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Inventory Fluctuations in Flaxseed and Linseed Oil

1926 - 1939

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Summary

The behavior of economic units engaged in the production and distribution of flaxseed and linseed oil in the United States is summarized in a system of linear stochastic equations. The latter contains such jointly dependent, observable economic variables as production, stocks, and consumption of the above commodities. The particular equation system studied reflects the investigator's desire to explain quarterly inventory fluctuations in these commodities and is formulated in the light of market conditions prevailing during the period 1926 - 1939. Primarily for the latter reason, no claim of predictive usefulness under present conditions is made for the estimates of these equations. The paper concludes with an evaluation of the estimates with reference to prewar years.

1. Historical background

The two important industrial raw materials furnished by the flax plant are flaxseed and flax fiber. However, the seed and fiber are not jointly produced; generally a different variety of plant is grown for seed than for fiber. The only important industrial use of flaxseed is in the production of linseed oil and linseed cake.

During the period studied, Argentina, the Soviet Union, and British India produced about 80 per cent of total world flaxseed production. Argentina alone produced about half of world production and exported about 80 per cent of the flaxseed exported. The United States, which typically imported about half of its annual consumption, obtained practically all its imports from South America, about 80 per cent of the imports originating in Argentina.

The Argentina harvest occurs in December, and the new crops is available for export in the first quarter of the calendar year. Domestic as well as Canadian seed is planted in the spring, harvested in August, and the bulk of it marketed upon harvesting. About 90 per cent of domestic flaxseed is grown in the North Central United States, although in the late thirties California became an important producing state. In the former region flaxseed competes most strongly with spring wheat and to lesser extent with other grains for the farmers' acreage at planting time. The comparative price of flaxseed and wheat is regarded as an important influence in the amount of flaxseed acreage planted. To some extent current prices may affect decisions underlying the yield from acreage planted. However weather experience is probably the most significant determinant of yield.

During the period, practically all the domestic crop of flaxseed was traded on the Minneapolis Grain Exchange. A large amount of purchases of Argentine and Canadian seed were made directly on the Buenos Aires and Winnipeg exchanges for flaxseed consumers, i. e. linseed oil producers, in the United States by their own agents. These purchases were then shipped directly to the consumers' plants. Also flaxseed was shipped on consignment to dealers on the East Coast by Argentine exporters for subsequent sales to consumers. The availability of foreign supplies of flaxseed prevented bottlenecks in linseed oil production, even in the years marked by severe crop failures in the United States.

The tariff on flaxseed did not serve to curtail imports of flaxseed in favor of imports of linseed oil. This was largely due to the existence of a compensatory tariff on linseed oil. Furthermore linseed oil production capacity was rarely fully utilized, particularly during the thirties. Exports of linseed oil were also negligible. Presumably oil produced in the United States could not compete in foreign markets in spite of the fact that exporters of flaxseed products received a drawback on the flaxseed tariff if the exported products were produced from imported flaxseed.

The technology of linseed oil production remained relatively constant during the period. Briefly, production consisted of crushing flaxseed under hydraulic pressure, yielding three-tenths to one-third of its weight in linseed oil and about two-thirds of its weight in linseed cake.[1] While some variation in yields was technically possible, the most important determinant of yield was the oil content of the seed, which itself depended on the climatic conditions under which the seed was grown. Linseed oil cake could be ground into meal, and the oil refined to various specifications, but oil and cake were the basic products of the production process.

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[1]Under fixed proportions of production, say three-tenths of a unit of oil and two-thirds of a unit of cake to one unit of seed and under a condition of zero profits with no costs but raw material considered, one would expect the relation

$$P_L = 10/3\ P_S - 20/9\ P_M$$

among the price per pound of linseed oil (P_L), of flaxseed (P_S), and of linseed meal (P_M), i.e. ground linseed cake. A least squares regression, using quarterly average prices per pound for the period 1926 - 1939, yields

$$P_L = 3.389\ P_S - 2.06\ P_M + .04;\ (R = .94)$$

The firms of the linseed oil industry have engaged in the production of other vegetable oils. However the vegetable oils were not competitors in production, given existing plant. Different techniques of production for the various oils and different sources of raw material supply have both tended to specialize plants to the production of particular oils.

While there was an organized market for trading in flaxseed, consumers of linseed oil contracted directly with producers for deliveries over a three-to-six month period for a price set at the time of contracting and were billed for the oil upon delivery. Approximately 90 per cent of linseed oil sales were handled in this manner.

In the United States, as abroad, linseed oil was used chiefly as a drying oil. Because the most important use of drying oils is in paints and varnishes, building activity was regarded as the chief determinant of linseed oil consumption. Other important industries whose activity affected linseed oil consumption were the automobile, furniture, linoleum, oiled-clothing, and rubber tire industries. Other drying oils used were tung and perilla oils, which were produced in the Orient. However the consumption of these oils as a percentage of total drying oil consumption was almost negligible. Partly this was a matter of their small supply, but also they had specialized uses for which linseeed oil was not well adapted and vice versa. The wide fluctuations of their relative prices during the period support to some extent the lack of interchangeability of these oils.

The by-product of linseed oil production, linseed cake, when ground into meal, is used as a livestock feed. Besides an export market, there was demand for the by-product in the agricultural Midwest. The price of linseed cake was tied closely to the price of other oil cake used as feed. Generally these prices followed those of feed grains because of substitutability among all feed materials. Domestically all forms of oil cake constituted less than 3 per cent of all feed materials. Exports of linseed cake to Europe were substantial. Europe, a deficit feed producer, has long had higher feed prices than the United States. These exports were largely originated by East Coast plants, which, operating on imported seed, could obtain a tariff drawback on the exported cake. Furthermore transportation costs were favorable to using the European market as an outlet rather than the principle feed markets of the Midwest.

2. The Explanation of Linseed Oil Consumption.

There was a marked seasonal pattern in linseed oil consumption. Peak's regularly occurred during the second quarter of the calendar year with troughs following by two quarters. This seasonal pattern was a reflection of the seasonality in the demand for drying oil products. The heaviest demand for drying oil products, particularly paints and varnishes, occurred in late spring and early summer, which was also the season of heavy building activity. There was similar but less marked seasonality in industrial production, as measured by the unadjusted FRB index. This could also be an important explanatory factor in linseed oil consumption, since there was industrial demand for drying oil other than that of the paint industry.

As was indicated in section 1, the demand for drying oils was essentially the demand for linseed oil. While this might serve to exclude prices of competitive drying oils from an explanation of linseed oil consumption, no strong a priori evidence exists for excluding all other prices, particularly the prices of drying oil products. Nevertheless it is always tempting to assume price inelastic consumption, if only to provide a simpler explanation.

The estimates of the following relation

2.1 $$C(t) = \alpha_1 B(t) + \alpha_2 Y(t) + \alpha_3 + U_1(t),$$

where

2.1.1 $C(t)$ = consumption of linseed oil in thousands of pounds during t^{th} quarter

2.1.2 $B(t)$ = average monthly construction contracts in thousands of square feet of floor area during t^{th} quarter.

2.1.3 $Y(t)$ = average of monthly FRB indices of industrial production (unadjusted) during t^{th} quarter (1926 = 100)

2.1.4 $U_1(t)$ = random residual,

met with some success.[2]

The use of "quarterly constants," i.e. a constant which is assumed non-zero in a given quarter and zero in the three others, in the consumption relation resulted in improvement. One might justify the use of such constants by the fact that there was a certain normal use of linseed oil products for each quarter. For example there was normally considerable repainting and maintenance work done after the end of winter in industry, on farms and in urban housing. As a rule the use of such devices as quarterly constants should be regarded as

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2 See section 5. Here and in the sequel the symbol t denotes the quarter as numbered from the beginning of the sample period. Random components of equations will always be denoted by the symbol U.

a first approximation to a systematic (and more complex) explanation of seasonality. However since the difference in fit of the two consumption relations is not too substantial, one might conclude that the more important elements of seasonality are attributable to the variables B and Y.

3. Pipeline Stocks of Linseed Oil.

Linseed oil consumers typically maintained a six to twelve week supply of oil on hand for their future needs. Such behavior would provide some explanation of the fact that peaks and troughs in linseed oil stocks normally occurred at the beginning of the quarter in which the peaks and troughs, respectively, in consumption occurred.

When approached on the subject of their inventory behavior, linseed oil producers claimed they produced oil on a to-order basis, maintaining a level of stocks just adequate to meet deliveries on their forward contracts. Actually producers' plant storage capacity for oil was not large and was quite small compared to the vast elevator capacity available for storing flaxseed. Flaxseed could be stored more economically than the oil or the by-product linseed cake, so that long term supplies of products could be carried in the form of flaxseed.

However producers did have certain inducements to hold a higher level of oil stocks than merely that required to meet current deliveries. Forward sales covering longer delivery periods and a heavier volume of such sales were typical of the fourth and first quarters, when the extent of the American and Argentine harvests became known. With this hedge available, producers could build up stocks in this period and thus avoid to some extent the diseconomies of adjusting production closely to seasonal fluctuations in deliveries.

The tendency of consumers to anticipate consumption and producers to anticipate deliveries by building up stocks of oil (the anticipations depending on the quarter in which they are formed) is reflected in equation (3.1) below by the presence of the variable

$$3.1.1 \quad \tilde{C}(t,\tau) = \begin{cases} \dfrac{B(t)}{B(t-4)} \left[C(t-3) + C(t-2) \right], & \tau = 1, 4 \\ \dfrac{B(t)}{B(t-4)} C(t-3), & \tau = 2, 3. \end{cases}$$ [3]

As a "predictor" variable, (3.1.1) might be interpreted as follows: the next period's consumption will be greater or less than last year's consumption in the corresponding period depending on whether current building activity is greater or less than that during the same time last year. Unfortunately more accurate measures of anticipations, for example, the volume of forward sales as a measure of anticipated deliveries, are unavailable.

In the examination of time series on linseed oil stocks, it appeared necessary to account for a phenomenon of importance in the depression years.[4] One observes large levels of end-of-quarter stocks in relation to the current and the succeeding quarter's consumption, in spite of sharp downturns in production. Historically the period was marked by repeated failures

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3. The symbol τ denotes the quarter numbered with reference to the calendar year.

4. Here and elsewhere this investigator has indicated a willingness to examine observations on variables before proposing models for their explanation. While this procedure invalidates assumptions made in various tests of significance of econometric models, the still primitive state of the "art of model building" does not leave much alternative. However, as far as that goes, most tests of significance in econometric work rest on equally shaky foundation.

linseed oil consumption.

Equation (3.1) gives no role to linseed cake and meal in the explanation of linseed oil stocks. Rising meal prices in relation to flaxseed prices might induce producers to hold oil stocks they would not otherwise have held currently in order to produce meal. However this possible effect was ignored on two grounds. Quantitatively the contribution of such an effect compared to total oil stocks would likely be small. On qualitative grounds the value of meal produced from a given quantity of flaxseed was typically a small portion of the cost of the flaxseed (and of the total value of the output). Also linseed oil was the most expensive of the commodities to store. Thus to produce oil that must be stored in order to obtain meal for marketing increases the holdings of a relatively expensive storage item for a relatively small return on current seed costs.

4. The demands for imports and stocks of flaxseed.

If exports and imports of linseed oil are ignored, linseed oil consumption and inventory equations determine linseed oil production. The latter determines flaxseed consumption, i.e. under the assumption of fixed proportions in production. In a model of price competition, there remain but two equations relating to the behavior of linseed oil producers to specify - a demand for flaxseed imports and a demand for flaxseed stocks.[5] Implicit in the latter equations

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5 Since linseed oil producers themselves hold title to the bulk of seed storage throughout most of the year and since they are also the final purchasers of of all imports, the total quantities of imports and end-of-quarter stocks will be treated as if determined by the decisions of linseed oil producers alone.

are factors which determined the distribution of flaxseed consumption.[6]

Most linseed oil plants in the United States were located on the East Coast and in states bordering the Great Lakes - particularly in States bordering Lakes Michigan and Superior. The East Coast plants of course had the easier access to imported seed supplies; the Midwestern plants, to domestic supplies. Because of the high cost of rail compared to Lake transport, virtually no flaxseed moved between the East and the Midwest during the period when Lake navigation was closed, i.e. about the last of November to the first part of April. Even severe seed shortages in the Midwest during winter typically did not result in Midwestern flaxseed prices rising sufficiently high for imported seed to reach that area. In such cases linseed oil production tended to shift from the area of seed scarcity to the area with relatively more favorable conditions of supply. In brief linseed oil moved by rail during winter rather than flaxseed. However during non-winter months flaxseed movements rather than production shifts tended to occur.

The problem of maintaining an adequate storage of seed after harvest time was particularly serious to linseed oil firms whose main operations were in the Midwest. If a seed shortage developed in the winter months, when sales were normally accelerating, these plants would lose sales to plants with a cheaper source of supply and/or produce at higher cost. During times of abundant harvests in relation to the annual needs of Midwestern plants (e.g. the

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6 It is unfortunate in this connection that observations on stocks and consumption are not available by area.

late twenties and scattered years during the thirties, Midwestern plants operated the year round on domestic seed with domestic seed moving as far as the East Coast during the months when Lake navigation was open. In years of small domestic crops (particularly during the thirties, marked by severe crop failures), imported seed moved to the Midwest even at the time of the harvest, and the Eastern plants operated on imported seed the year round.

The seed storage facilities available to Eastern plants was negligible compared to the tremendous elevator capacity of the Midwest. But Eastern plants had far less requirement for flaxseed storage. An import supply was available throughout the year, and in fact during the winter months when Midwestern seed was unavailable except at comparatively high cost, supplies of Argentine seed were most plentiful. Then during the rest of the year, Midwestern and imported seed costs were in closer alignment for Eastern plants, and the latter might be purchases in both the domestic and foreign markets.

In the following discussion it is assumed that only Midwestern plants purchased seed for stocks, that purchases of seed for stock by Eastern plants were comparatively unimportant.

For a given linseed oil production decision, the decision to import flaxseed for consumption would presumably be an increasing function of the cost of domestic seed relative to the cost of imported seed for a plant in the Midwest or in the East. In addition a decision to import seed for stocks by Midwestern plants would be an increasing function of anticipated future seed requirements - anticipations dependent on the quarter in which they are formed.

The aggregate of these decisions is summarized in equation

$$(4.1)\quad I(t) = \gamma_1 \frac{p(t)}{p_a(t)} + \gamma_2 F(t) + \gamma_3 \hat{C}(t, \tau) + \gamma_4 t + \gamma_5 + U_3(t),$$

where

4.1.1 $I(t)$ = U. S. imports of flaxseed in thousands of pounds during t^{th} quarter.

4.1.2 $\frac{p(t)}{p_a(t)}$ = ratio of quarterly average Minneapolis price of flaxseed per bushel (1926 = 100) to quarterly average Buenos Aires price of flaxseed in bushels during t^{th} quarter (1926 = 100).

4.1.3 $F(t)$ = U. S. consumption of flaxseed in tons during t^{th} quarter.

$$4.1.4\quad \hat{C}(t, \tau) = \begin{cases} \frac{B(t)}{B(t-4)} C(t-3) & , \tau = 1,2 \\ \frac{B(t)}{B(t-4)} [C(t-3) - C(t-2) + C(t-1)], & \tau = 3 \\ \frac{B(t)}{B(t-4)} [C(t-3) + C(t-2)], & \tau = 4. \end{cases}$$

A lagged price ratio appears in (4.1), as it was felt that part of imports during a quarter would reflect lagged decisions. In connection with anticipated seed requirements, the latter are assumed to be an increasing function of $\hat{C}(t, \tau)$ and a decreasing function of $H(t)$. However the latter variable does not appear explicitly in (4.1). Its contribution is reflected to some extent by the presence of the variable $F(t)$; a priori its exclusion should reduce the contribution of $F(t)$ in the equation.

The rate of disappearance of flaxseed stocks in the Midwest depended on imports for stocks and also on the distribution of flaxseed consumption. The latter in turn depended on the price of domestic seed relative to the price of imported seed.[7/] Relative price movements which induced Midwestern plants to curtail consumption could be viewed as the inducement for holders of stocks to conserve supplies in the Midwest and contrary movements, as an inducement to make more of the existing supplies available for consumption. The level of anticipated seed requirements would play a similar role in conservation and depletion of stocks.

The behavior underlying the disappearance of seed supplies in the Midwest is summarized in

$$(4.2)\quad D(t) \equiv S(t-1) + P(t, \tau) - S(t) = \alpha_1 \frac{p(t)}{p_a(t)} + \alpha_2 H(t) + \alpha_3 \hat{C}(t, \tau) + \alpha_4 t + U_4(t),$$

where

(4.2.1) $D(t)$ = disappearance of existing seed supplies in tons during t^{th} quarter.[8/, 9/]

7/ As mentioned earlier, competition among plants in all areas tended to shift linseed oil production to sources of cheapest seed supply.

8/ As was suggested in section 1, domestic seed production is assumed to be exogenous. Also since the harvest occurs in late July and early August domestic production was added in the definition of existing supplies for the third quarter.

9/ Unplanned inventory accumulation in linseed oil would mean unplanned disappearance of flaxseed stocks. To draw a parallel with (3.1), unplanned end-of-quarter linseed oil stocks, or rather the explanation of this unobservable variable, should be substituted for $H(t)$ in (4.2). Then a variable denoting the flaxseed equivalent of unplanned linseed oil inventory accumulation should be subtracted from the right-hand side of the equality in (4.2) to obtain an equation involving unplanned disappearance of stocks as a component. However the present formulation gives an adequate role to oil stocks, planned and unplanned, in the explanation of the actual disappearance of flaxseed stocks.

(4.2.2) $S(t)$ = end-of-quarter flaxseed stocks in tons.

(4.2.3) $P(t,\tau)$ = annual U. S. flaxseed production in tons,

(= 0 for τ = 1, 2, 4).

Instead of equation (4.2) a "demand for stocks" equation

$$(4.3)\quad S(t) = \varepsilon_1 \frac{p(t)}{p_a(t)} + \varepsilon_2 H(t) + \varepsilon_3 \hat{C}(t,\tau) + \varepsilon_4[S(t-1) + P(t,\tau)] + \varepsilon_5 t + \varepsilon_6 + U_4(t)$$

was actually estimated - without restricting ε_5 to the value one. A priori, the estimates of coefficients of (4.3) were expected to be compatible with the rationale underlying (4.2), i.e. $\varepsilon_1, \varepsilon_3, > 0$; $\varepsilon_2 < 0$; and $\varepsilon_4 \approx 1$.

5. The equations and their estimates.

For convenience the equations specified in the preceding sections and two alternative equations are listed below:

$$5.1\quad C(t) = \alpha_1 B(t) + \alpha_2 (t) + \alpha_3 + U_1(t)$$

$$5.1'\quad C(t) = \alpha_1' B(t) + \alpha_2' Y(t) + \sum_{i=1}^{4} \alpha_{3i}' \delta_{i\tau} + U_1'(t)$$

$$5.2\quad H(t) = \beta_1[H(t-1)-C(t)] + \beta_2 \tilde{C}(t,\tau) + \beta_3 t + \beta_4 + U_2(t)$$

$$5.2'\quad H(t) = \beta_1'[H(t-1)-C(t)] + \beta_2' \tilde{C}_b(t) + \beta_3' \tilde{C}_y(t) + \beta_4' t + \beta_5' + U_2'(t)$$

$$5.3\quad I(t) = \gamma_1 \frac{p(t)}{p_a(t)} + \gamma_2 F(t) + \gamma_3 \hat{C}(t,\tau) + \gamma_4 t + \gamma_5 \frac{p(t-1)}{p_a(t-1)} + \gamma_6 + U_3(t)$$

$$5.4\quad S(t) = \varepsilon_1 \frac{p(t)}{p_a(t)} + \varepsilon_2 H(t) + \varepsilon_3 \hat{C}(t,\tau) + \varepsilon_4[S(t-1) + P(t,\tau)] + \varepsilon_5 t + \varepsilon_6 + U_4(t) ,$$

where

$$\delta_{i\tau} = \begin{cases} 0 , & i \neq \tau \\ 1 , & i = \tau \end{cases} , \qquad \tau = 1,2,3,4.$$

In addition the definitions

5.5 $S(t) = S(t-1) + P(t, \tau) + I(t) - F(t)$

5.6 $F(t) = \eta[C(t) + H(t) - H(t-1)]$,

where η is the conversion ratio between flaxseed and linseed oil, form part of the equation system.

The variables C, H, I, F, S and p/p_a were assumed to be endogenous. For purposes of estimation, all other variables were assumed to be predetermined, i.e. to have a probability distribution independent of the unobservable random disturbances U_i. The U_i were assumed to have a joint distribution independent of t with zero means and finite variances and covariances. All equations, except of course the identities, were estimated by least squares. With the exception of (5.1) and (5.1'), each equation was estimated by the limited-information-single-equation method with the assumption in each case that only the variables B and Y appear elsewhere in the system.[10/] The estimates appear in Table 1.

For given values of the predetermined variables and the random disturbances, the values of the endogenous variables are completely determined, i.e. the equation system is "complete."

10/ This involved ignoring overidentifying restrictions in each case. The possible consequences of this and the assumption that Y was a predetermined variable will not be discussed here.

This points up an interesting feature of the system. The system states that the relative price of domestic and foreign flaxseed is determined solely by behavior equations relating to linseed oil producers and consumers in the United States. Viewed from the standpoint of foreign markets, the system implies that the United States demand for foreign flaxseed is determined independently. However this aspect of the equations is consistent with the basic assumption underlying much in their specification, namely, that linseed oil consumption was unaffected by price - at least within the range of experience in the period studied. In view of the discussion of sections 1 and 2, the latter assumption appears valid as a first approximation. Over the longer period including world War II and postwar experience, technology in the use of drying oils has changed so that other oils can be used where once linseed oil had no substitutes. Indeed prewar dependence in linseed oil may have been the very stimulus to these developments in technology.

NOTE: This is the concluding installment of CCDP: Economics No. 2061. Errata for the previous installment are:

page 12, line 16: Change "purchases" to "purchasers".

page 15, line 7 : Change " ε_5" to " ε_4".

page 17, line 16: Change "in" to "on".

Table 1. Estimates of parameters

	Name of estimate	Type of estimate*	Estimates								Coefficient of correlation	Standard error of u
Eqn. (5.1') and (5.1)	Variable		C(t)	B(t)	Y(t)	1						
	Mean		138796.	39832.	95.236							
	Coefficients	LS	1.0	1.4326	430.28	40755.					.885	21053.
	Standard error											
	Variable		C(t)	B(t)	Y(t)	δ_{11}	δ_{22}	δ_{33}	δ_{44}			
	Coefficients	LS	1.0	1.1925	627.57	20482.	57682.	37256.	9909.		962	10708.
	Standard error											
Eqn. (5.2') and (5.2)	Variable		H(t)	$\frac{H(t-1)}{-C(t)}$	$\tilde{C}(t,\tau)$	$\tilde{C}_b(t)$	$\tilde{C}_v(t)$	t	1			
	Mean		137671.	.253.13	218582.	139607.	140575.	0				
	Coefficients	LS	1.0	.41039	.17992	0	0	-594.08	98447.		.790	22213.
	Standard error											
	Coefficients	LI	1.0	.12177	.18236	0	0	-311.87	97840.			24571.
	Standard error											
	Coefficients	LS	1.0	.50799	0	.15634	.34852	-170.47	66980.		.749	24258.
	Standard error											
	Coefficients	LI	1.0	.42653	0	.15869	-.10849	335.82	130876.			24438.
	Standard error											
Eqn. (5.3)	Variable		I(t)	$\frac{p(t)}{p_a(t)}$	F(t)	$\hat{C}(t,\tau)$	$\frac{p(t-1)}{p_a(t-1)}$	t	1			
	Mean		239427.	1.1275	204891.	260203	1.1274	0				
	Coefficients	LS	1.0	-32788.	1.4861	-.06973	246370.	1593.	-287666.		.646	84349.
	Standard error											
	Coefficients	LI	1.0	52339.	2.5262	-.01585	401171.	2884.	-785292.			97191.
	Standard error											
Eqn. (5.4)	Variable		S(t)	$\frac{p(t)}{p_a(t)}$	H(t)	$\hat{C}(t,\tau)$	S(t-1) + P(t, τ)	t	1			
	Mean		233992.	1.1275	137671.	260203.	330073.	0				
	Coefficients	LS	1.0	83087.	-20912.	.11631	.80533	623.14	-126980.		.970	38521.
	Standard error											
	Coefficients	LI	1.0	93518.	-.71082	.11568	.80006	456.59	- 67772.			42827.
	Standard error											

*LI = limited information;
LS = least squares.

6. An evaluation of the estimates.[11]

Only two limited information estimates of coefficients are disappointing in view of a priori expectations concerning their signs, namely, the estimated coefficient of $\tilde{C}_y$ in (5.2') and of $\hat{C}$ in (5.3). The average contribution of both these variables (the product of the estimated coefficient and the mean of the variable) is small compared to the mean of the "explained" variable.

There is a surprising similarity between the least squares and limited information estimates of equations. In the case of either (5.2) or (5.2'), an assumption that the residual in the equation and the residual in the consumption equation are uncorrelated would make the least squares and limited information estimates equivalent. In all equations the small difference between estimates of standard errors of residuals by the two methods is particularly surprising in view of the fact that the sum of squares of the least squares residuals is smallest by construction and that the limited information method used is relatively inefficient.[12] The estimated standard errors are also indicative of how well the equations "fit" the observations of the sample period. The standard errors for residuals in the estimated equations were about one-sixth the mean of the variable explained except in the case of (5.1'), which displays a very good fit, and (5.3), which displays a very poor one.[13]

The next point in evaluating the estimates concerns their performance in a post-sample period. This task was approached without too much hope.

11/ At this writing, computations useful in evaluating certain aspects of the estimates, e.g., the significance of estimated coefficients and serial correlation of residuals, are not complete.

12/ Cf., n. 10, p. 16.

13/ It should be borne in mind that there is considerable variation in the quarterly means of the variables.

The structural equations specified simply do not admit the wider range of economic experience of the World War II and postwar periods. The technology of production as well as the influences underlying consumption of linseed oil changed with the onset of preparation for war in the United States. Also the beginning of the war in Europe completely changed the conditions of world flaxseed supply. In 1940, an immense effort was begun to make the United States self-sufficient in flaxseed production, and in the postwar period the United States has even become a net exporter.

Nevertheless two types of comparison were made concerning the performance of the limited information estimates during the year 1940.[14] The first was a comparison of the 1940 residuals calculated from the limited information estimate of an equation and values of σ^*, the estimated standard error of the residual in the least squares estimate of that equation, the standard error computed according to the conventional formula. This comparison is not based on any exact theory of testing hypotheses. However, the procedure was motivated by the desire to test a given equation directly for a change in structure. The general theory underlying tests of this kind is unsatisfactory in at least two respects: first, the tests are not independent of the normalization rule used in the equation, and second, the assumptions underlying the (asymptotic) distribution of residuals calculated from the limited information estimate of a structural equation are not sufficiently general to employ the theory in the present case.[15] The $t_{.05}$ intervals based on σ^* (and given in Table 2) must be viewed merely as an attempt to determine whether or not the limited information estimates fared as well in 1940 as the least squares estimates could be expected to perform.

14/ These comparisons were motivated by procedures suggested in [1] and in private discussion with Clifford Hildreth.

15/ Cf. [2].

Table 2. Residuals Calculated from Structural Equations - 1940

Equation	Quarter	LS	LI	$t_{.05}\sigma^*$
5.1	1	-20578	-	-
	2	3667	-	-
	3	-30827	-	-
	4	-60907	-	-
5.1'	1	-14379	-	22294
	2	-23945	-	22214
	3	-36995	-	22397
	4	-37592	-	22839
5.2	1	31209	29865	46387
	2	17946	11223	46572
	3	9580	-4599	47238
	4	-9972	-30540	50712
5.2'	1	18542	18664	-
	2	-17049	-19686	-
	3	-18709	-29943	-
	4	27423	7516	-
5.3	1	-29372	-67273	183317
	2	-95924	-107992	181136
	3	-153906	-206247	196665
	4	-279439	-480934	219963
5.4	1	12543	34711	84517
	2	-66740	-64068	82894
	3	-13592	-18426	105994
	4	-98690	-87270	109758

They did not in the case of the limited information estimate of the import equation nor in the case of the least squares estimate of the consumption equation.[16/] The fact that the residuals were large and negative is revealing.[17/] The "explaining" variables indicated large imports of flaxseed when transoceanic shipping was being curtailed by the war. Production and building activity, stimulated by rearmament, indicated large linseed oil consumption when much of this activity was probably unrelated to the demand for drying oil products.

The second type of comparison was motivated by the desire to test the predicting value of the limited information estimates for 1940. The predictions which can be made from the estimated structure (the coefficients given in Table 1) are as follows. Let $y(t)$ and $z(t)$ denote row vectors of the endogenous and predetermined variables respectively, and let $u(t)$ denote the vector of random residuals. The equation system, composed of (5.1'), (5.2), (5.3),...,(5.6), may be rewritten as

$$(6.1) \qquad By'(t) = \Gamma z'(t) + u'(t),$$

where B is a non-singular matrix of order six and Γ is a six by thirteen rectangular matrix. Then the predictions $y^*(t)$ of $y(t)$ are given by

$$(6.2) \qquad y^{*'}(t) = +(B^*)^{-1}\Gamma^* z'(t),$$

where the elements of B^* and Γ^* are the limited information estimates of

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16/ Under the distribution assumptions of section 5, the least squares and limited-information-single-equation estimates of (5.1) and (5.1') are equivalent.

17/ The definition of the calculated residual $u^*(t)$ is:

$$u^*(t) = Y(t) - \sum a_i X_i(t)$$

where $Y(t)$ denotes a variable to the left of the equality sign in an equation of section 5 and $X_i(t)$ a variable to the right with estimated coefficient a_i.

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Table 3. Predictions of Endogenous Variables for 1940

Quarter	Variable	1940 value	Predictions* (a)	Predictions* (b)	1939 value
1	C(t)	119454	133833	133833	119625
2		168743	192688	192688	155960
3		153175	190170	190170	151947
4		153527	191119	191119	136322
1	H(t)	173200	139484	148403	161387
2		132840	123447	142313	130250
3		115146	129202	129309	112629
4		153804	198269	130254	142457
1	I(t)	268000	313702	345493	358000
2		177000	365722	475802	245000
3		73000	521299	586889	173000
4		144000	1039568	808736	122000
1	S(t)	253793	297318	302951	150120
2		109508	228680	248183	64288
3		816503	975248	986869	524167
4		606130	951398	910499	340572
1	$\frac{p(t)}{p_a(t)}$	1.157	1.753	1.865	1.256
2		1.059	1.540	1.929	1.099
3		1.332	2.833	3.064	.943
4		1.564	4.520	3.706	1.055

* Predictions (a) are based on a system consisting of (5.1'), (5.2), (5.3),...,(5.6); predictions (b), on (5.1'), (5.2'), (5.3),...,(5.6).

the corresponding elements of B and Γ.

The predictions obtained according to (6.2) are compared in Table 3 with predictions from a "naive" model.[18] The latter states that the predicted value of a variable for a given quarter is the value of that variable in the corresponding quarter of the previous year.

18/ The term is Andrew Marshall's. For a discussion of naive models, see [1].

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