have about the consequences of alternative selections. As an illustration of how the nature of the problem faced by the agents depends on the type of information they are assumed to have, consider the following four cases.

Case I: Agents of the group know the individual strategies and the environmental strategy, say $\alpha = \alpha^o$, $\omega = \omega^o$. In this case to each δ there corresponds a unique history given by

(8) $$x^o_\delta = \xi(\alpha^o, \omega^o, \delta).$$

In this case the agents of the group require criteria for choosing a particular history from a class of attainable histories. The attainable class may be indicated

(9) $$X^o_\Delta = \bigcup_{\delta \in \Delta} \xi(\alpha^o, \omega^o, \delta).$$

Case II: The agents know that individual strategies that will be employed come from a certain class $\hat{A}$ and that the strategy of the environment belongs to a class $\hat{\Omega}$. To each δ there corresponds a class of histories, $\hat{X}_\delta$, given by

(10) $$\hat{X}_\delta = \bigcup_{\substack{\alpha \in \hat{A} \\ \omega \in \hat{\Omega}}} \xi(\alpha, \omega, \delta).$$

In this case the agents need criteria for choosing among alternative classes of possible histories.

Case III: The agents have an a priori probability distribution $p(\alpha, \omega)$ of the relevant strategies. Choice of a particular δ determines a corresponding probability distribution of histories, say $p_\delta(x)$.

Case IV: A class of probability distributions, say $P(\alpha, \omega)$ is known. To each δ there corresponds a class of probability distributions $P_\delta(x)$ of possible histories.

Since the knowledge of the consequences of the group strategy varies with the initial information of the agents we might expect that criteria formulated to guide the selection under one state of information would not necessarily apply to a different state. For example, if we somehow aggregated individual preferences among alternative histories into a social preference, this would be sufficient to order the alternative group strategies in Case I but not in the other cases.

The remainder of the discussion relates to Case II. The discussion consists of speculation about how certain ethical values that commonly enter informal discussions of group choice might be formulated so as to apply in this context. The values considered are Individual Liberty, Security, and Equality. In each case several of many possible formulations are suggested. Pending further examination they should be regarded as largely illustrative.

Individual Liberty

Alternative concepts based on sets of actions possible or available to individuals are outlined. What is possible or available depends on the kinds of restrictions considered. We first observe that under the assumptions of Case II, the actions of individuals (denoted by a) will belong to the following set.

(11) $$\hat{A}_{\Delta} = \bigcup_{\substack{\alpha \in \hat{A} \\ \omega \in \hat{\Omega} \\ \delta \in \Delta}} \xi_a(\alpha, \omega, \delta)$$

When a group strategy δ has been selected, it is known that a belongs to

(12) $$A^{(1)}_{\delta} = \bigcup_{\substack{\alpha \in \hat{A} \\ \omega \in \hat{\Omega}}} \xi_a(\alpha, \omega, \delta)$$

which is a (generally proper) subset of $\hat{A}_{\Delta}$. Assuming that δ has been selected, $A^{(1)}_{\delta}$ represents actions that cannot be ruled out by the information and strategy of the group's agents. One way to formulate the concept of individual liberty would be to associate liberty with the set $A^{(1)}_{\delta}$ and to regard it as defining a partial order on Δ given by

(13) $$\delta^{o} \; L^{(1)} \; \delta^{*} \Longleftrightarrow A^{(1)}_{\delta^{o}} \supseteq A^{(1)}_{\delta^{*}}.$$

$\delta^{o} \; L^{(1)} \; \delta^{*}$ is read, "δ^{o} is at least as good as δ^{*} by criterion $L^{(1)}$".

If $\delta^{o} L^{(1)} \delta^{*}$ and $\delta^{*} L^{(1)} \delta^{o}$ both hold, δ^{o} and δ^{*} are equivalent by criterion $L^{(1)}$; if neither holds they are incomparable. If $\delta^{o} L^{(1)} \delta^{*}$ holds but $\delta^{*} L^{(1)} \delta^{o}$ does not hold, i.e., $\delta^{*} \delta^{o}$, then δ^{o} is better than δ^{*} by criterion $L^{(1)}$.

The fact that $L^{(1)}$ gives only a partial ordering means that, in general, this criterion would have to be strengthened or applied jointly with other criteria to give a unique best strategy. To the extent that agents of the group apply $L^{(1)}$ they might be loosely described as minimizing the extent to which group strategy interferes with individual actions. This criterion might be called permitted liberty.

The actions of the i-th individual are contained in a projection of $A^{(1)}_{\delta}$ which may be indicated

(14) $$A^{(1)}_{i\delta} = \bigcup_{\substack{\alpha \in \hat{A} \\ \omega \in \hat{\Omega}}} \xi_{a_i}(\alpha, \omega, \delta).$$

Some might contend that this set is not very significant since many of the individual actions it contains could only be realized by the individual if the environment and other individuals would cooperate. This suggests an alternative criterion which we might call guaranteed liberty.

To outline this concept we assume another piece of information in the possession of the agents. Let $\tilde{A}_i$ be a set of strategies that are known to be available to the i-th person, i.e., it is known that $A_i \supseteq \tilde{A}_i$. Assume such information for every individual and let $\tilde{A} = \tilde{A}_1 \times \tilde{A}_2 \times \cdots \times \tilde{A}_n$. Consider the set

(15) $$A^{(2)}_{i\delta} = \bigcap_{\substack{\alpha_{(i)} \in \hat{A}_{(i)} \\ \omega \in \hat{\Omega}}} \bigcup_{\alpha_i \in \tilde{A}_i} \xi_{a_i}(\alpha, \omega, \delta)$$

where $\alpha_{(i)}$ is a specification of a strategy for everyone except i and $\hat{A}_{(i)}$ is a set of strategies for the n-1 other individuals known to contain their actual strategies. It is obtained from $\hat{A}$ by deleting the set of possible strategies for individual i. The interpretation of the set $A^2_{i\delta}$ that it contains the actions known to be available to individual i regardless of the strategies of other persons and the environment. Let

$$(16) \qquad A^{(2)}_{\delta} = A^{(2)}_{1\delta} \times A^{(2)}_{2\delta} \times \ldots \times A^{(2)}_{n\delta}$$

be the Cartesian product of the $A^{(2)}_{i\delta}$. Consider the partial ordering given by

$$(17) \qquad \delta^{o}\, L^{(2)}\, \delta^{*} \Leftrightarrow A^{(2)}_{\delta^{o}} \supseteq A^{(2)}_{\delta^{*}} \quad .$$

To the extent that $L^{(2)}$ is applied, the group tries to enlarge the set of actions known to be available to individuals rather than to reduce the set of actions known to be unavailable.

A criterion that is, in a sense, intermediate between $L^{(1)}$ and $L^{(2)}$ is indicated below.

$$(18) \qquad \delta^{o}\, L^{(3)}\, \delta^{*} \Longleftrightarrow A^{(3)}_{\delta^{o}} \supseteq A^{(3)}_{\delta^{*}}$$

where

$$(19) \qquad A^{(3)}_{\delta} = \bigcap_{\omega \in \hat{\Omega}} \; \bigcup_{\alpha \in \mathcal{A}} \; \xi_{a}(\alpha, \omega, \delta).$$

Whereas applying $L^{(2)}$ involves an effort to protect the individual's range of available actions from both the possibility of unfavorable strategy by the environment and possibly unfavorable strategies by other individuals, $L^{(3)}$ involves guarding against an unfavorable strategy by the environment only.

Many other formulations of this concept are possible. For example, some may feel that each person's range of actions should be protected against certain types of strategies of other persons and the environment but not necessarily all. This might require the development of concepts of fairness to be combined with concepts of liberty.

Security

The constructions given here are intended to express the idea of security as an assurance against undesirable actions. The concepts underlying the two constructions presented are distinguished by calling them unconditional and conditional security. Individual preferences are used to identify which actions are undesirable.

Following Arrow's notation we let R_i indicate the preferences of the i-th individual. $a_i^o \, R_i \, a_i^*$ means that actions a_i^o are preferred or indifferent to actions a_i^*. The possible actions of the individual are assumed to be simply ordered by R_i. To use R_i in the present context would be to implicitly assume that R_i was known to the agents. Instead it is assumed that a partial ordering $\hat{R}_i$ that agrees with R_i is known to the agents. $a_i^o \, \hat{R}_i \, a_i^*$ means that a_i^o is known to be preferred or indifferent to a_i^*.

Let $\hat{A}_{i\Delta}$ be the projection of $\hat{A}_\Delta$ which contains possible actions for the i-th person and let $B_{i\delta}^{(1)}$ be the subset of $\hat{A}_{i\Delta}$ containing all elements for which

$$(20) \qquad b_i \in B_{i\delta}^{(1)}, \; a_i \in A_{i\delta}^{(1)} \implies a_i \, \hat{R}_i \, b_i$$

where $A_{i\delta}^{(1)}$ is defined by (14).

$B_{i\delta}^{(1)}$ consists of elements known to be inferior or indifferent to all the elements that can materialize if δ is selected. An increase in $B_{i\delta}^{(1)}$ may thus be associated with an increase in security for individual i. Let

$$(21) \qquad B_\delta^{(1)} = B_{1\delta}^{(1)} \times B_{2\delta}^{(1)} \times \cdots \times B_{n\delta}^{(1)} .$$

A partial ordering of the group strategies is given by

$$(22) \qquad \delta^o \, S^{(1)} \, \delta^* \iff B_{\delta^o}^{(1)} \supseteq B_{\delta^*}^{(1)}$$

where $S^{(1)}$ means " is as good according to criterion $S^{(1)}$ as". By this criterion the agents try to exclude as many undesired outcomes as possible regardless of individual choices. It might therefore be said to be concerned with unconditional security.

To consider an alternative let $B_{i\delta}^{(2)}$ be the subset of $\hat{A}_{i\Delta}$ consisting of those elements that are known to be inferior or indifferent to some element of $A_{i\delta}^{(2)}$ as defined in (15).

(23) $$b_i \in B_{i\delta}^{(2)} \Rightarrow \exists\, a_i \in A_{i\delta}^{(2)} \ni a_i \hat{R}_i b_i.$$

The agents known that by proper choice of strategy, the i-th person can achieve actions at least as satisfying as any actions in $B_{i\delta}^{(2)}$. If the agents seek to make $B_{i\delta}^{(2)}$ large they are seeking to assure individual i a favorable chance. Favorable actions will depend partly on a favorable selection of strategy by the individual and in this sense we might describe the underlying concept as conditional security.

To take all individuals into account let

(24) $$B_{\delta}^{(2)} = B_{1\delta}^{(2)} \times B_{2\delta}^{(2)} \times \ldots \times B_{n\delta}^{(2)}$$

and define a partial ordering by

(25) $$\delta^{o}\, S^{(2)}\, \delta^{*} \Longleftrightarrow B_{\delta^{o}}^{(2)} \supseteq B_{\delta^{*}}^{(2)}.$$

Equality

Just mentioning equality suggests the question, "Equality of what?". The most extreme version would be equality of actions, $a_i = a_j$ for all i, j. This is, of course, impossible and undesirable. Equality of income has sometimes been considered. If it were seriously proposed then the appropriate definition of income would cause some concern. While income does not explicitly enter our model most any definition of income that might be chosen could be used to specify an individuals income as a function of his actions, say $I(a_i)$. The requirement of equality would be $I(a_i) = I(a_j)$

for all i, j. This illustrates the possibility that, while equal actions are impossible, equality of certain parameters of actions may be possible and desirable.

Equality of opportunity is often proposed. One version of this could be expressed as follows. Let $a^{[i\ j]}$ be the array of actions obtained from a by interchanging the actions of the i-th and j-th individuals and let $\alpha^{[i\ j]}$ be the array of strategies obtained from α by interchanging the i-th and j-th strategies. Equality of aopportunity might be said to be achieved if

(26) $$\xi a_i (\alpha, \omega, \delta) = \xi a_j (\alpha^{[i\ j]}, \omega, \delta)$$

for all $\alpha \in A$ and all $\omega \in \hat{\Omega}$. In particular this would mean that persons who selected identical strategies would have identical actions.