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Organized Decision-Making [1]

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Note: Sections I and II of this paper correspond to Sections I and II of CCDP Economics No. 2029 ("On Optimal Communication Rules for Teams," by J. Marschak and D. Waterman) which are here revised and amplified. In particular the concept of "organization," of which a "team" is a special case, is defined here, in Section I, by the existence of separate utility functions (bonus functions) for the individual members. Section II revises and thus supersedes the analysis of the team model introduced in the previous paper.

While the team model provides for decentralized decision-making within centrally prescribed action rules, the organization model studied in Section III of this paper provides for decentralized rule-making.

Some readers may find it useful to start with the examples of Sections II and III before proceeding to the general definitions of Section I. Five charts are placed at the end of the paper.

The author owes much to A. Newell's and Joseph B. Kruskal's "Organization Theory in Miniature" (ONR Contract N7 onr-419 Task order 4), and to C. B. Tompkins' working paper "Notes on Organization" (George Washington University, Logistic Research Project, 2 January 1951). Oral communications by A. Bavelas and D. Rosenblatt (on "networks") are reflected in paragraph 25. The cooperation of D. Waterman and suggestions of M. Beckmann, G. Debreu, K. Faxèn, I. Herstein, T. Koopmans and R. Radner have been very helpful. Further criticisms are invited.

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I. General.

1. We shall define an organization as a group of persons (called members), each of whom pursues his individual goal but whose actions are restricted by a set of rules designed to serve a certain goal called the organization goal. We shall call a team an organization whose goal completely determines the individual goals of its members.

2. More precisely, consider n members, $i=1, \ldots, n$. The i-th member finds himself in a certain "situation," or "state of information" x_i, an element of the set X_i of all possible states of his information. Let A_i be the set of all possible actions a_i that the i-th member can undertake upon receiving this information. (This action may itself consist of imparting information to another member, or asking him for information.) An individual rule of action for the i-th person, γ_i is a correspondence (one-to-one or many-to-one) between the elements of X_i and those of A_i:

$$(2.1) \qquad \gamma_i : X_i \rightarrow A_i , \text{ or } \quad a_i = \gamma_i (x_i).$$

3. In an organization, γ_i is an element of a set Γ_i: the set of rules permitted to the i-th member. An organization is called centralized if each Γ_i consists of one element only: in this case, it is prescribed in advance how each member shall respond to each situation. In the general case, each member can work out his own rules of action, subject to prescribed restrictions.

4. The Cartesian products $X=X_1 x X_2 x \ldots x X_n$; $A=A_1 x A_2 x \ldots x A_n$; and $\Gamma = \Gamma_1 x \Gamma_2 x \ldots x \Gamma_n$ represent, respectively, the set of joint situations,

the set of joint actions and the organizational statute. The set of n rules $\gamma_1, \ldots, \gamma_n$ used by the members will be denoted by γ, an element of Γ. (γ can be called the joint rule.) See Figure I for the case where a_i, x_i, γ_i are real members, and n=2.

5. Let r be the physical result of a joint situation x in X and a joint action a in A. Then by (2.1), r depends on γ and x:

$$r=r(\gamma, x),$$

say. In general, this dependence is a random one. We shall call prospect, and denote by p, the probability distribution of r. If x is given, p depends on γ. But, in general, x is a random variable. Let its distribution be F. Then p depends on γ and F:

$$p=p(\gamma, F).$$

6. Let P be the set of all possible prospects. The existence of individual interests of the organization members has the following meaning: for each i(=1, ..., n) there exists a relation Δ_i (read: "at least as desirable for i as") which completely orders all elements of P; moreover, if the sets γ and γ' are identical except for their i-th elements, γ_i and γ_i', and if, for given F,

$$p(\gamma, F)\ \Delta_i\ p(\gamma', F); \text{ \underline{not} } p(\gamma', F)\ \Delta_i\ p(\gamma, F),$$

then the i-th member will choose rule γ_i in preference to γ_i'.

7. Since the rules at the i-th member's choice must be within the permitted set Γ_i, and similarly for other members, the set γ chosen by all will depend on the organizational statute Γ. Let this set be $\gamma^{(\Gamma)}$. The existence of an organizational goal has the following meaning: the set P is

completely ordered by a relation Δ_0 (read "at least as desirable for the organization as"); moreover, if Γ and Γ' are two statutes and, for a given F,

$$p(\gamma^{(\Gamma)}, F)\ \Delta_0\ p(\gamma^{(\Gamma')}, F);\ \underline{\text{not}}\ p(\gamma^{(\Gamma')}, F)\ \Delta_0\ p(\gamma^{(\Gamma)}, F),$$

then statute Γ will be chosen in preference to Γ'.

8. In a completely centralized organization (defined in 3 above) the individual preference relations $\Delta_i (i=1, \ldots, n)$ are irrelevant, and the organizational statute Γ is chosen on the basis of Δ_0 alone. In a <u>team</u>, the individual preference relations are identical with the organizational preference relation: $\Delta_0 = \Delta_1 = \ldots = \Delta_n$.

9. Great simplification is achieved if the ordering relations $\Delta_0, \Delta_1, \ldots$ between prospects are endowed with some (rather weak) properties which imply the "Neumann-Morgenstern theorem": there would then exist, for every j=0, 1, ... a "utility function" of the result (unique up to a linear transformation), to be denoted by

$$w_j(r) = w_j[r(\gamma^{(\Gamma)}x)] = u_j(\gamma^{(\Gamma)}, x) = u_j,\ j=0, 1, \ldots, n,$$

say, and having the property that j prefers higher expected values of w_j to lower ones. We shall also write

$$u_j = u_j(\gamma^{(\Gamma)}, x; w_j),$$

to emphasize that utility can be regarded as a function of the rules and of the situation, and as a functional of the utility function. To each prospect P corresponds an expected utility

$$Eu_j = \int_X u_j(\gamma^{(\Gamma)}, x; w_j)\ dF(x) = U_j(\gamma^{(\Gamma)}, F; w_j)$$

say. For example, if the organization is a firm, U_0 may represent the expected profit, while U_1 may be the expected income (salary, bonus, etc.) of the member 1 of the firm.

10. In the case of a team, (and similarly in the case of a centralized organization), the subscript j can be dropped and $\gamma^{(\Gamma)}$ can be replaced by γ (a unique element of Γ). An "efficient rule" $\hat{\gamma}$ will be one that maximizes U:

$$\underset{\gamma}{\text{Max}}\, U(\gamma, F; w) = U(\hat{\gamma}, F; w).$$

Hence $\hat{\gamma}$ depends only on F, the distribution of situations, and on w, the utility function of the team:

$$\hat{\gamma} = \hat{\gamma}(F, w),$$

say. The determination of $\hat{\gamma}$ — viz., the best rule of action for team members, including rules of communication — will be studied in Section II, using a particularly simple example, and pointing out to further details of interest, still to be investigated.

11. The case of decentralized organization, on the other hand, presents many unsolved problems of more fundamental nature; a special example, to be studied in Section III will merely help to throw some light on those problems.

II. Example of a team.

12. Consider a firm which consists of two partners: one of them keeps himself informed of the market price of the firm's (unique) product; the other, of the market price of the (unique) raw material used by the firm. Each of these prices has rectangular probability distribution, between a known maximum and a known minimum. Each partner can commit the firm in the following way only: upon his order, the firm sells exactly one unit of product and at the same time buys the amount of raw material necessary to produce one unit of

product. (To avoid irrelevant complications, we may assume that the firm is speculating on commodity exchanges or is able to draw on infinite inventories.) In the light of his special information, each partner can either (1) do nothing; or (2) telephone the other partner, at a certain cost to the firm; or (3) commit the firm. In the case (2), the partners exchange information and either commit the firm to the purchase and sale of two units, or do nothing, -- depending on whether the price of the product exceeds or does not exceed the cost of raw material. A "rule" prescribes to each of the partners when to take actions (1), (2), (3).

Problem: find the rule that will maximize the expected profit.

13. Let y_1= price of product; y_2= cost of raw material needed to make one unit of product; d= cost of telephoning. As in Section I, let a_i be the action of the i-th partner (i=1,2). The variable a_i has three values: 0(= do nothing), p(= phone), q(= commit). The firm's profit (u', say) =

$$(13.1)\quad \begin{cases} 0 & \text{if} \quad a_i=0 \quad (i=1,2) \\ 2(y_1-y_2) & a_i=q \quad (i=1,2) \\ y_1-y_2 & a_i=0,\ a_j=q\ (i,j=1,2 \text{ or } 2,1). \\ -d & \left.\begin{array}{l} y_1 \leqslant y_2 \\ y_1 > y_2 \end{array}\right\} \text{and } a_i=p \ (i=1 \text{ or } 2) \\ 2(y_1-y_2)-d & \end{cases}$$

14. For the sake of mathematical simplicity we introduce the assumption that max y_1=max y_2=$\bar{y}$, min y_1=min y_2=$\underline{y}$, and define

$$(14.1)\quad \begin{cases} \bar{y} - \underline{y} = g(\text{range});\ c = d/g; \\ x = \dfrac{y_1 - \underline{y}}{g}\ ;\ x = 1 - \dfrac{y_2 - \underline{y}}{g}\ ; \end{cases}$$

then $\max x_i = 1$, $\min x_i = 0$ $(i=1,2)$ so that, in the notation of Section I, $x_i \in X_i = [0,1]$, the unit interval; the space of joint situations = X = unit square. (See Fig. I.) The rule is denoted by $\gamma = (\gamma_1, \gamma_2)$. Since a_i has three values only, γ_i is a partitioning of $X_i = [0,1]$ into three subsets X_i^o, X_i^p, X_i^q such that

(14.2) $$a_i = k \text{ if } x_i \in X_i^k\ ;\ k = 0, p, q.$$

Finally, the (rectangular) distribution $F(x) = F(x_1, x_2)$ has, over the domain $(0,0) \leqslant (x_1, x_2) \leqslant (1,1)$, the density

(14.3) $$f(x_1, x_2) = 1.$$

15. With the help of (14.1), (14.2) the profit u' defined in (13.1) can be rewritten in a symmetric fashion as a function of x_1, x_2, c. We have:

$u' = u \cdot g$, where $u = u(x_1, x_2) =$

(15.1) $$\begin{cases} 0 & \text{if} \quad x_i \in X_i^o \quad (i=1,2) \\ 2(x_1+x_2-1) & \quad x_i \in X_i^q \quad (i=1,2) \\ x_1+x_2-1 & \quad x_i \in X_i^o,\ x_j \in X_j^q \quad (i,j=1,2 \text{ or } 2,1) \\ -c & \quad x_1+x_2 \leqslant 1 \\ 2(x_1+x_2-1)-c & \quad x_1+x_2 > 1 \end{cases} \left.\vphantom{\begin{matrix}a\\b\end{matrix}}\right\} \text{and } x_i \in X_i^p \ (i=1 \text{ or } 2).$$

We can call u the firm's utility function; it is proportional to profit and depends on x_1, x_2, c and on the rule γ that partitions the sets X_1, X_2. The best rule, $\hat{\gamma}$, will maximize the expected value of u, which is equal to

(15.2) $$\begin{aligned} U &= \int_0^1\!\!\int_0^1 u(x_1, x_2; \gamma, c)\, dF(x_1, x_2)\, dx_1\, dx_2 \\ &= \int_0^1\!\!\int_0^1 u(x_1, x_2; \gamma, c)\, dx_1\, dx_2, \text{ by (14.3).} \end{aligned}$$

Thus $\hat{\gamma}$ will depend on c only.

16. It is plausible intuitively (<u>but has not been proved rigorously</u>) that the optimal rule δ_i must belong to the set of rules that partition the unit interval into three intervals, and not into any other kinds of subsets; and that the lower values of x_i must correspond to "doing nothing," the higher values to "committing" and the intermediate values to "phoning." On this basis, the function (15.1) is tabulated on Fig. II, where the intervals are

(16.1) $$[0, \alpha_i) = X_i^o; \quad [\alpha_i, 1-\alpha_i'] = X_i^p; \quad (1-\alpha_i', 1] = X_i^q; \; i=1,2.$$

We thus confine our search for the best rule to the set of rules each of which is defined by the vector $\delta = (\alpha_1, \alpha_1', \alpha_2, \alpha_2')$, where

(16.2) $$(0,0) \leqslant (\alpha_i, \alpha_i') \leqslant (\tfrac{1}{2}, \tfrac{1}{2}); \; i=1,2.$$

17. The following special case will prove useful. Suppose it is known that

(17.1) $$\alpha_1 = \alpha_1' = \alpha_2 = \alpha_2' = \alpha,$$

say; so that the 4 rectangles on Fig. II become equal squares, and the diagonal segment joins two of their corners. Then (15.2) becomes

(17.2) $$U = U(\alpha, c) = -\tfrac{2}{3}\alpha^3 + 4c\,\alpha^2 - c + \tfrac{1}{3},$$

to be maximized with respect to α subject to the inequalities $c \geqslant 0$; $0 \leqslant \alpha \leqslant \frac{1}{2}$. One obtains, for the best value of α,

(17.3) $$\hat{\alpha} = \min(4c, \tfrac{1}{2}) = \hat{\alpha}(c).$$

The function $\hat{\alpha}(c)$ will be shown on Fig. IV. Thus, when $c=0$, the best rule is always to phone. When $c \geqslant \frac{1}{8}$, the best rule is: to do nothing if $x_i < \frac{1}{2}$, to commit if $x_i > \frac{1}{2}$, and never to phone. When $0 < c < \frac{1}{8}$, the best rule is to do nothing if $x_i \leqslant 4c$, to commit if $x_i \geqslant 1-4c$, and to phone in the intermediate case.

18. If (17.1) is not known to be true, the straightforward maximization of U in (15.2) with respect to the vector $\gamma=(\alpha_1, \alpha'_1, \alpha_2, \alpha'_2)$, subject to the inequalities (16.2) and to $c \geq 0$, is quite involved. The problem has not been solved rigorously. Preliminary studies by Mr. Dan Waterman of the Cowles Commission seem to permit, however, the conjecture that the optimal values for the α's do satisfy (17.1) and therefore (17.3). In any case, the solution (17.3) will prove of value in the discussion of Section III.

19. The gaps in the mathematical solution, indicated in 16 and 18 (doubly underlined phrases), must be filled. In addition, the problem should be generalized in various directions, as follows.

20. In 14, the prices of the product and the raw material (per unit of product) were assumed to have common range and common minimum. This led to a convenient symmetry. If the assumption of common range is dropped, we shall expect the best rule to create a "hierarchy" within the team: the partner who specializes in the less fluctuating price will more often ask his fellow for information than be asked by him. Moreover, the assumption of common minimum and range has implied non-positive expected profit, and this is not realistic, at least for this particular economic example. The assumption should be dropped.

21. The assumption of rectangular distribution of each price did not allow for correlation. This is not realistic; and even aside from the particular economic interpretation, it is interesting to check upon the following conjecture: the stronger the correlation between x_1 and x_2 the less need is there for communication, i.e., the smaller the best sets X_1^p.

22. The model can be generalized to make the quantity bought and sold a continuous variable (it has only values 0 and 1 in the present model).

23. The cost of communication, c, may be made to rise continuously with the "amount of information": it is more costly to communicate a more precise information (e.g., an estimate with a smaller variance). This, together with the suggestion in 22, may make the mathematical solution easier.

24. The problem can be extended to n partners.

25. One can discuss the choice of a communication network, represented by a matrix $\mu=[\mu_{ij}]$ where $\mu_{ij}=0$ or 1 according as i can or cannot send messages to j. (Further refinements are possible.) The cost of constructing the network being a known function of μ, one maximizes expected profit with respect to both μ and the action rule α.

26. One should proceed to the "dynamic" case: a (stochastic) process consisting of situations, actions, situations, actions,... .

27. By introducing separate utility functions for the participants, one attacks the "organization" problem, more general than that of a "team." We shall do this in Section III.

III. Example of an organisation.

28. We maintain all the assumptions of Section II (excluding the generalizations of paragraphs 20-26), but assume that each member i devises his individual rule α_i so as to maximize the expected value of his "bonus function" prescribed by the firm: a function of the outcome of the member's action. The firm has to choose the two bonus functions in such a way as to maximize the expected profit. (In terms of Section I, the bonus functions define the organizational statute, Γ; the expected utility to the firm was called there U_0, but will be called here U).

29. To fix ideas, we shall limit the bonus functions to the following set (which may or may not contain the absolutely best bonus functions). The bonus u_i will be the profit that arises to the firm from the i-th member's action, with the true cost of communication, c, replaced by a "shadow cost" β_i. The problem is thus to find the values $\beta_i = \hat{\beta}_i$ (i=1,2) that will maximize the expected profit, or its linear transform U, if each member responds to the situation x_i by an action a_i that maximizes his expected bonus $U_i = \int_0^1 u_i(x_1, x_2, a_i)dx_j = U_i(x_i, a_i)$, $i \neq j$. We have:

$$
(29.1) \qquad
\begin{aligned}
u_i &= 0 && \text{if } a_i = 0\\
&= x_1 + x_2 - 1 && \text{if } a_i = q\\
&= x_1 + x_2 - 1 - \beta_i && \text{if } a_i = p \text{ and } x_1 + x_2 > 1\\
&= -\beta_i && \text{if } a_i = p \text{ and } x_1 + x_2 \leqslant 1
\end{aligned}
$$

Therefore to the three actions correspond three curves, indicated on Figure III by 0, q, p, respectively:

$$
(29.2) \qquad
\begin{aligned}
&0: \quad U_i(x_i, 0) = 0\\
&q: \quad U_i(x_i, q) = x_i - 1 + \int_0^1 x_j \, dx_j = x_i - \tfrac{1}{2}\\
&p: \quad U_i(x_i, p) = -\beta_i + \int_{1-x_i}^1 (x_i + x_j - 1)dx_j = -\beta_i + \frac{x_i^2}{2}
\end{aligned}
$$

The p-parabola crosses the 0-line (from below) at $x_i = \sqrt{2\beta_i}$ and crosses the straight q-line (from above) at $x_i = 1 - \sqrt{2\beta_i}$, provided $\sqrt{2\beta_i} \leqslant \frac{1}{2}$, $\beta_i \leqslant \frac{1}{8}$.

If $\beta_i > \frac{1}{8}$, the parabola lies everywhere below the other two lines (i.e., he will never phone). Let $\delta_i = \min(\sqrt{2\beta_i}, \frac{1}{2})$. Then the best action $\hat{a}_i$ and the resulting expected bonus $\hat{U}_i = \max_{a_i} U(x_i, a_i)$ will depend on x_i in the following fashion:

$$\text{If} \quad x_i \in [0, \delta_i] \quad \text{then } \hat{a}_i = 0, \ \hat{U}_i = 0$$

$$(29.3) \qquad x_i \in (\delta_i, 1-\delta_i) \text{ then } \hat{a}_i = p, \ \hat{U}_i = \frac{1}{2} x_i^2 - \beta_i$$

$$x_i \in [1-\delta_i, 1] \text{ then } \hat{a}_i = q, \ \hat{U}_i = x_i - \frac{1}{2} \ .$$

(The interval in the middle line may be empty if $\delta_i = \frac{1}{2}$, i.e., the shadow cost β_i is prohibitive, i.e. $> \frac{1}{8}$. Otherwise the size of this interval (the probability of the member's taking the initiative in calling up) is $1-2\sqrt{2\beta_i}$.

The contribution — $v_i(x_i)$, say, — of the i-th member to the firm's expected profit, for a given x_i, is identical with $\hat{U}_i$ in (29.3), except that, in the middle line, $(\frac{1}{2} x_i^2 - \beta_i)$ must be replaced by $(\frac{1}{2} x_i^2 - c)$ since the firm bears the true not the shadow cost. The expected value of this contribution, $V_i = \int_0^1 v_i(x_i)dx_i$ is

$$(29.4) \qquad V_i = \frac{1}{3}(\sqrt{2\beta_i})^3 + \frac{1}{6} + (2\sqrt{2\beta_i}-1)\,c \qquad \text{if } \beta_i \leq \frac{1}{8} ,$$

$$= \frac{1}{8} \qquad\qquad \beta_i \geq \frac{1}{8} .$$

The upper of these two expressions for V_i is maximized at $\beta_i = c$. Call the value of V_i at this point, $V(c)$; $V(\frac{1}{8}) = \frac{1}{8}$, and $V(c) > \frac{1}{8}$ for $0 \leq c < \frac{1}{8}$ (See Fig. V).

The best shadow cost (the same for both members of the firm) is

$$\hat{\beta} = \hat{\beta}(c) = c \quad \text{if } c \leq \frac{1}{8}\text{, and}$$
$$\geq \frac{1}{8} \quad \text{if } c \geq \frac{1}{8}\text{;}$$

$$\hat{\delta} = \hat{\delta}(c) = \min(\sqrt{2c}, \tfrac{1}{2})\text{;}$$

i.e., shadow cost = true cost unless the latter exceeds $\frac{1}{8}$, in which case any $\hat{\beta} \geq \frac{1}{8}$ will achieve the same (and best) result by prohibiting communication.

30. On Fig. IV, the function $\hat{\delta} = \hat{\delta}(c)$ is compared with the function $\hat{\alpha} = \hat{\alpha}(c)$, which was shown — in paragraph 17 — to be optimal in the case of the teams when the partition rule is to be picked from the set of all symmetrical partitioning [see (17.1)-(17.3)]. The length of the middle interval, i.e., the quantity $(1-2\hat{\delta})$ or $(1-2\hat{\alpha})$, is the probability for the organization member to initiate a conversation. It changes inversely with $\hat{\delta}$ and $\hat{\alpha}$. So does the probability that communication will be started by any of the two members (it is measured by the cross-shaped area between the four rectangles on Fig. II). Since $\hat{\delta} > \hat{\alpha}$ for $0 < c < \frac{1}{8}$, and $\hat{\delta} = \hat{\alpha} = \frac{1}{2}$ for $c \geq \frac{1}{8}$, communication is less frequent in the case of autonomous rule-making by the two partners than in the case of the team. Moreover, since it was proved, at least for the case of symmetrical partitioning, that $\hat{\alpha}(c)$ did maximize expected profit, and since the best autonomous rule-making has proved to imply symmetrical partitioning, the autonomous rule-making is less efficient than the team rule, except when the true communication cost is prohibitive, in which case the two systems are equally efficient. This is illustrated on Fig. V, where $\hat{U}$ is the highest expected profit in the case of a team (assuming symmetrical partitioning), and $2\hat{V}_1$ is the aggregate of the highest contributions of the two members of the firm with autonomous rule-making.

31. These results cannot claim generality since the model studied is quite special, for example because of the linear features of its bonus function or for other reasons.

32. Note in particular that each member, in devising his rule of action, was assumed to ignore the possibility of his being called up by the other man. We can only say here that this possibility would open up a game-theoretical problem. On the other hand, to exclude this possibility, one would have to replace the two-way exchange of information permitted by the "telephone," by a more restricted communication form (such as the "library"), permitting only quest for information. Distinctions between "telephone," "library," "telegraph," etc. would be a matter for "network" studies such as those indicated in paragraph 25. . One may also think of "scheduled messages" ("wait for your partner's call till 10 a.m., then phone if necessary"); this will possibly lead to a study of stochastic processes, indicated in paragraph 26.

Fig. I

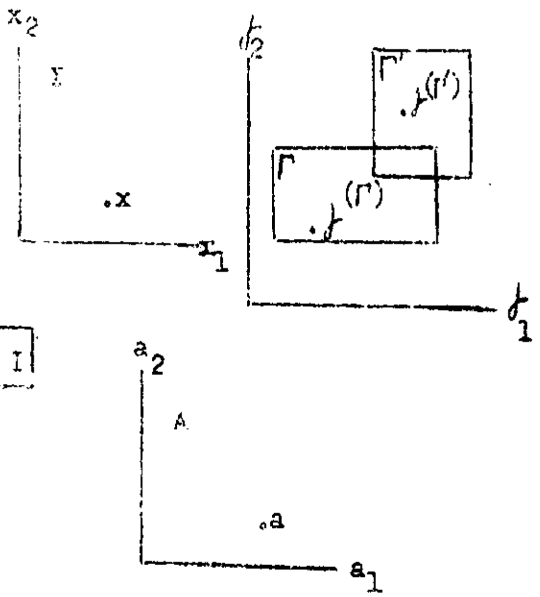

Fig. II

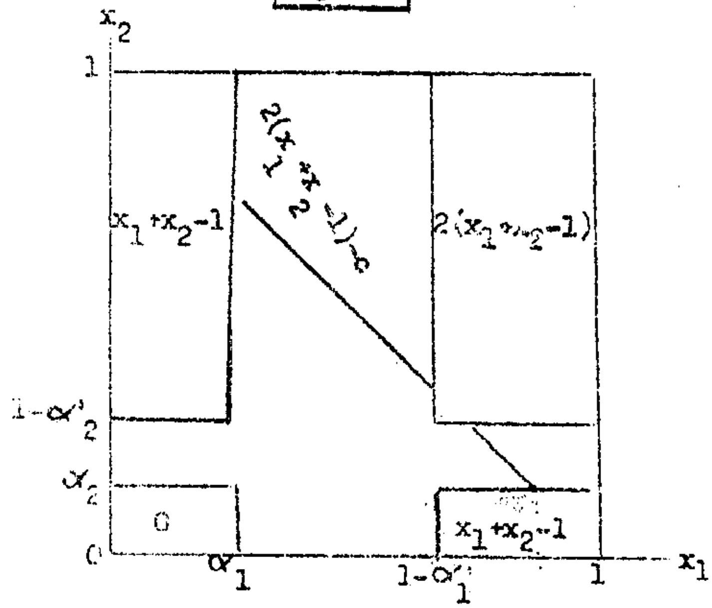

Fig. III

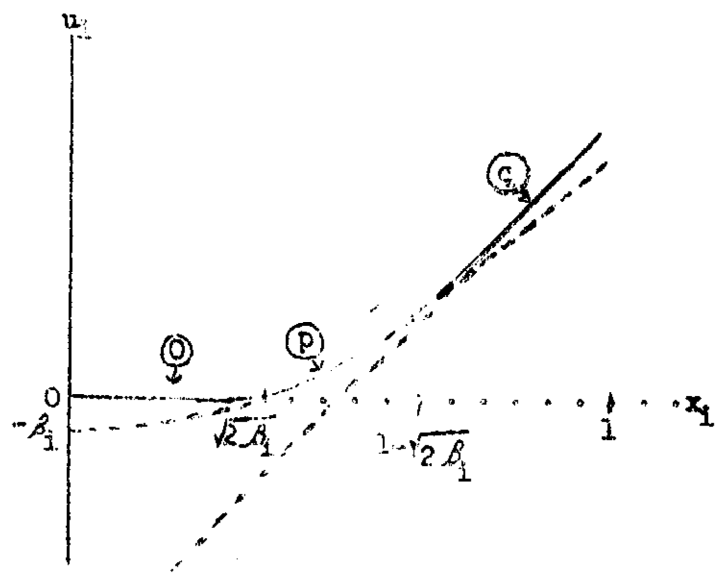

Fig. IV

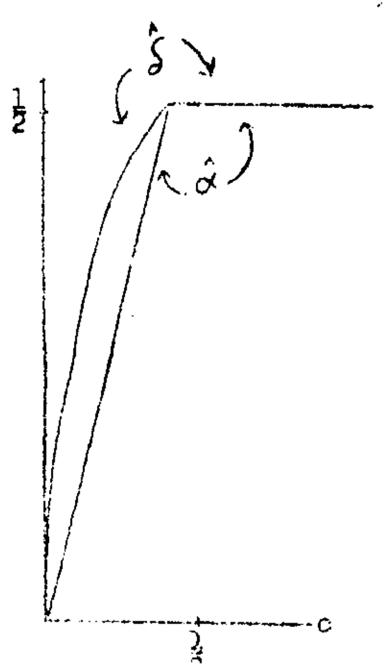

Fig. V