COWLES COMMISSION DISCUSSION PAPER: ECONOMICS 2047

Economical Routing of Empty Railroad Freight Cars*

Kirk Fox

July 1, 1952

This paper reports an attempt at empirical application of certain activity analysis (linear programming) techniques to the problem of finding an optimal overall routing plan for the movement of empty railroad freight carriers from points of availability to points of need. The theoretical model for this work was developed by Tjalling C. Koopmans [1947], [1951]. As he pointed out, finding such an optimal routing plan also provides important information relevant to marginal cost rate setting. There are two parts to the cost of transporting a freight consignment, the direct cost of loading, moving, and unloading the freight, and the indirect cost of moving the empty car to where it is needed for further use. On the basis of the optimal routing plan a potential may be found for each shipping point. The differences between each pair of these potentials give indexes of the indirect cost (for instance could be multiplied by cost per mile of moving empties to convert them to costs). These potentials will change if the pattern of needs and availabilities changes enough to alter the set of routes which makes up the

* Research undertaken under contract between the Cowles Commission for Research in Economics and the RAND Corporation.

optimal routing pattern.[1] In other words, the potentials are a set of relative prices for empty carriers at different points of the map.

The analysis has been given a somewhat dynamic aspect by the simultaneous examination of eight separate but adjoining periods embracing all seasons and two years, one of the latter a year of adjustment to semimobilization.

While it may be hoped that profit motives are quite effective forces for bringing about efficient movements of empty carriers within local areas or within the domain of any particular road, this does not imply that the overall efficiency of the railroad system has been served. Furthermore, adjustment of car flows to new situations of supply and demand for empty carriers must reach its efficient program by a trial and error process over time. This adjustment period means extra costs to the system. By the techniques below and using computing apparatus now available this trial and error procedure can be short-circuited and performed on paper and within electric brains without one car being moved. If there are overall efficiencies which can be found then the savings to the system can be used in part to compensate those who must operate at something different from their own most efficient local program.

The first motivation for this scheme was that it should be a tool for efficient transportation mobilization under war conditions, but if it does mean real savings to the system a cooperative application of its methods by the railroads would be worthwhile in peacetime as well as in

---

1. For instance, below, for the second and fourth quarters of 1949 there is considerable difference between the patterns of surpluses but the routes of the optimal graph and the potentials for all centers remain the same for both periods.

wartime.[2]

Once the routing authority (U. S. Government or A.A.R.) has determined its optimal routing plan it may <u>order</u> the movement of empties from one place to another according to its plan. Alternatively, and with probably much less policing and coercion, it would perhaps be possible to adjust freight rates according to the indirect cost differentials (potentials) and let profit motives direct movements of cars in accordance with the optimal program.[3] [4]

If the <u>changes</u> in the pattern of needs for and availabilities of empty carriers are <u>not</u> great from period to period their routing pattern will remain essentially the same and no very costly trial and error program will be necessary. One purpose of this study has been to see if <u>seasonal</u> changes in the pattern do force significant changes in the overall empty routing pattern.

That special events <u>can</u> force drastic changes on the routing pattern can be well remembered from the World War II shift in 1945 from concentrated effort in Europe to concentrated effort in Asia. The year 1950 is examined

---

2. That even modest efficiences in handling empties would be very profitable is indicated by the following from I.C.C. Bureau of Accounts and Cost Finding, Rail Carload Cost Scales by Territories as of Jan. 1, 1951. Statement No. 1-51.

"The ratio of empty to loaded car miles varies from 1.00 for Stock and tank cars to 0.32 for box cars. The effect of decreasing the empty return ratio by ten percentage points would result in decreasing out-of-pocket costs, for shipments having the average length of haul, by approximately 3%."

3. These adjustments of freight rates to induce proper movements of empties could possibly be handled so that on the average the status quo with respect to rate structure could be maintained, in other words, so that none of the vested interests who make up the balance of power for our present rate structure would take exception to this scheme on the ground that it would upset the "excellent" rate schedule now in effect.

4. The rate technique would probably involve greater lags in execution than the direct order technique. This might either restrict the frequency of rate changes over the year or demand fairly advanced predictions of needs and future rates.

here to see if an event of considerably smaller magnitude, the shift to semimobilization, can bring short run changes which would make these computing techniques fruitful.

The sense in which the routing plan of this study is optimal is that for a given program of loaded movements the total distance moved by all empty carriers is made as small as possible. Clearly, distance moved is not the only criterion of cost or efficiency in the handling of empties,[5] but it has been used here to represent whatever might be considered most suitable, and is probably not too bad an index.

A simplifying assumption of this method is that there are concentration points for empty cars and that all cars move between these points and are not picked up or put off along the way. This crude approximation to reality is quite harmless where the data are for a fairly large number of real railroad centers, but in this example, where only state data are used and these are further aggregated into twelve areas, considerable discrepancy from reality is unavoidable.

For the methods used the ideal model would be one where each area is served by a single center which handles all movements of carriers to and from itself, thereby acquiring in any given time period a surplus of empty carriers (positive or negative). Further, each center would be joined to each other center by a transportation route.

The ideal model could be closely approximated by including small enough cities and towns but the computation for such a disaggregated

---

5. Time lost while empty is possibly often more important than distance moved empty, and where the shortest routes are congested, empties may be sent 'round Robin Hood's Barn to save time. Another case in point might be that in which the shortest route in miles involves mountains or other use of extra equipment and a more circuitous valley route means lower costs in either time or money. These more sophisticated problems are concurrently being treated by Martin Beckmann of the Cowles Commission.

system would be an almost hopeless task (assuming data available) and would not be necessary for a good approximation to the solution of the overall optimal routing plan. A plan for the movements of empties between the fifty to seventy-five major railroad points would probably be very adequate, and with new high speed computing equipment would be quite manageable.

For this paper a very much less adequate aggregation scheme had to be used. Published data were only available by states, as indicated in footnote 7, and only manual computing equipment was available.

It was arbitrarily decided that ten or twelve groups of states would be manageable by the limited computing facilities. These were selected to correspond as closely as possible to the ideal type area. That is, the rail center for each area should be a true clearing center for as much as possible of the local traffic for the area and for as little as possible of the local traffic for other areas. This was desirable so that when the carloads originated in an area were subtracted from the carloads terminated in the area the difference between the two would be as accurate an approximation as possible to the actual surplus (positive or negative) of empty cars for the area center. (For example many towns in the Illinois area are short-hauled to from the centers of another area, the statistics showing a car surplus (±) in the Illinois area, whereas actually this surplus is in the area of the center serving the towns, say, St. Louis.)

Since the theoretical model treats each area as having a single center, the actual case of multiple important centers was a further handicap to the analysis, especially where the real centers were at extreme ends of an area and divided its traffic so that it would more properly have been divided into two areas, e.g., the California area.

In actually working out the solutions a table of distances between area centers was needed. For this purpose the center for each area was either the natural existing one, e.g., Chicago, or else a semi-intuitive center of gravity of the multiple centers of the area, weighted in part by geographical and rail line considerations and in part by the relative importance of the multiple centers in amount of traffic carried.

Clearly, the availability of only state data and the aggregation into a relatively small number of areas forced some very arbitrary compromises.

The needs for empty carriers which will actually be provided and the available empties which will actually be moved must be found for each area. More exactly, only the net surpluses (positive or negative) need be known since freight movements will provide for all but these amounts of empties. The sum of the positive surpluses must of course equal the sum of the negative surpluses, that is, what is provided is not more than what is available.

Another simplifying assumption is that all carriers are homogeneous. Because of this assumption each type of car must be considered separately.[6] This study has restricted itself to box cars.

The following maps are in pairs by quarters. On the first of each pair are shown (in boxes) the twelve surpluses of empty boxcars, each at the center of its area. Joining the centers are the routes which have

---

6. This, of course, still doesn't satisfy the homogeniety assumption since there is considerable variation among cars of any particular type, and further since even identical box cars may become contaminated or for some other reason are not interchangeable for all box car carried commodities.

been found to be optimal (minimize total distance moved by empties) by computations using the information on surpluses and the table of distances between area centers.[7/] The routes shown are the only ones over which inter-area movements of empties will be made, the amount of traffic over each being also shown.[8/] The iterative techniques used here were those of Koopmans/ and Reiter [1951]

---

7. The greatest task involved in this entire study was to find the approximations to the values for the surpluses (positive and negative) which appear on the eight maps above.

Whereas, we desired to have the actual numbers of empty cars of a particular type which were in surplus for each of several major railroad centers, we actually had (a) the number of tons of revenue freight (b) of each commodity type (c) reported by (d) only Class I U. S. railways to have been, (e) originated and (f) terminated (g) in each state and Canada. To help we had (h) Interstate Commerce Commission Bureau of Transport Economics and Statistics, Carload Waybill Analysis, 1949, Distribution of Carloads by Weight Brackets for Each Commodity Class by Type of Car, Statement No. 5058, November 1950, which showed as an average for the whole country for an entire year for a 2% sample what percent of each commodity type was distributed to each type of car and also (j) Association of American Railroads, File: 148-10, Carload Traffic Originated and Tons per Car, June 9, 1950, which showed as an average for the whole country and for the entire year how many tons of each commodity were carried per car.

The methods used were as follows: (k) (e) was subtracted from (f) for each commodity type. (l) The (k)'s were aggregated into groups of states according to the scheme above. (m) The (l)'s were multiplied by (j) to convert tons of commodities to cars of commodities. (n) The (m)'s were multiplied by (h) to convert cars of commodities into box cars. (o) The (n)'s were summed for each area to give positive and negative box cars for each area. Chiefly because of faulty reporting (c) but also because (d) only class I railroads reported and because (d) and (g) only U. S. roads reported and Mexico was not included, the sum of the negatives did not equal the sum of the positives. (p) This was crudely adjusted by assuming that the percentage error by which the positives were too small, x, was equal to the percentage error by which the negatives were too large, x, yielding the following relation.

$(1 + x)\ \Sigma\ (\text{positives}) + (1 - x)\ \Sigma\ (\text{negatives}) = 0$

whose solution gave correction factors $1 + x$ and $1 - x$, to be multiplied by each of the (o)'s.

8. That these routing plans are indeed optimal may be checked using the necessary and sufficient conditions below from the literature cited, i.e., that each routing plan has associated with it a set of potentials, p_i, such that

$$(1)\ p_i \leq p_j + s_{ij} \quad \text{for all } (i, j),$$

$$(2)\ p_i = p_j + s_{ij}, \text{ if } x_{ij} > 0,$$

where s_{ij} are the distances between centers and x_{ij} are the flows between centers i and j.

and involved only very simple arithmetical operations. For a more realistic less aggregative solution the number of such arithmetical calculations would become overpowering and electronic computations would have to replace human ones.[9/] The second map of each pair shows the potentials (relative prices) for empty boxcars at each of the centers.

---

9. In the activity analysis model in the case of only four areas:

Activities - Moving Empties From:

	x_1 A to C	x_2 A to D	x_3 B to C	x_4 B to D
Surplus A = α	1	1	0	0
at: B = β	0	0	1	1
Negative C = γ	-1	0	-1	0
Surplus at: D = Δ	0	-1	0	-1
Miles = M	m_{AC}	m_{AD}	m_{BC}	m_{BD}

Yielding:

$$\alpha = x_1 + x_2$$

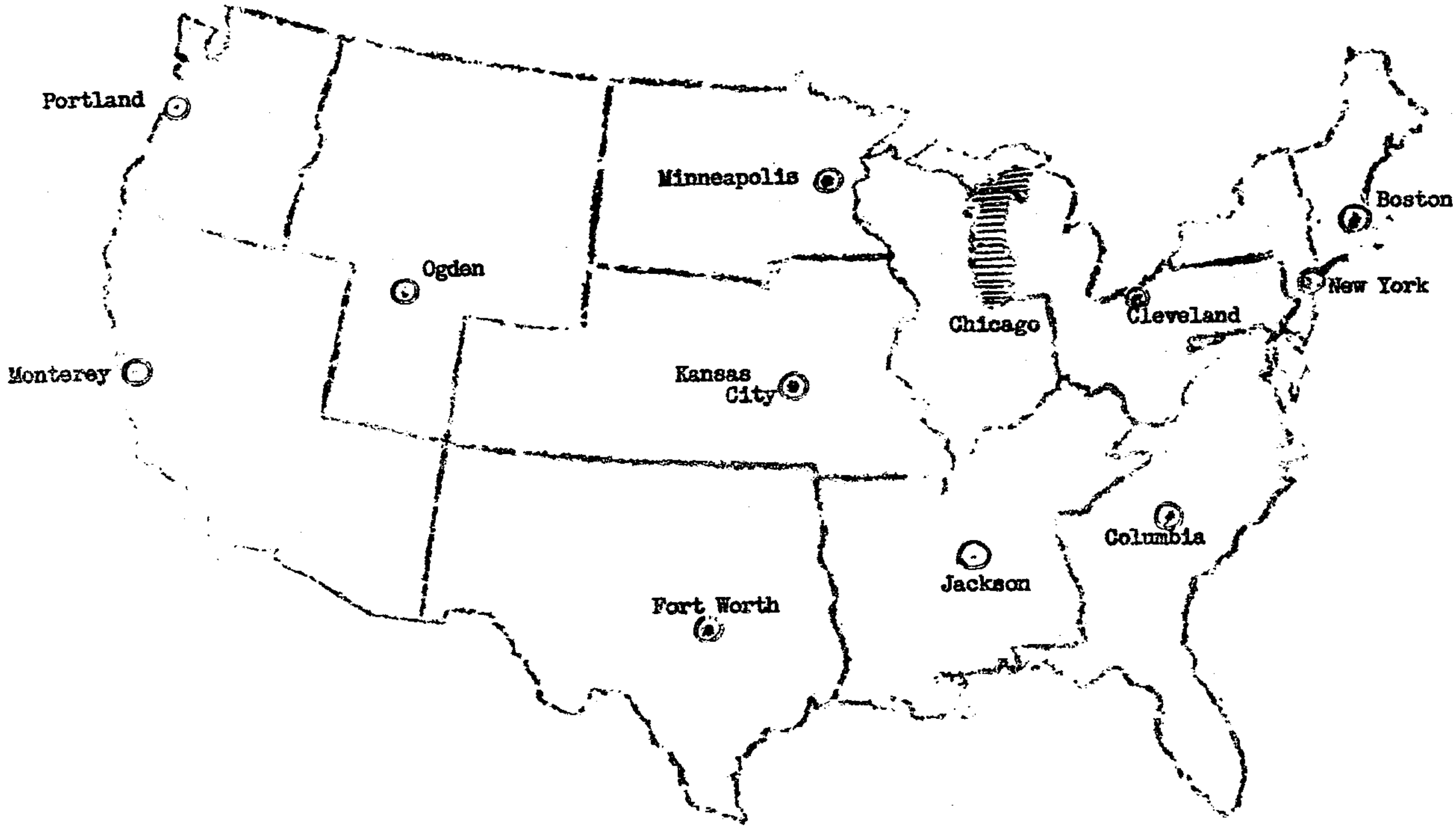
$$\beta = x_3 + x_4$$

$$\gamma = -x_1 - x_3$$

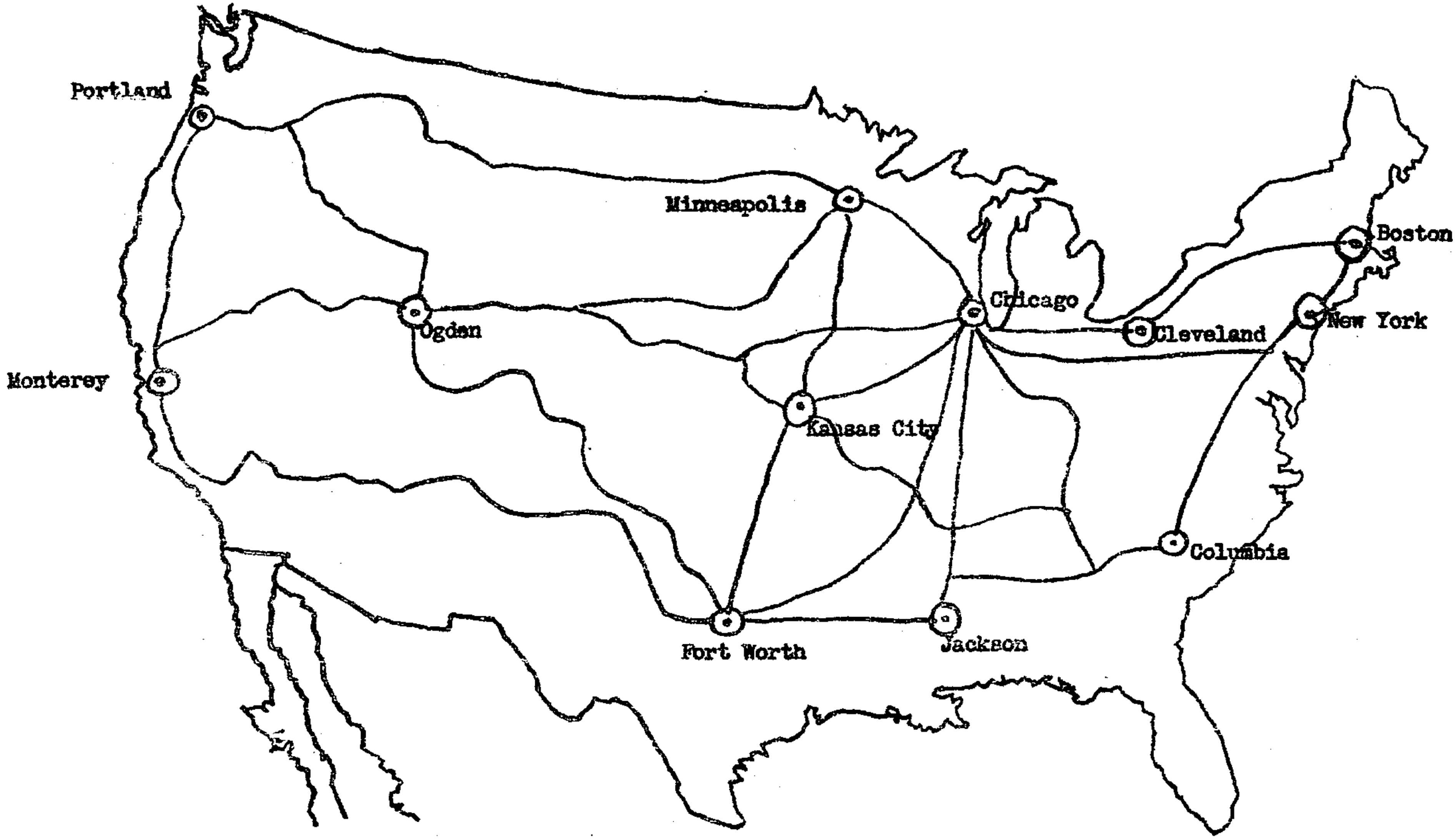
$$\Delta = -x_2 - x_4$$

$$M = m_{AC}x_1 + m_{AD}x_2 + m_{BC}x_3 + m_{BD}x_4,$$

where the last function is to be minimized subject to the first four conditions. Of course many of the x_i could be seen to be zero by gross geographical intuitive comparisons. For computation techniques and suggestions see Cowles Commission Monograph 13 referred to in Koopmans and Reiter [1951].



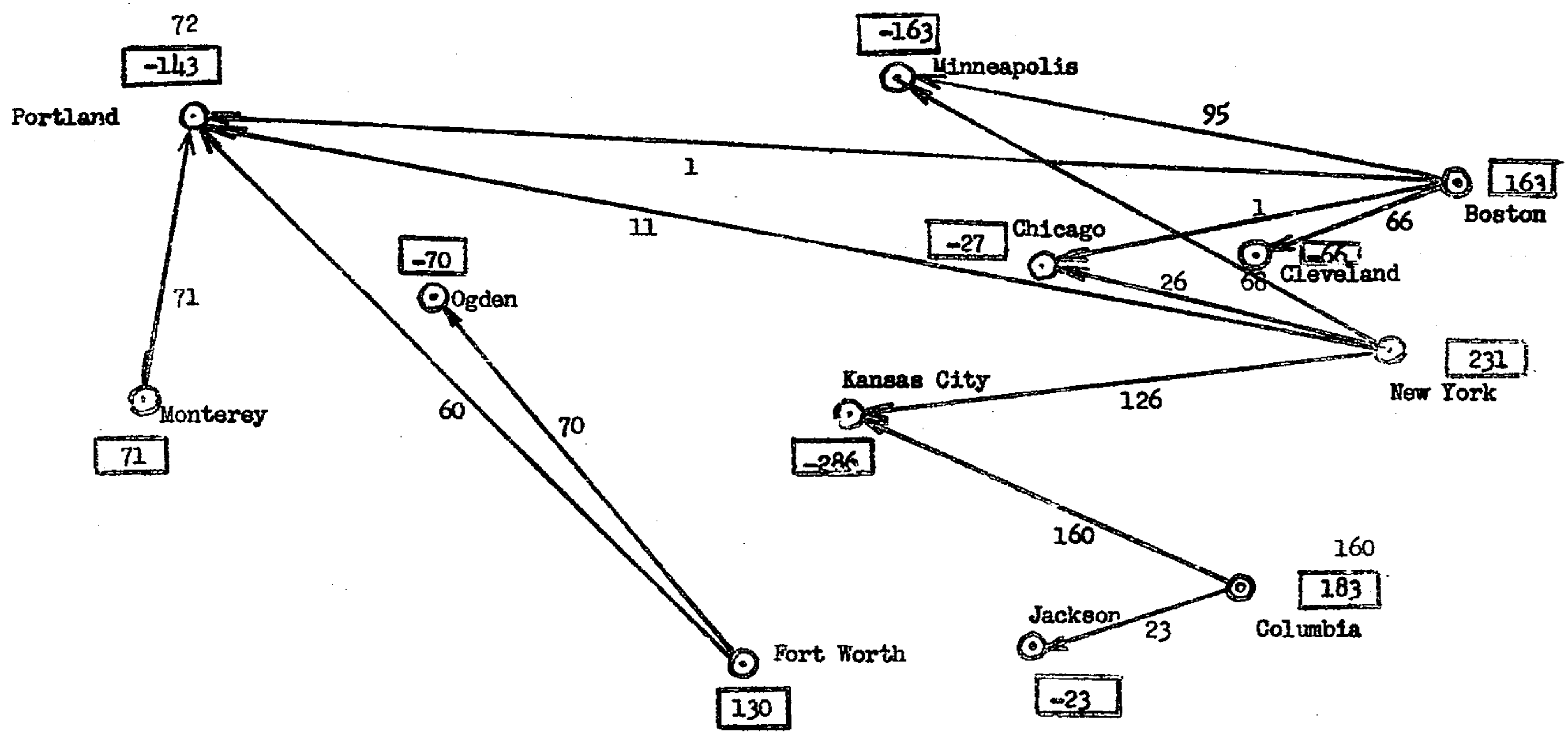
Map Showing
Areas and Centers



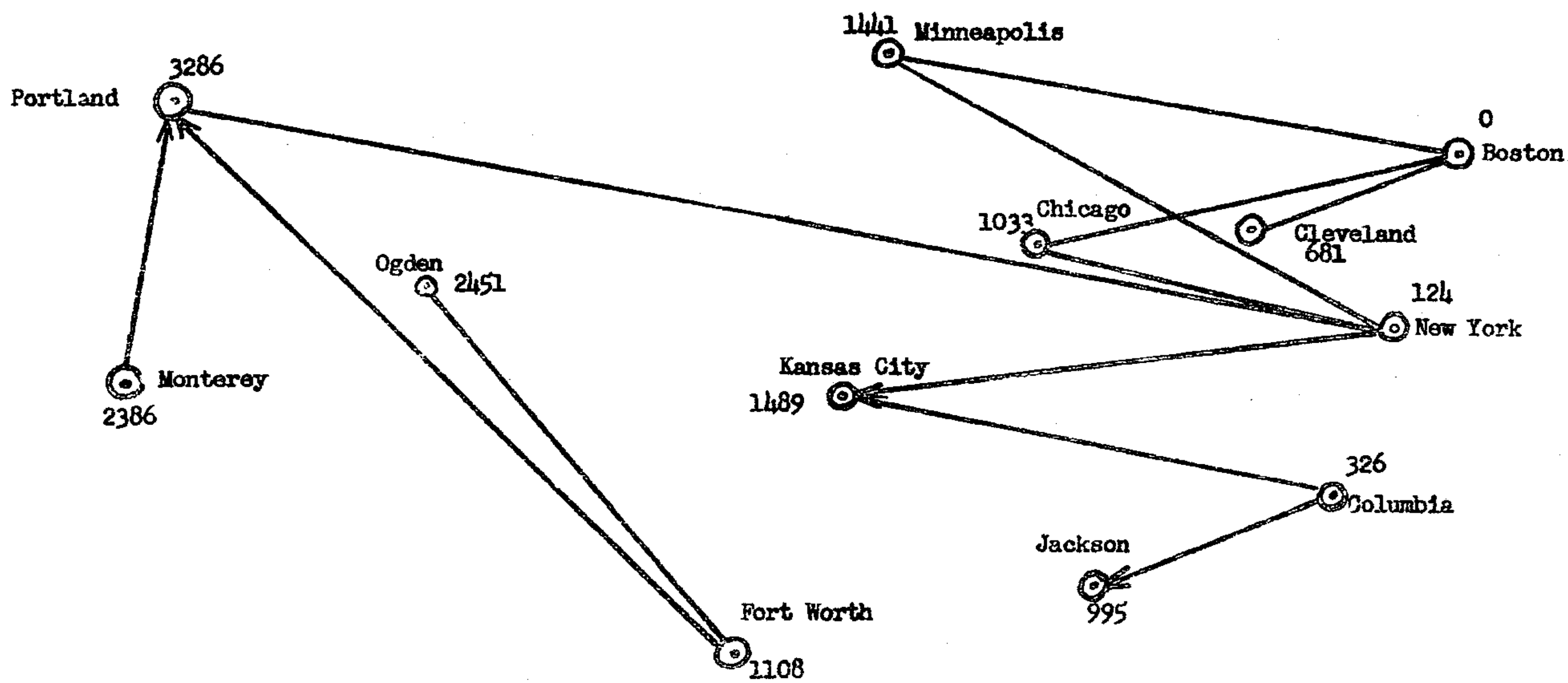
Rail Network Serving
Twelve Selected Centers

	Boston, Mass.	New York, N. Y.	Columbia, S. C.	Cleveland, Ohio	Jackson, Miss.	Chicago, Ill.	Minneapolis, Minn.	Kansas City, Mo.	Fort Worth, Texas	Ogden, Utah	Portland, Oregon	Monterey, Calif.
Boston, Mass.	X	232	935	681	1470	1033	1441	1513	1937	2521	3286	3432
New York, N. Y.	232	X	703	579	1239	909	1317	1365	1773	2397	3162	3308
Columbia, S. C.	935	703	X	863	669	886	1294	1163	1106	2265	3050	3132
Cleveland, Ohio	681	579	863	X	992	357	765	786	1256	1845	2610	2756
Jackson, Miss.	1470	1239	669	992	X	747	1123	726	437	1780	2615	2247
Chicago, Ill.	1033	909	886	357	747	X	408	480	1004	1488	2253	2399
Minneapolis, Minn.	1441	1317	1294	765	1123	408	X	501	1007	1381	1815	2292
Kansas City, Mo.	1513	1365	1163	786	726	480	501	X	536	1169	1954	2080
Fort Worth, Texas	1937	1773	1106	1256	437	1004	1007	536	X	1343	2178	1810
Ogden, Utah	2521	2397	2265	1845	1780	1488	1381	1169	1343	X	851	911
Portland, Oregon	3286	3152	3050	2610	2615	2253	1815	1954	2178	851	X	900
Monterey, Calif.	3432	3308	3132	2756	2247	2399	2292	2080	1810	911	900	X

Table of Railroad Distances

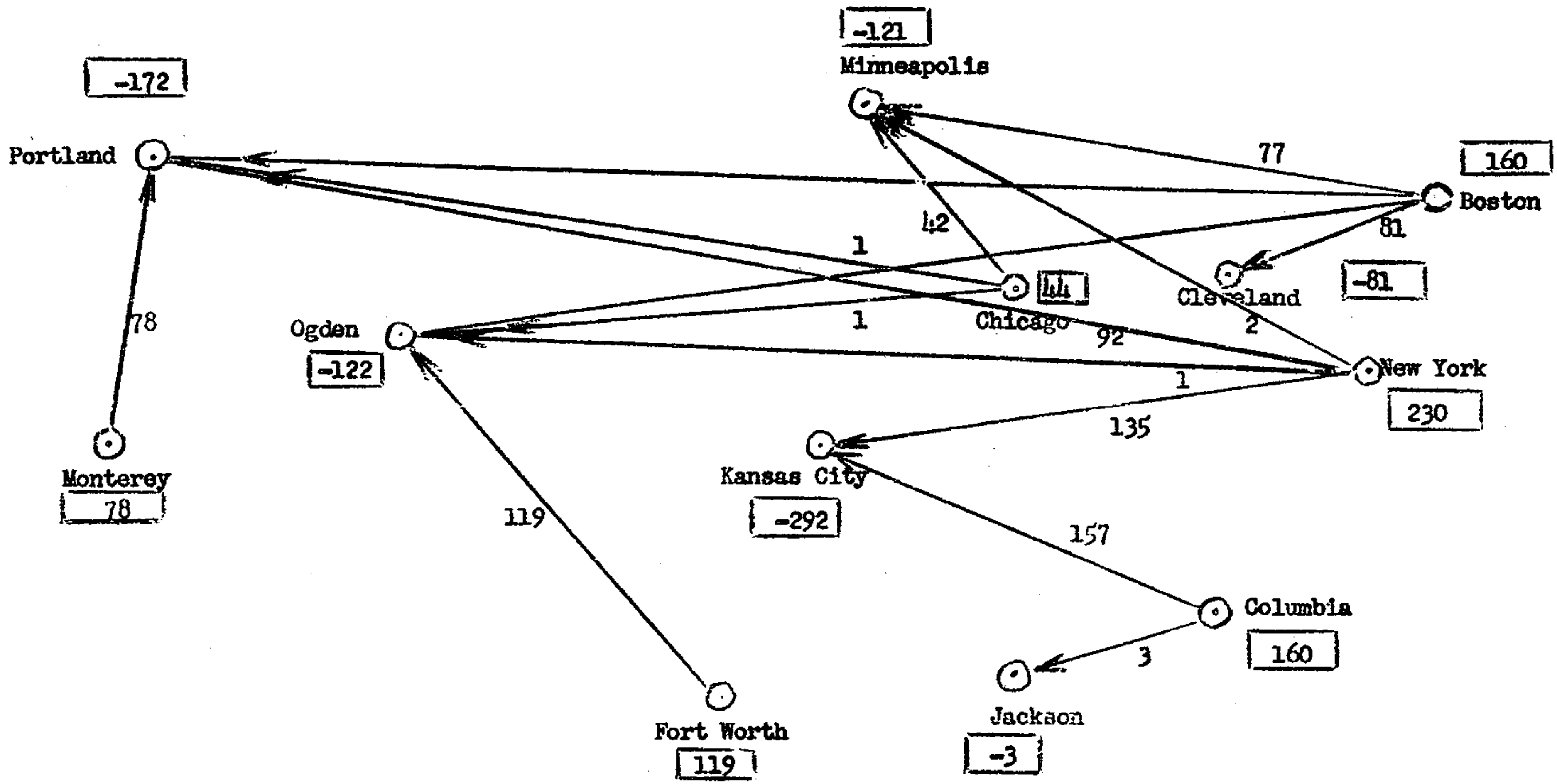


Surpluses and Optimal Routing Plan
First Quarter, 1949

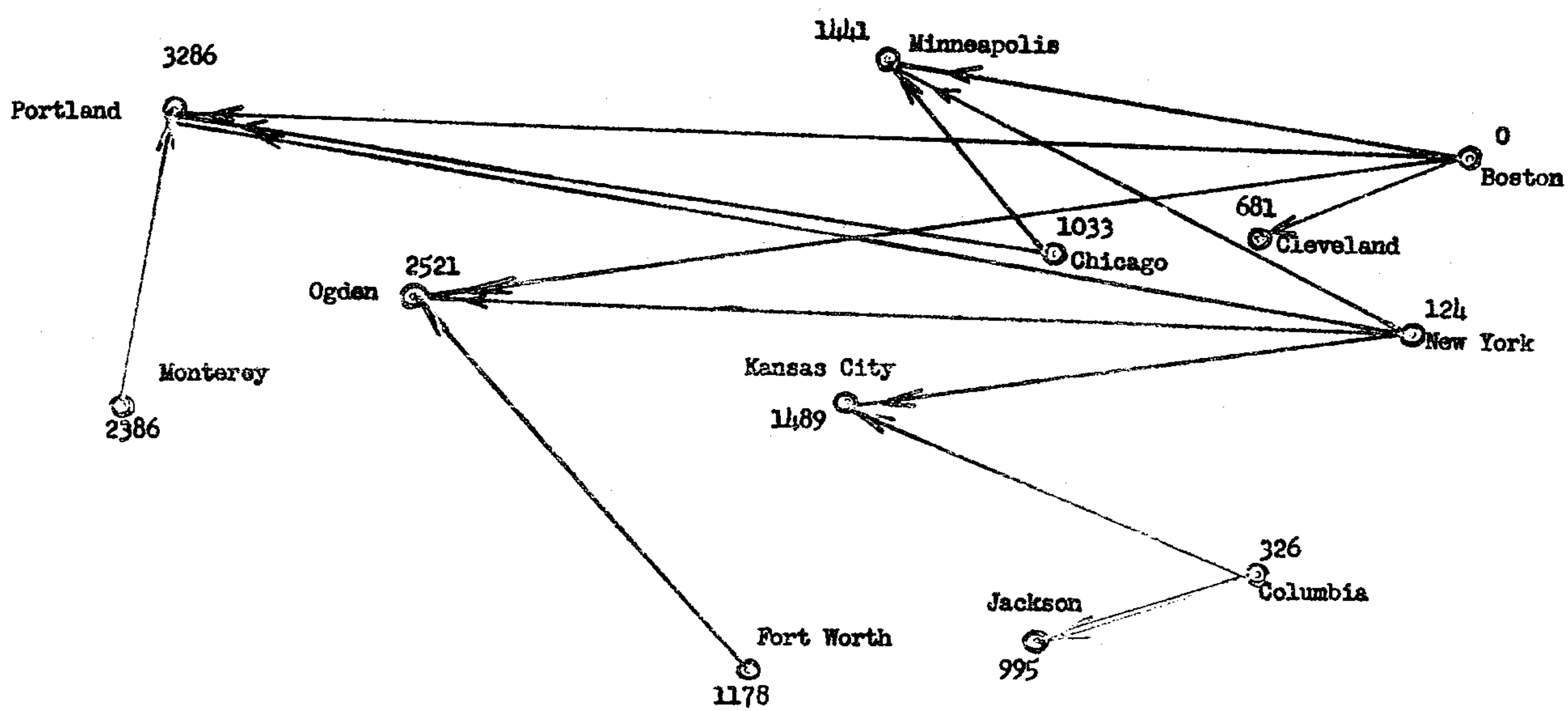


Potential Map

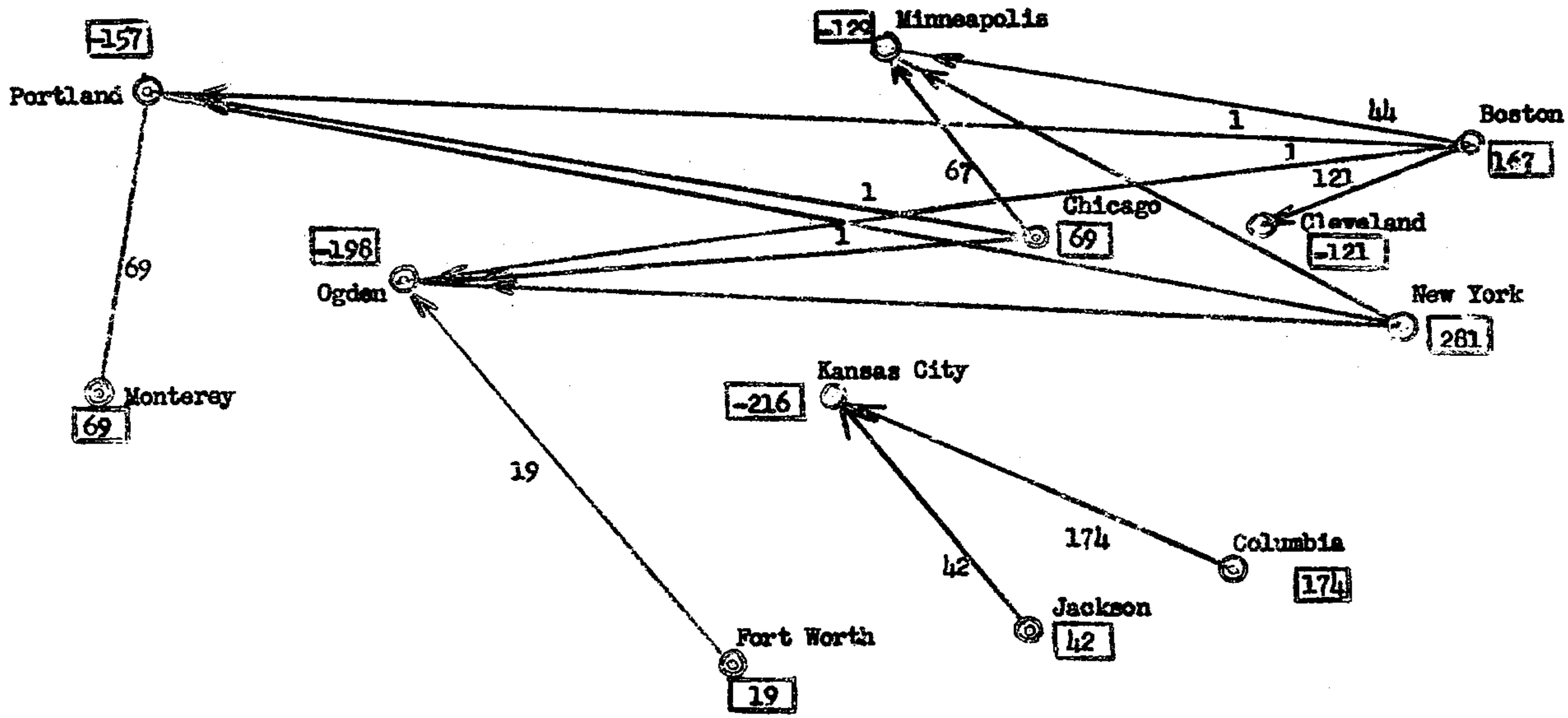
First Quarter, 1949



Surpluses and Optimal Routing Plan
Second Quarter, 1949

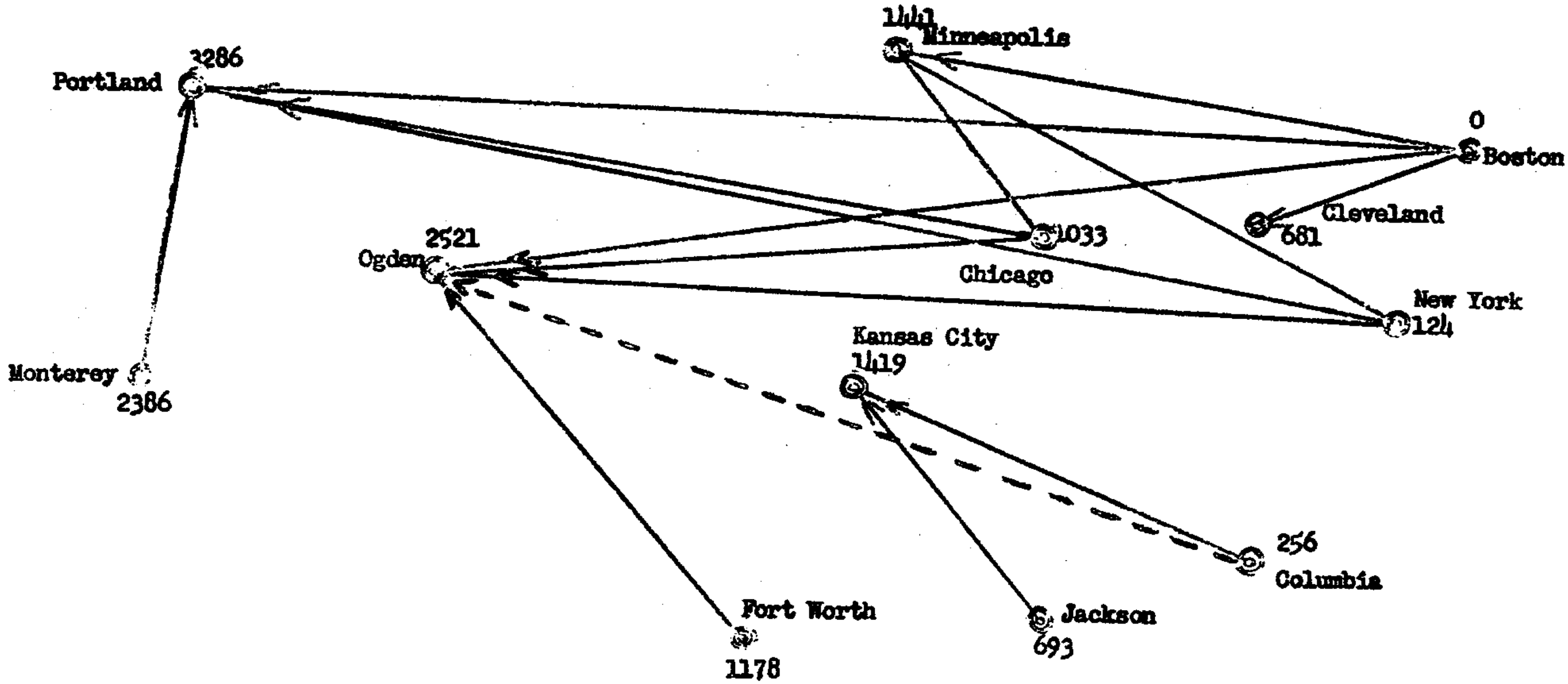


Potential Map
Second Quarter, 1949



Surpluses and Optimal Routing Plan

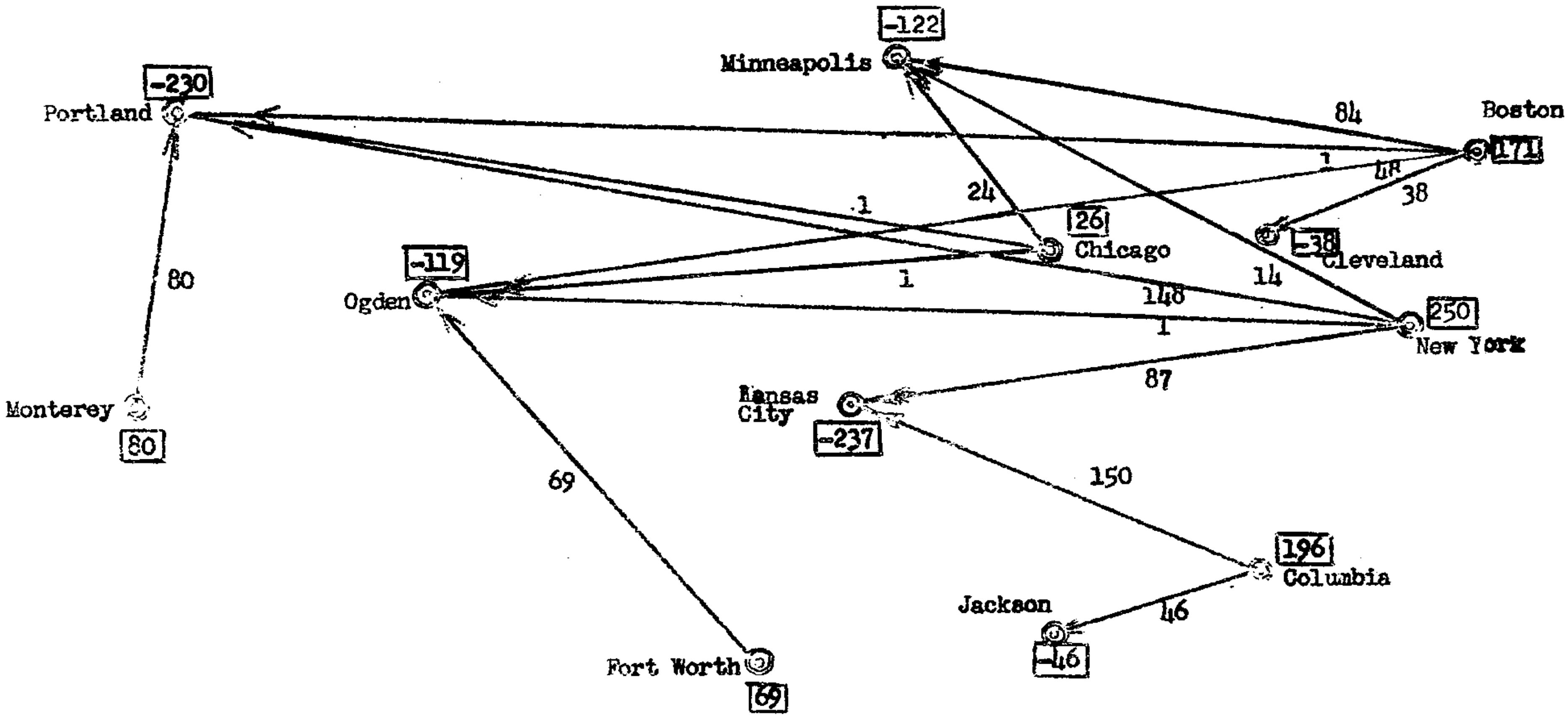
Third Quarter, 1949



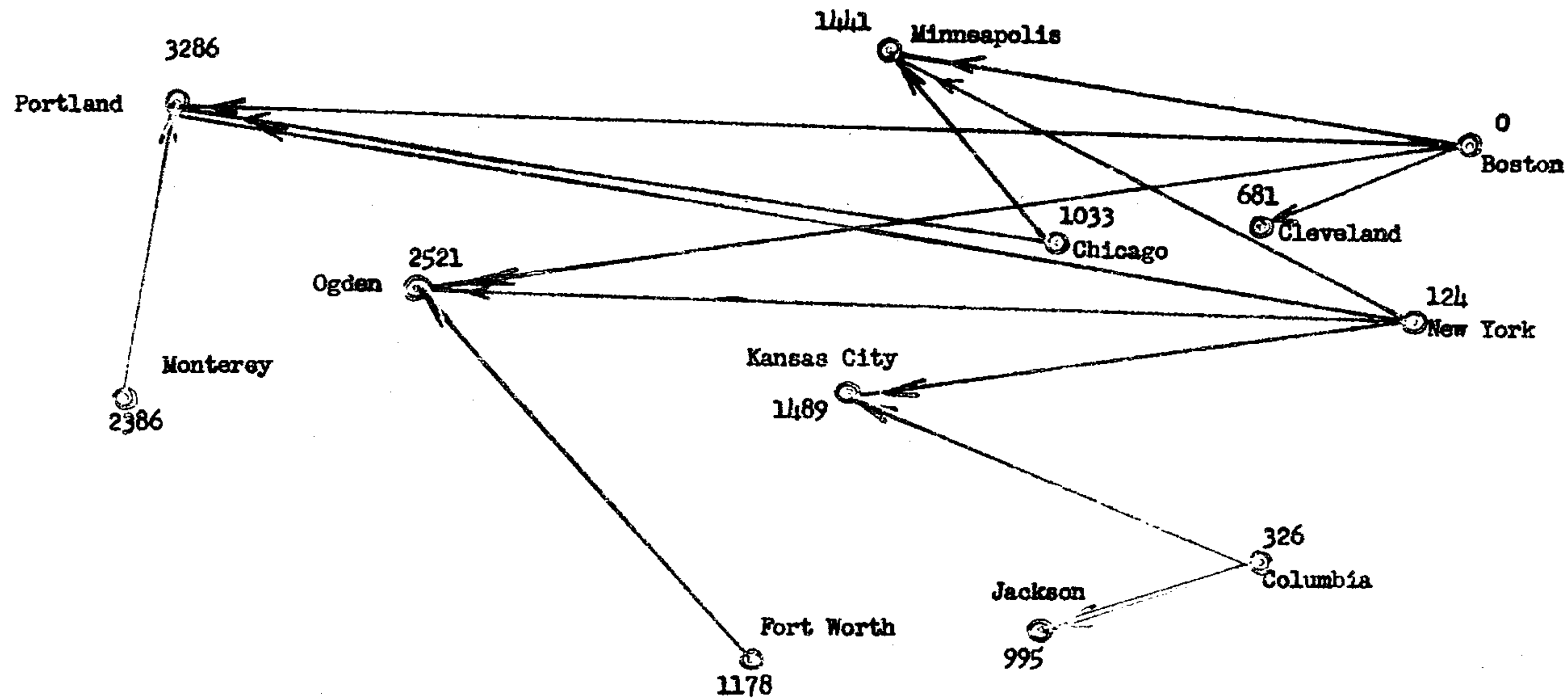
Potential Map

Third Quarter, 1949

Note: If surplus figures had not been rounded a very small number of cars would have moved from Columbia to Ogden.



Surpluses and Optimal Routing Plan
Fourth Quarter, 1949



Potential Map
Fourth Quarter, 1949

Each map is drawn on the "assumption" of indefinite continuation of the quarterly program of surpluses on which it is based.

If a plan is optimal it need not be unique. In real situations there may frequently be a number of solutions all associated with the same total mileage minimum. This situation occurs whenever the routing program contains a neutral circuit. A neutral circuit is one involving at least four centers (two with negative and two with positive surpluses) with distances between them of just such magnitude that, from the point of view of minimizing total distance moved, it is indifferent which of the positive surplus centers provides empties for which of the negative surplus centers, provided, of course, that needs are exactly satisfied and availabilities are exactly utilized. A neutral circuit will always arise when routes must pass a common point.

A matrix may be made with rows designated by positive surplus centers and columns designated by negative surplus centers, the elements being the flows between the paired designators. If by any interchanging of rows and columns it is possible to obtain a submatrix none of whose elements vanish, the designators of the rows and columns of this submatrix are involved in a neutral circuit. (The converse is not true. Since the entries in the submatrix are quite arbitrary some could have been made zero.) Such a table is shown here for the routing plan for the Second Quarter of 1949. In this case movements from New York, Boston, and Chicago to Minneapolis, Ogden and Portland all pass through Chicago, and are therefore involved in a neutral circuit.

Optimum Flows Second Quarter 1949

	Cleveland	Jackson	Kansas City	Minneapolis	Ogden	Portland
Boston	81	0	0	77	1	1
New York	0	0	135	2	1	92
Chicago	0	0	0	42	1	1
Columbia	0	3	157	0	0	0
Fort Worth	0	0	0	0	119	0
Monterey	0	0	0	0	0	78

The year 1949 embraced a very sharp decline in freight carloadings. This might have been expected to cause important shifts in distribution of loaded carmovements and therefore in movements of empties. It is interesting to note that the optimal routing pattern and the potential maps changed very little in this period. The position of Lake Michigan as an obstacle to the straight line movement of the greater part of the east to west movement of empty carriers accounts for the relative stability of the potentials under considerable change in the pattern of surpluses. Indeed the fact that so many centers are involved in this single neutral circuit makes the solution of this problem at once simpler and less interesting. If the seasonal changes we have found for 1949 are typical (i.e., no greater than for most years) then it is perhaps not worthwhile to make monthly or quarterly determinations of the optimal routing patterns and potentials, unless greater disaggregation is also introduced. A very simple rule for holders of empty cars would have been applicable for the quarters and centers examined here:

Move all empty boxcars to the nearest points of need until either the need is satisfied or the supply of empties is exhausted according to the following priorities, first, northwest; if no near needs in that direction or when needs satisfied west; if no near needs in that direction or when needs satisfied north; if all such needs are satisfied then next point further west.

Bibliography

Koopmans, Tjalling C., [1947], "Optimum Utilization of the Transportation System" in Proceedings of the International Statistical Conferences, 1947, Washington, D. C., Vol. 5 (Vol. 5 reprinted as Supplement to Econometrica, Vol. 17, 1949, pp. 136-146; Abstract in Econometrica, Vol. 16, January, 1948, pp. 66-68).

Koopmans, T. C., and Reiter, S. [1951], "A Model of Transportation" in Activity Analysis of Production and Allocation, Tjalling C. Koopmans, Ed., New York, John Wiley and Sons, Inc., 1951, pp. 222-259.