COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 2056

Some Game Theoretic Points of View on Scientific Research[1/]

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November 17, 1952

The question of how we could get a model for the activity of scientific research, and how this model perhaps could be used for a clarification of the problems of decision making, that must be solved by starting, directing and stopping a specific research activity and in allocating the research activity of an organized set of researchers on specific subsets of activities and by allocating economical resources to different sets of researchers, seems to me to be a problem worth discussion. The broader problem of the usefulness or harmfulness of attempts to direct the research activity of a set of organized sets of research workers may also be of interest in this connection. A discussion of the social pressure field influencing the research workers will also perhaps be necessary if we try to get a realistic picture of the conditions prevailing in that adventurous game that is played by subjects influencing the research activity going on in the world, an activity mostly slowly but profoundly influencing the activities and well-being of all human beings.

I used the term game as a name for that broader class of activities to which, I think, the research activity belongs. I did it only tentatively to direct my attempts to get a suitable model for the scientific

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1. Research undertaken by the Cowles Commission for Research in Economics under contract Nonr-358(01), NR 047-006 with the Office of Naval Research.

research activity. Game theoretic models have been made fashionable by many distinguished research workers during the last ten years. I am prepared to expect that the clothes made by the theory of game will give only a very poor fit if not specially tailored for the purpose. But before we can make any more or less serious attempts to put such clothes on, we will try to clarify the concept of scientific research.

According to my surely limited experience of scientific research it seems to me to be an activity, by which we are seeking new ways to some interesting goals, by means of a set of more or less firmly organized research workers following simultaneously or sequentially a set of promising ways belonging to a network of ways expanding during the research, to find out where they lead. Mostly the set of ways followed lead outside the set of goals we had in mind when starting the research. In some cases they lead to other goals more interesting and valuable than the goals first thought of, perhaps also to many goals of minor interest but sometimes we really find paths to the original goals. A large part of the research activity seems to produce nothing of value, except the information that nothing interesting has been found by following some specific part of the network of possible paths. This information could, however, be useful for directing the efforts of other researchers into other paths than those already used. The negative findings are thus also valuable but primarily only for research workers and would be research workers. The positive findings are often very valuable to a large set of subjects. Subjects outside the set of research workers, are mostly only interested in results that can be used for improving their activity, by opening new ways to increase their incomes. The set of research workers are however mostly more interested in the question of solving that jigsaw puzzle (as Michael Polanyi

called it) of putting together the pieces of information they have produced, by inventing new hypothesis and by adjustments of old ones to a system of theories that will cover the whole set of scientifically significant information produced, than in the question of the economic importance of special pieces of information. Thereby they satisfy directly a human need of understanding the external world and the abstract world's creatable by the human mind by using rules of logic. But indirectly this is also an efficient way to produce practically useful results, by giving the research workers tools for their work and by suggesting promising paths for future research.

The network of communication between different scientists and between scientists and other subjects are also important parts of the information producing machinery. Nonnegligible forces in the game of scientific research are also the feelings of pride the research workers knows, when he has been able to contribute to the system of scientifically important knowledge, especially if it will be acknowledged in some form by distinguished scientists, who already have done acknowledged contributions to science. A research worker is thus only in a relatively low degree directed and directable by economic motives, but nevertheless economic resources are necessary conditions for continuing and expanding scientific research.

The problem of how to allocate economic resources, by their suppliers, to different subsets of scientific research is thus an important one. We will now try to formulate this problem in game theoretic terms (see also the attached appendix).

We think of the set A of decision makers. The decision maker A_α may, at point of time t, belong to some of the four subsets: $A^{(0)}$ = nature, $A^{(1)}$ = the research workers producing and communicating information to each other and to non-researchers and getting in exchange economic resources

from the set $A^{(2)}$ of research supporters. The rest $A^{(3)}$ of A are interested in the communicated information but do not pay more than the costs for communicating information already produced by $A^{(1)}$. The subjects $A^{(2)}_{\alpha} \subset A^{(2)}$ have a more or less altruistic indicator of goodness, in the sense that $A^{(2)}_{\alpha}$ takes into account also benefits rendered to decision makers in $A^{(3)}$, when $A^{(2)}_{\alpha}$ makes decisions to support research workers $A^{(1)}_{\alpha} \subset A^{(1)}$.

The set of all results R obtained or obtainable by means of activities of subjects $A_{\alpha} \subset A$ may be divided in three subsets:

$$R = C \cup I \cup J \ .$$

The subset C contains results called economic resources, the second useful information and J products of the imagination and nonsense giving perhaps entertainment but being of only negligible importance for good decision making. The economic resources C in the possession of A_{α} at t can be increased by production $C_{t\rightarrow}^{(\overset{\frown}{\alpha})}$ or import $C_{t\rightarrow}^{(\rightarrow\alpha)}$ and decreased by export $C_{t\rightarrow}^{(\alpha\rightarrow)}$ from A_{α}. Information can be increased by research work $I_{t\rightarrow}^{(\overset{\frown}{\alpha})}$, and communication from others $I_{t\rightarrow}^{(\rightarrow\alpha)}$ but cannot be decreased by export to other decision makers than to nature. This export $I_{t\rightarrow}^{(\alpha\rightarrow 0)}$ is usually called forgetting. The results J cannot be possessed at all, they produce only entertainment during production or communication. The production and communication of I and J are possible only by combining them with a flow of economic resources C, they are by themselves immaterial. The set of all future results of interest to A_{α} we denote by $R_{t\rightarrow}^{(\alpha)}$. The results in the possession of A_{α} at t we denote:

$$R_{\dashv t}^{(\alpha)} = (C_{\dashv t}^{(\overset{\frown}{\alpha})} \cup C_{\dashv t}^{(\rightarrow\alpha)} - C_{\dashv t}^{(\alpha\rightarrow)}) \cup (I_{\dashv t}^{(\overset{\frown}{\alpha})} \cup I_{\dashv t}^{(\rightarrow\alpha)} - I_{\dashv t}^{(\alpha\rightarrow 0)}).$$

The flows of economic resources and information can be subdivided in subsets according to which subject is the importer, respectively exporter, and according to the combinations of economic resources (C) and information (I) that are communicated as results of the same decision. The decision maker A_α is assumed to have an indicator of goodness $g^{(\alpha)}_{t_1 P_{t\alpha}}$, which is some kind of average of the value $g^{(\alpha)}_t(R^{(\alpha)}_{t\vdash})$ evaluated in "dollars," he assigns to different possible subsets of future results $R^{(\alpha)}_{t\vdash}$. The estimated probability $P(g^{(\alpha)}_t(R^{(\alpha)}_{t\vdash}) \geq g) = P_{\alpha t}(g)$, that the value of the future results exceeds g is modifiable by decisions influencing the distribution of the results $R^{(\alpha)}_{t\vdash}$. We assume further that the average $g^{(\alpha)}_{t_1 P_{t\alpha}}$ is calculable by a formula of the form:

$$U^{(\alpha)}_t(g^{(\alpha)}_{t_1 P_{\alpha t}}) = \int_{-\infty}^{+\infty} U^{(\alpha)}_t(g)\, dP_{\alpha t}(g) = \int_{-\infty}^{\infty} P_t(g)\, u^{(\alpha)}_t(g)\, dg$$

where $U^{(\alpha)}_t(g)$ is a monotonically increasing function of g, and $u^{(\alpha)}_t(g) = \frac{dU^{(\alpha)}_t(g)}{dg}$ the sensibility function, studied more in detail in the appendix distributed.

The supporters $A^{(2)}_\alpha \subset A^{(2)}$ of research have as already said usually a more or less altruistic kind of indicator of goodness and acts often as a representative for a whole circle $A^{(\alpha)}$ of subjects. Such an "altruistic" indicator of goodness may perhaps be considered as an estimate of a weighted sum of the indicators $g^{(\alpha\beta)}_{t_1 P_{\alpha\beta t}}$ for the different subjects $A^{(\alpha)}_\beta$ belonging to $A^{(\alpha)}$:

$$g^{(A^{(\alpha)})}_{t_1 P_t(A^{(\alpha)})} = \sum_\beta \gamma^{(\alpha)}_\beta \cdot g^{(\beta)}_{t_1 P_{\alpha\beta t}}; \quad 1 \geq \gamma^{(\alpha)}_\beta > 0 \text{ if } A_\beta \in A^{(\alpha)}.$$

The system of indicators of goodness

$$\left\{ g^{(A^{(\alpha)}}_{\alpha \neq 0 \; {}_tP_t(A^{(\alpha)})} \right\} = \text{the payoff}$$

can be considered as describing the position of a ball to be played in the research game. The players A_α try to push the "ball" forward, everyone in his own direction of increasing values of $g^{(A^{(\alpha)})}_{{}_tP_t(A^{(\alpha)})}$ by acts modifying the distribution of future results $R_{t|-}$. Nature can be considered as a player which during the game is pushing the "ball" mostly backward as a result of "forgetting if information," and of consumption and other decreases of the value of economic resources. Sometimes nature pushes the ball "forward" by a stochastic increase in results exported from nature to the other players in the game. Nature A_o is also the player, which puts constraints on the modifiability of the probability distributions the other players try to modify by their actions. The "strategy" of nature seems, however, to be invariant in time and this fact makes it possible for the research workers $A^{(1)}_\alpha$ to increase the information $I^{(\alpha)}_t(A_o)$ about the "strategy of A_o" thereby increasing their possibilities to increase their indicators of goodness without at the same time pushing the indicator of goodness for some other player A_β ; $\beta \neq 0$ back. This fact makes scientific research especially valuable to the whole set of decision makers.

Up to now we have assumed that the supporters of scientific research are very clever and really can construct a good altruistic indicator of goodness, which they try to maximize when they make decisions to export economic resources $C^{(\alpha \rightarrow \beta)}_{t-1\, t+\tau}$ combined with some instructions $I^{(\alpha \rightarrow \beta)}_{t-1}$ to the research workers about the goals for the research work supported and the allowed domain for re-export of these resources from the research worker A_β. Those decisions are usually preceded by incoming information

$I_{\overrightarrow{t}}^{(\beta \to \alpha)}$ asking for support for some research program. The task of constructing a more or less altruistic indicator of goodness, which could guide the decision maker to make good decisions is, however, so difficult that it is very probable that he will make decisions that could be considered as clearly nonoptimal by a critical observer.

There is, I think, a tendency to give too little attention to research activity of basic character necessary for the development of the whole system of knowledge, that makes it possible for the set of research workers to produce those particular results the supporter is interested in as applications of more fundamental principles belonging to the set of pure science. It is therefore, I think, a good thing if supporters interested in the well-being of very large groups of human beings compensate this tendency by supporting effectively that part of the research activity, which is most difficult to evaluate in dollars and cents. These results assist namely the whole set of research workers in their activity, without directly resulting in research products of economic importance. They give however direct intellectual satisfaction of the human need to understand the external world and the abstract worlds the human mind is able to create.

If the suppliers of resources, having in mind goals, which have nothing to do with the scientific goal to increase as rapidly as possible the value of the set of scientifically significant information, use such strategies, which interfere with the duties of the research workers to judge if some results claimed to be new pieces of scientific knowledge is a discovery or not, (that is if $R \subset I$ or $R \subset J$) such a strategy would be very harmful and could destroy the incentives to work for large groups of scientific research workers, thereby slowing down the progress of science. In applied research

aiming at a goal to find cheaper ways to produce something, the businessman is often competent to judge whether the way found really is such a way, but in uncertain cases it would be better to decide it by experiments, and use of statistical inference from the observed results. I think, it is also foolish strategy to try to hurry up the research activity to find a solution to some practical problem, because the largest increase to the set of valuable information, is usually found if the research workers proceed very carefully, paying attention also to a whole set of other goals, which perhaps can be reached as by-products during the seeking of a way to the goal the supplier of the resources have in mind. Often the byproducts are such, which would be very difficult to reach when they are assigned as primary goals for a special research activity. It would perhaps be a good help for the research workers to have an extensive list of goals considered as economically important, so that they do not miss them if they could be reached as byproducts during a research activity not primarily designed for the purpose to find them.

When I now look back at the flow of sentences I have produced, I see that I have not been able to fully reach my goal to invent a good game theoretic model for that complicated activity we call scientific research, but I hope the byproducts are valuable enough to compensate you for the trouble to follow me on the way. I hope that the points of view expressed will stimulate you to a lively discussion, which perhaps will open new ways to attack the problem. I think that the scientific research activity itself is a very interesting field for scientific research.

The Payoff for the Scientific Research Game.

Decision Makers and their Attributes

(1) A_α = The decision maker , $A_\alpha \in A$, A_0 = Nature.

(2) $\left\{(\cdot)_\alpha\right\}_\alpha = \left\{(\cdot)_1, (\cdot)_2, \cdots, (\cdot)_\alpha, \cdots\right\}$ = a system of symbols obtained when α varies through all α; $A_\alpha \in A$.

(2.1) α = an index telling that $(\cdot)_\alpha$ refers to A_α.

(3) $\alpha\rightarrow$ = an attribute, used to give the information that the symbol, to which it is attached corresponds to some results outgoing from A_α.

(3.1) $\rightarrow\alpha$ = incoming to A_α.

R = Results of Activities

(4) R_μ = a result of kind μ of an activity, $R_\mu \subset R$. R includes both physical and mental results.

(5) $R^{(\alpha)}$ = Results available to A_α a union of R_μ.

(6) $R^{(\alpha\rightarrow)}$ = Results of A_α 's activity "exported" to other subjects.

(7) $R^{(\widehat{\alpha})}$ = Results "produced" by A_α but not "exported."

(8) $R^{(\alpha\rightarrow\beta)}_{\mu,\ \rightarrow t}$ = Results exported from A_α to A_β of kind μ until t.

Economic Resources $C \subset R$

(9) $C^{(\alpha)}$ = Economic resources in the possession of A_α ,

(10) $C^{(\widehat{\alpha})}$ = Economic resources produced by A_α not exported.

(11) $C^{(\rightarrow\alpha)}_{\rightarrow t}$ = Economic resources "imported" to A_α until t.

(12) $C^{(\alpha\rightarrow)}_{\rightarrow t}$ = Economic resources exported from A_α until t

(13) $C^{(\alpha)} = C^{(\widehat{\alpha})}_{\rightarrow t} \cup C^{(\rightarrow\alpha)}_{\rightarrow t} - C^{(\alpha\rightarrow)}_{\rightarrow t}$ = The characteristic property of economic resources.

Information $I \subset R$

(14) $I_{\dashv t}^{(\alpha \rightarrow)}$ = information communicated from A_α = export of information to other subjects than A_o.

(15) $I_{\dashv t}^{(\rightarrow \alpha)}$ = information communicated to A_α = import of information.

(16) $I_{\dashv t}^{(\widehat{\alpha})}$ = information produced by A_α .

(17) $I_{\dashv t}^{(\alpha \rightarrow 0)}$ = information lost because of imperfect memory.

(18) $I_t^{(\alpha)}$ = information available to A_α at t.

(19) $I_t^{(\alpha)} = I_{\dashv t}^{(\rightarrow \alpha)} \cup I_{\dashv t}^{(\widehat{\alpha})} - I_{\dashv t}^{(\alpha \rightarrow 0)}$.

The characteristic property of information (18) says that information can not be decreased by sharing it with others only by "forgetting."

(20) $I_{s,t} = \bigcup_\alpha I_{s,t}^{(\alpha)}$ = Scientifically significant information = "statistics" reliable describing world events $x_{\dashv t}$ or conditional probabilities for future world events in case some $A_\alpha \in A$ uses a specific plan of action (high valued quality controlled mental products). They are subsets of the above mentioned results I. It is of special importance to take them in account by evaluating the results R of human activity.

Mental Products other than I belong to a subset of R we could call J. These products can be neglected by decision making.

Evaluation of Future Results $R_{t\dashv}^{(\alpha)} \subset R$.

(21) $R_{t\vdash}^{(\alpha)} = R_{t\vdash}^{(\rightarrow \alpha)} \cup R_{t\vdash}^{(\alpha \rightarrow)} \cup R_{t\vdash}^{(\widehat{\alpha})} \cup R_t^{(\alpha)}$ = future results of the activity of A_α given $R_t^{(\alpha)}$.

(22) P_α = assumed probability distribution of $R_{t\vdash}^{(\alpha)}$, <u>modifiable</u> by acts changing the distribution of $R_{t\vdash}^{(\alpha\rightarrow)}$ and $R_t^{(\widetilde{\alpha})}$ over possible subsets.

(23) $g_{tP_\alpha}^{(\alpha)}$ = indicator of goodness of P_α interpretable as gains worth $g_{tP_\alpha}^{(\alpha)}$ $ (of constant purchasing power) at t to A_α.

(24) $U_t^{(\alpha)}(g)$ = a monotonically increasing real valued (bounded) function of g.

(25) $u_t^{(\alpha)}(g) = \frac{dU_t^{(\alpha)}(g)}{dg}$ = the "sensibility" of A_α at t to changes in g.

(26) $u_t^{(\alpha)}(g) = \sum_\lambda \gamma_{\lambda t} \frac{1}{\sigma_{t\lambda}^{(\alpha)}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{g - m_{t\lambda}^{(\alpha)}}{\sigma_{t\lambda}^{(\alpha)}}\right]^2}$ = a plausible hypothesis about the form of $u_t^{(\alpha)}(g)$ (see diagrams).

(27) $g_t^{(\alpha)}(R_{t\vdash}^{(\alpha)})$ = value of the indicator of goodness to A_α at t in case $R_{t\vdash}^{(\alpha)}$ is hypothetically given.

(28) $g_t^{(\alpha)}(R_{t\vdash}^{(\alpha)}) = g_{1t}^{(\alpha)}(I_{t\vdash}^{(\rightarrow\alpha)}; R_t^{(\alpha)}) + g_{2t}^{(\alpha)}(C_{t\vdash}^{(\rightarrow\alpha)}; I_{t\vdash}^{(\alpha\rightarrow)}; R_t^{(\alpha)}) + g_{3t}^{(\alpha)}(R_{t\vdash}^{(\widetilde{\alpha})}; R_t^{(\alpha)}) - g_{4t}^{(\alpha)}(C_t^{(\alpha\rightarrow)}; I_{t\vdash}^{(\rightarrow\alpha)}; R_t^{(\alpha)})$

= a plausible hypothesis about the form of $g_t^{(\alpha)}(R_t^{(\alpha)})$. Only the first mentioned set is evaluated under the condition that the others are known.

(29) $U_t^{(\alpha)}(g_{tP_\alpha}^{(\alpha)}) = E_{P_\alpha} U_t^{(\alpha)}(g_t^{(\alpha)}(R_t^{(\alpha)})) = \int_{g=-\infty}^{\infty} P(g_t^{(\alpha)}(R_t^{(\alpha)}) \geq g)\, u_t^{(\alpha)}(g)\, dg$

a plausible hypothesis about the form of $g_{tP}^{(\alpha)}$.

(30) $\left\{g_{tP_\alpha}^{(\alpha)}\right\}_\alpha$ = the payoff in the research game.

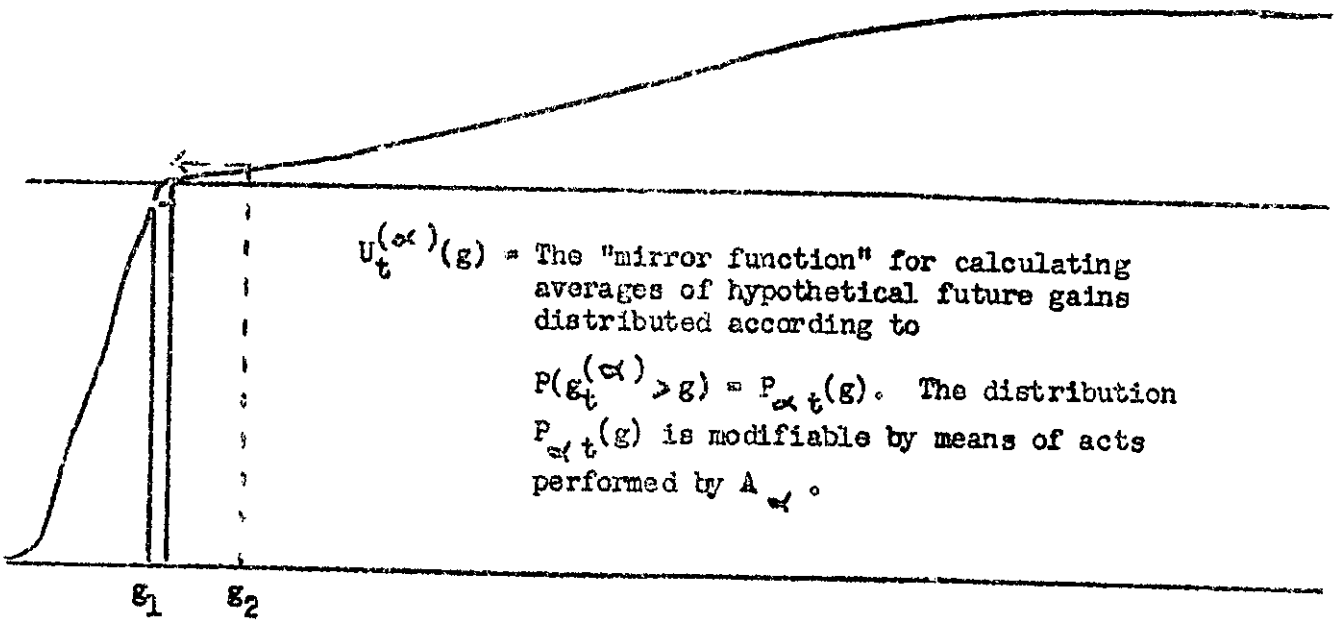
α = metaphorically "the position of the ball the players A_α try to push each in the direction of increasing $g_{tP_\alpha}^{(\alpha)}$ by modifying P_α.

(31) $g^{(A)}_{tP_A} = \sum_{\alpha} g^{(\alpha)}_{tP_\alpha}$ = a convenient (conventional) indicator of goodness for an "altruistic" well informed spectator judging the goodness of the whole game played by players $\subset A$.

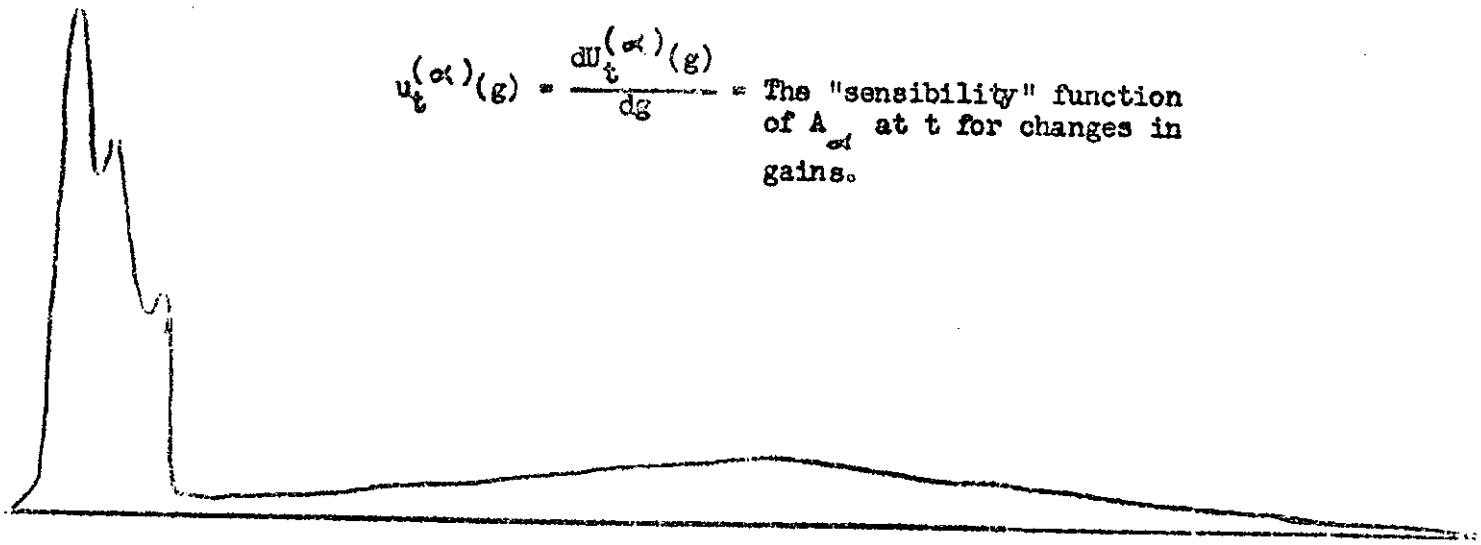
(32) The research workers $A' \subset A$ try to increase $g^{(A)}_{tP_A}$ by efforts to produce and export information and import economic resources.

(33) The suppliers $A'' \subset \dot{A}$ of economic resources for the research game export economic resources C and some information about goals to A', to get increased import of high valued information from A' in exchange. Other subjects than the suppliers pay usually only for the physical products ($\subset$ C) used as a mean for communicating I. The suppliers A" often act as representatives for larger subsets of A than those included in A". They have namely often an "altrustic" indicator of goodness, that is, some weighted sum of estimates of $g^{(\alpha)}_{tP_\alpha}$.

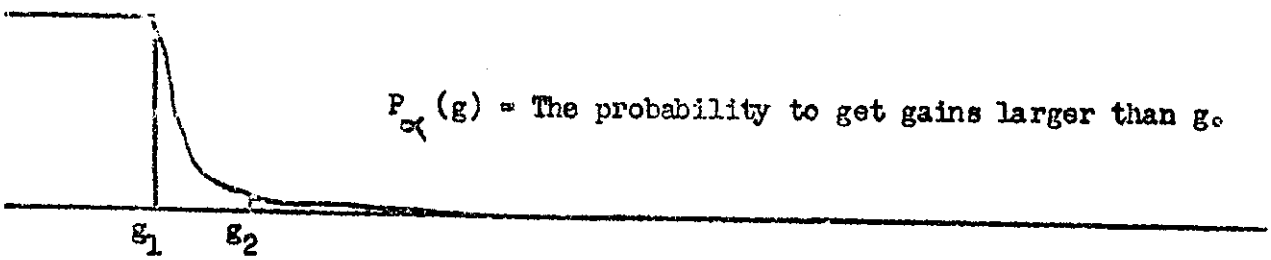
DIAGRAM I



$U_t^{(\alpha)}(g)$ = The "mirror function" for calculating averages of hypothetical future gains distributed according to $P(g_t^{(\alpha)} > g) = P_{\alpha t}(g)$. The distribution $P_{\alpha t}(g)$ is modifiable by means of acts performed by A_α.



$u_t^{(\alpha)}(g) = \frac{dU_t^{(\alpha)}(g)}{dg}$ = The "sensibility" function of A_α at t for changes in gains.



$P_\alpha(g)$ = The probability to get gains larger than g.

$$U_t^{(\alpha)}(g_{t\hat{P}_{\alpha t}}^{(\alpha)}) = E\, U_t^{(\alpha)}(g_t^{(\alpha)}) = \int_{+\infty}^{-\infty} U_t^{(\alpha)}(g)\, d\, P_{\alpha t}(g) = \int_{-\infty}^{\infty} P_{\alpha t}(g)\, u_t^{(\alpha)}(g)\, dg.$$

$g_{t\hat{P}_{\alpha t}}^{(\alpha)}$ = Average gain mirrored through $U_t^{(\alpha)}(g)$.

For the above pictured distribution $P_\alpha(g)$ and $U_t^{(\alpha)}(g)$, $g_{t\hat{P}_{\alpha t}}^{(\alpha)} \approx 0$ in spite of the fact that the arithmetical mean is clearly > 0.

DIAGRAM II

Decomposition of $U_t^{(\alpha)}(g) \approx \sum_{\lambda=1}^{4} U_{\lambda t}^{(\alpha)}(g) = \sum_{\lambda=1}^{4} r_\lambda \Phi\left[\frac{g - m_{\lambda t}^{(\alpha)}}{\sigma_{\lambda t}^{(\alpha)}}\right]$

$U_{1t}^{(\alpha)}(g) \approx r_1 \Phi\left[\frac{g - m_{1t}^{(\alpha)}}{\sigma_{1t}^{(\alpha)}}\right]$

g=0

$U_{1t}^{(\alpha)}(g)$ increases with the expectation of A_α not to be "broken down" if he gets a gain = g and lives very carefully and decreases with the decrease in well-being necessary if $g < 0$.

$U_{2t}^{(\alpha)}(g) \approx r_2 \Phi\left[\frac{g - m_{2t}^{(\alpha)}}{\sigma_{2t}^{(\alpha)}}\right]$

g=0

$U_{2t}^{(\alpha)}(g)$ = increases with the expectation of A_α to get more credit if he asked for it, if he has to take a loss = -g.

g=0 $U_{3t}^{(\alpha)}(g) \approx r_4 \Phi\left[\frac{g - m_{3t}^{(\alpha)}}{\sigma_{3t}^{(\alpha)}}\right] \approx$ increases with the expectation to be able to "live as usual" without borrowing money if A_α has to take a loss = -g.

$U_{4t}^{(\alpha)}(g) \approx r_4 \Phi\left[\frac{g - m_{4t}^{(\alpha)}}{\sigma_{4t}^{(\alpha)}}\right]$

$U_{4t}^{(\alpha)}(g) \approx$ increases with A_α's expected ability to improve his physical well-being if he gets a gain = g.