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A Proposed Inquiry into some Markets with Forward Trading

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Economic dynamics is mainly concerned with the joint effects of three complications in the static scheme:

1. Some commodities can be used at alternative times.
2. Production and consumption cannot be adjusted instantaneously to changes in external conditions.
3. Future events are not known with the same certainty as past or present events.

Any pair of these complications call for nontrivial extensions of static theory, but in most practical problems all three will arise. The third point has proved to be the most troublesome one and despite considerable research, some of it on a very high level, a generally accepted and empirically valid approach has not yet emerged (cf. Arrow [1]). In the circumstances of an investigation of situations in which these three elements are conspicuous may help towards the development of realistic theories in the dynamics of uncertainty. It could hardly be hoped that this would lead to a definite theory of expectations, but some sorting out of more and less important aspects of the problem would already be a contribution.

The value of such an empirical investigation will of course be greatly enhanced if the sector to be studied is interesting in its own right. In

that case our knowledge of the economy may be increased both directly and indirectly. The inquiry would be still more valuable if it provides additional experience with recent statistical techniques, notably those expounded in [11].

An important device, developed in the practice of business to meet the three difficulties mentioned above, is that of forward trading, i.e., trading for future delivery. The best-known instances are commodity futures and forward exchanges (futures in foreign currency); slightly more farfetched examples are loans with fixed interest and time charters in tramp shipping. The theoretical importance of forward trading has been recognized by some writers on dynamic economics, especially by Hicks [6], who relies heavily on Keynes' theory of "normal backwardation" [10]. According to this theory future prices will usually be less than expected spot prices by the amount of a premium which sellers of futures (presumed to be hedgers) pay to speculators taking over the risk of holding stocks. In its original form this theory will not bear examination, and it has been rightly attacked on various counts, but it nevertheless contains a grain of truth as we shall see below.

There are future markets in a wide variety of commodities, even in such surprising items as onions. The most important markets in the U.S. are those in grains (esp. wheat and corn) and in cotton; these are at the same time the crops on which the prosperity of American agriculture largely depends and on which a great deal of statistical information is readily available. Wheat and cotton are much affected by international factors that are not directly relevant to our problem. This does not apply to corn, which has other technical advantages over wheat in that there is a clearer distinction between old-crop and new-crop futures and that trading is more centralized, viz., in Chicago. On the other hand the

demand for corn, being a feed grain, has a much more complicated pattern than the demand for wheat. These complications, which result from variations in livestock inventories, are in themselves of great interest to dynamic theory and may lead to comparisons between perishable and non-perishable commodities. There is moreover a futures market in lard, which is a corn product.

As a first approach it may therefore be convenient to devote special attention to corn and the commodities related to it. Any conclusions of apparently wider applicability that may emerge can then subsequently be tested with reference to other commodities, and the underlying theory should accordingly be formulated with such a generalization in mind.

Since futures prices are but one of the several interrelated aspects of the corn market it is obviously necessary to study them as part of a substantially complete model, which will have to cover the relevant features of the whole feed grain-livestock economy. The principal variables in that model will be production, consumption, inventories and prices.

Corn production in any year is determined by the acreage seeded and the yield per seeded acre. The latter can be regarded as a predetermined, even a random, variable for our purposes, being almost entirely dependent on the weather. The acreage seeded, however, is consciously selected, presumably on the basis of expected money returns per acre. As the total acreage of cropland in the U.S. is roughly fixed, or at any rate varies less than the acreages planted of individual crops, these acreages will depend largely on relative prices. Disregarding yield fluctuations for the moment, we may assume that for each piece of land there is a vector of normal yields (in bushels per acre) y_i for each crop i ($i=1, \dots, n$). If the price (less marginal cost) of a bushel of crop i is expected to be p_i then, under perfect competition, the destination of each piece of

land will be such as to maximize total return.

$$R = \sum p_i x_i y_i \quad x_i \geq 0; \quad \sum_{i=1}^n x_i = 1 \quad (1)$$

where x_i is the proportion planted with crop i . Some additional rule for cases of indeterminacy is necessary. It can easily be seen that (wherever these derivatives exist)

$$\frac{\partial x_i}{\partial p_j} \begin{cases} \geq 0 & \text{if } i = j \\ \leq 0 & \text{if } i \neq j \end{cases} \quad (2)$$

and

$$\sum_{j=1}^n p_j \frac{\partial x_i}{\partial p_j} = 0 \quad (3)$$

Similar properties hold more generally for directional derivatives.

In practice neither p_i nor y_i are known with certainty so that a much more complicated problem arises. Assuming risk aversion we may conjecture that the values of x_i given by (1) will be too high for crops whose prices or yields are more than usually uncertain (cf. [7]) if we regard p_i and y_i as means of probability distributions. Pending further research on this matter it will be necessary to regard expectations as certain.

With this restriction we still have to make assumptions as to the values of p_i and y_i actually occurring in (1). For y_i a long-term average might be used but for the fact that in some crops technological changes have taken place, such as the introduction of hybrid corn and the improvement in soybeans. The use of a moving average to represent expected yields might therefore be preferable. The situation with regard to p_i shows the same features in a more marked degree. If price fluctuations from year to year were considered to be random it would not be rational to relate acreage seeded to any particular past price, but in fact producers will be unable to say whether recent variations in prices are due only to changes in

supply or whether they reflect shifts in demand as well. This being the case a larger weight will have to be given to recent prices, and the problem is how far back one has to go. In an early study by Bean [1a] prices of one and two years ago were found to account for most variations in acreage; although his methods and results are now naturally obsolete a similar approach still seems to be appropriate.

The supply function for the i -th crop would then be as follows

$$L_i = f_i(p_{1t-2} \bar{y}_{1t}, p_{1t-1} \bar{y}_{1t}, \dots, p_{nt-2} \bar{y}_{nt}, p_{nt-1} \bar{y}_{nt}) \quad (4)$$

where L_i is acreage seeded and \bar{y}_{it} is a moving average of yield per acre terminating in year $t-1$. How many different crops have to be included cannot be determined in advance, but the number may be substantial as relatively minor crops such as soybeans, grain sorghums and flaxseed are highly variable in acreage. Some multicollinearity is also to be expected.

Many points are ignored in the above discussion, e.g., the influence of Government regulations, varying input ratios or crop rotation. Where necessary and possible suitable amendments to (4) will be introduced.

The situation on the demand side is much more complicated, as corn is largely a production good. About 90 percent of corn consumption is fed to livestock, of which about half to hogs and the remainder to cattle (about 20 percent of total), poultry (15 percent) and horses (10 percent); these shares are crude averages referring to the thirties. The remainder goes to various industrial uses (human consumption is negligible); exports are relatively small. Most corn does not leave the farms where it is grown and is fed directly to animals. The interrelations resulting from this pattern of demand in addition to those connected with forward trading, make corn an excellent object of dynamic analysis.

The formulation of a workable model of the feed grain-livestock market will have to await further examination of a number of comparatively technical points. Here we shall confine ourselves to a highly simplified scheme

which may serve to illustrate the principal phenomena that have to be taken into account. We assume that there is only one feed grain (corn) and one type of livestock (hogs), which is held exclusively for meat production. No stocks of corn are held. The variables entering the model will be

C_t = price of corn in year t

D_t = consumer demand for pork in year t

F_t = consumption of corn by hogs in year t

H_t = price of hogs in year t

I_t = hog inventory at end of year t

L_t = acreage planted with corn for harvest in year t

O_t = nonfeed consumption of corn in year t

P_t = production of corn in year t

Q_t = production of hogs in year t

S_t = hogs slaughtered for consumption in year t

Y_t = consumer income in year t

Z_t = corn yield per acre of harvest in year t .

Identities:

$$I_{t-1} + C_t = S_t + I_t \quad (5)$$

$$F_t + O_t = P_t \quad (6)$$

$$L_t Z_t = P_t \quad (7)$$

$$D_t = S_t \quad (8)$$

Technical relation:

$$F_t = f_1(Q_t) \quad (9)$$

Corn and hog production:

$$L_t = f_2(C_{t-2}, C_{t-1}) \quad (10)$$

$$Q_t = f_3(H_{t-1}, C_{t-1}, I_{t-1}) \quad (11)$$

Supply of hogs:

$$S_t = f_4(Q_t, H_t, C_t) \quad (12)$$

Demand functions:

$$D_t = f_5(E_t, I_t) \quad (13)$$

$$Q_t = f_6(C_t, Y_t) \quad (14)$$

The functions $f_1 - f_6$ also depend on random disturbances. At the beginning of year t the variables F_t , L_t and Q_t are determined by past values of all the variables. Since Z_t can be regarded as exogenous, P_t is also given and consequently C_t . We also consider Y_t to be exogenous (at any rate in this illustrative model) and hence C_t is determined by Q_t , while H_t will be such as to equate D_t and S_t , which at the same time yields I_t .

It will be seen that in this model the price of corn is determined on the nonfeed market; this to some extent reflects the fact that corn for feed does not normally leave farms so that in principle corn need not have a market price at all for that purpose. Nevertheless it may be argued that corn prices are still too prominent in the hog supply relations; it might be preferable to replace (11) by

$$Q_t = f_3(H_{t-1}, P_t, I_{t-1}). \quad (11')$$

Although this would not affect the interdependence of the various equations, it would change the behaviour of the system over time in that there is a better adjustment of hog production to corn supply. Among the many difficulties concealed in this model is the problem of dating.

There is one corn crop each year, viz., in October, whereas pig marketings are heaviest in the spring and in the fall. The calendar year will probably not be the suitable time unit for this analysis.

The statistical properties of the model will depend on the actual form of the equations, several of which will have to be nonlinear. It will be

seen that the equations as they stand look promising from the point of view of identifiability.

Once the coefficients have been estimated conclusions about the periodicity and stability can be drawn, which may be related to theoretical work of Goodwin [5]. The data show a distinct pig cycle with a period of four or five years and an evident connection with the hog-corn price (or production) ratio.

Finally we come to corn stocks, which have been purposely left out of the account in the preceding discussion because they are closely related to future prices. One of the two economic functions of the futures markets is in fact the distribution of demand over time by means of the carry-over; the other function is the transfer of price risks by means of hedging.

Theoretical discussions of forward markets have usually stressed either one of these two functions; one may even distinguish a British school (Keynes [10], Hicks [6], Kaldor [8], [9], Dow [3], [4], [9], Hawtrey [9], Elan [2]) which has especially studied the risk phenomena and an American school (Working [14], [15] and several papers in Wheat Studies; also Williams [13]) which emphasizes the carry-over. We have no unified theory to offer yet; instead we shall outline some of the considerations from which such a theory should be developed.

In a free market for a perishable commodity the price will always equate current production and current consumption. The two functions of the futures markets we have referred to arise entirely out of the durability of the commodity; we shall deal with risks first.

Owners of stocks can escape from part of their price risk by selling futures to the extent of their holdings and similarly those who expect to need supplies at a future date can largely ignore subsequent developments

in prices if they buy futures. The qualifications are necessary because of divergences between cash and futures markets into which we cannot enter here. In practice the first category of futures traders is more important than the second one, since it is easier to delay the start of new production processes than to interrupt processes already started. Hence buyers of futures have to be found outside the circles of those taking part in the actual movement of the commodity from producers to consumers. These buyers will have to be bulls, i.e., people who are sufficiently certain that the price of the future will rise. Hedgers of stocks on the other hand, no matter what they expect subsequent prices to be, act as bears. Consequently in a futures market where some sellers of futures are hedgers the price of the future will be lower than in a market composed entirely of speculators.

This is also the conclusion of the "theory of normal backwardation" which was mentioned earlier in this paper ("backwardation" exists if the future price of a future is below the currently quoted spot price). The argument used there is somewhat different: it is noted that owners of stocks will be prepared to pay a risk premium in order to be relieved of their uncertainty. Speculators, it is claimed, will only be willing to undertake risks if they expect to gain by a sufficient margin. Hence a risk premium will actually be paid, and this will keep speculators in business. Keynes [10] even put this premium at a normal figure of 10 percent per annum, though no substantial evidence is advanced. Considering the nature of speculators, particularly in the U.S., it is doubtful whether they need a premium to enter the field, and still more so whether they actually receive one. Cross-section studies of speculation (esp. [12]) indicated heavy and consistent losses of nonprofessional speculators in grain. According to one investigation of cotton futures [4] the average

the post risk premium was about zero, but another study of that market [16] showed small net losses by hedgers, which might well constitute a risk premium.

In any event it seems clear that the quantitative importance of the risk premium for the formation of futures prices is small and ambiguous. We prefer a formulation in terms of supply and demand for futures, as used above, which will also enable us to evade the controversies about expectations which the theory of normal backwardation has stirred up. By this Casselian approach we deprive ourselves of a possibly useful additional source of information, and at a later stage of the inquiry it may be profitable to go back to this fundamental problem.

Let us now consider the carry-over, which is intended to even out fluctuations in demand and especially in supply. For convenience in presentation we shall speak only of variations in supply, though it must be realized that shifts in demand may be even more difficult to foresee. If demand and production conditions do not change much (as is approximately the case in wheat, but less so in corn) the carry-over mechanism would work perfectly if, irrespective of the actual size of the crop, prices always stayed within narrow limits determined by storage costs. Such a mechanism has been described by Williams [13]. It implies that during periods of relative abundance the distant futures sell at higher prices than the near futures, since otherwise stockholders would sell spot and buy futures rather than incur storage costs.

A price pattern of this kind persisted in wheat from the summer of 1928 to the end of 1934 (with the exception of 1931 when the Federal Farm Board cornered May wheat) and in corn during the years 1932-34, though at the same time the level of prices fluctuated violently. Given the fact that operators in the grain market (including the Government) were unable to say when harvests were abundant and when short the existence of a substantial

carry-over did not by itself promote price stability; the reverse might even be true. Williams' theory deals essentially with the effect of a single shock, in the form of a large crop, on an otherwise stable environment; the conclusions which the author draws from his analysis therefore appear rather rash. Thus he claims that "a speculative carry-over tends to be found in most years," which means that in those years the distant futures must be at a premium. During the 16 years 1922-38 however there were only 6 in which this was true for wheat, and during the 14 years 1926-39 only 5 with premiums on new-crop corn. Post-war experience has also pointed to the importance of "inverse carrying charges."

The explanation of this phenomenon is the principal task of the theory of forward markets; discounts on new-crop quotations are not an anomaly as has often been suggested. "Normal backwardation" would provide an explanation of these discounts if it could be argued that risks are greater when crops are small, but there is more reason to think that the argument works the other way. According to our analysis of the supply and demand for futures hedging may depress futures, but if the supply is short stocks, and consequently hedging pressure, will be small (cf. also [8]). Moreover, even if we admit a risk premium it cannot account for more than a small part of the large discounts that have sometimes occurred, as Working [15] has rightly remarked.

A complicating factor in years of small supplies is the emergence of corners in old-crop futures, but these mainly influenced prices immediately before maturity; this is one of the reasons why it is better to observe spreads between futures during a period before the month of delivery. Corners are a symptom rather than a cause of small supplies.

The explanation frequently advanced for inverse carrying charges (cf. e.g., [8] and [15]) is based on the "convenience yield" of stocks. A

distinction is made between working stocks, which are necessary for the continuity of industrial and commercial operations and therefore have a yield irrespective of their expected prices and speculative stocks which are held only in hope of price advances. The marginal convenience yield of stocks will be a decreasing function of the amounts held and will vanish if total stocks exceed working stocks. Hence the formula (cf. [8])

$$P_{t+1} - P_t = i + c - q(X) \quad (15)$$

where p_{t+1} is the price of a new crop future, p_t the price of an old crop future, i is the rate of interest, c warehouse cost and $q(X)$ the marginal convenience yield of a stock of X units. The relation between $P_{t+1} - P_t$ and X in the case of wheat has been studied by Working (in Wheat Studies and in [15]); remarkably close correlations are found both there and in corn.

Nevertheless (15) is not entirely satisfactory for three reasons: the "convenience yield" is a somewhat nebulous concept, warehouse cost is irrelevant if $p - p$ is negative and it is not clear at what time X has to be observed. An interpretation in terms of time preference may overcome these objections. Each actual or prospective holder of stocks has a choice as to the moment at which he will dispose of his stocks, by using them as an input in some productive process (speculative stocks will disappear if carrying charges are inverted). This process will require complementary goods, which may remain idle if no corn (or whatever the commodity is) is available. If the process can still be delayed these complementary goods need not be bought or can be used otherwise, hence corn now will be preferred to corn later. (Similar reasons were invoked above to explain the effect of hedging; cf. p. 9). Moreover "later" means here: after the new crop is available. If supplies are ample this time preference will not

show up, but if supplies are short the new-crop discount will reserve supplies to those whose need is sufficiently urgent, while others will have to wait until after the new crop.

The discount resulting from the scale of time preferences of the different corn consumers is directly related to the so-called commodity rate of interest (cf. [6]); the latter also includes the rate of interest, of interest / as buyers of old-crop corn will have to pay earlier than buyers of new-crop corn. If t is a date in the current crop year, $t+1$ the date the new crop, and V_{t+1} consumption between those dates, then

$$V_{t+1} = f_7(p_t, p_{t+1}, i) \quad (16)$$

p_{t+1} will be determined by expectations as to supply and demand in the new crop year, which will in turn be influenced by previous prices, crop reports, etc. The market rate of time preference p_t/p_{t+1} will depend on prices and sales prior to t and on stocks I_t at that date.

We may remark in passing that the above approach also holds if there are several old-crop futures. Specifically, if May corn is at a premium over December corn (the first new-crop future), then July corn and September corn will be at smaller premiums over December. We also stress again that the above is not yet a formal theory, but merely a possible approach to one. Attempts will be made to develop it further, so as to include non-seasonal commodities and groups of related goods (e.g., cotton and cotton-seed, soybeans and soybean oil), and to test it empirically in a number of instances if this is possible without going too deeply into other aspects of the commodity concerned.

The incorporation of the carryover and futures prices into the simple model consisting of equations (5)-(14) requires substantial revisions of that model. As this calls for further study of the basic observations the extended model cannot yet be discussed here. The principal modifications will probably involve the use of half-yearly instead of yearly data; we

had seen already that this was desirable because of the pattern of hog production. It is now all the more necessary since corn is seeded in the spring and speculation on the new crop does not start until then. Some of the lagged relations might also be made more realistic by this alternation. There should not be too many difficulties with the statistical material, most of which is available on a monthly basis.

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REFERENCES

- [1] Arrow, K. J., "Alternative Approaches to the Theory of Choice in Risk-taking Situations," Econometrica, Vol. 19 (1951).
- [1a] Bean, L. H., "The Farmers' Response to Price," Journal of Farm Economics, Vol. 11 (1929).
- [2] Blau, G., "Some Aspects of the Theory of Futures Trading," Review of Economic Studies, Vol. 11 (1943-44).
- [3] Dow, J. C. R., "A Theoretical Account of Futures Markets," Review of Economic Studies, Vol. 7 (1939-40).
- [4] _____., "The Inaccuracy of Expectations," Economica, N.S., Vol. 8, (1941).
- [5] Goodwin, R. M., "Dynamical Coupling with Special Reference to Markets Having Production Lags," Econometrica, Vol. 15, (1947).
- [6] Hicks, J. R., Value and Capital, 2nd edition, Oxford, 1916, esp. p. 135-42.
- [7] Johnson, D. G., Forward Prices for Agriculture, Chicago, 1947, esp. Ch. IV.
- [8] Kaldor, N., "Speculation & Economic Stability," Review of Economic Studies, Vol. 7 (1939-40).
- [9] _____., Dow, J.C.R. and Hawtrey, R.G., "A Symposium on the Theory of the Forward Market," Review of Economic Studies, Vol. 7, (1939-40).
- [10] Keynes, J. M., Treatise on Money, London, 1930, esp. Vol. II, p. 142-44.
- [11] Koopmans, T. C. (ed.), Statistical Inference in Dynamic Economic Models, New York, 1950.
- [12] Stewart, B., An Analysis of Speculative Trading in Grain Futures, U.S. Department of Agriculture Technical Bulletin, 1001, Washington, 1949.

- [13] Williams, J. B., "Speculation and the Carryover," Quarterly Journal of Economics, Vol. 50 (1936).
- [14] Working, H., "Quotations on Commodity Futures as Price Forecasts," Econometrica, Vol. 10 (1942).
- [15] _____, "Theory of the Inverse Carrying Charge in Futures Markets," Journal of Farm Economics, Vol. 30, (1948).
- [16] Yamey, B. S., "An Investigation of Hedging on an Organized Produce Exchange," Manchester School, Vol. 19 (1951).