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The Analysis of Quality Variations in Consumption

(Outline)

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In another paper (outlined in CCDP Economics 2030) some complications arising out of the non-negativity of quantities bought were dealt with by means of a technique which was much closer to linear programming than to differential calculus. Because of non-negativity the usual infinitesimal approach to consumption theory has only local validity and requires that the list of the commodities actually consumed is given in advance. By introducing "quality" as an additional variable for each item of consumption it is possible to make the application of calculus methods less local than they would otherwise be.

For each good i the different qualities v_i (assumed to be continuous within an interval) are defined by their prices under some basic price-system. We consider only linear price changes, which change the cost of a consumption (x_i, v_i), where x_i is quantity, from $x_i v_i$ into $x_i(a_i + b_i v_i)$, where a_i and b_i may be called the quality price and the quantity price respectively. This implies a definition of "commodity" and "quality".

The problem then becomes to maximize

(1) $$u(x_1, \ldots, x_n, v_1, \ldots, v_n)$$

Subject to

(2) $$\sum_{i=1}^{n} x_i(a_i + b_i v_i) = M$$

where M is money income. We assume that the maximizing values will all turn out to be non-negative.

The analysis is based on a combination of "revealed preference" and the Hotelling-Roy or "indirect" utility function, which enables us to write down the more important second-order inequalities and symmetry conditions without the use of determinants. This function $f(a_1, \ldots, a_n, b_1, \ldots, b_n, M)$ is always at a minimum for compensated changes in the parameters a_i, b_i and M. In the classical problem (without the v_i) this technique incidentally yields the theorem

(3) $$\left(\frac{\partial}{\partial p_i} + \frac{x_i}{x_j}\frac{\partial}{\partial p_j}\right)\frac{x_i}{x_j} < 0$$

which rules out perfect complementarity.

In the quality problem we obtain results such as the following

(4) $$\frac{\partial x_i}{\partial a_i} + x_i \frac{\partial x_i}{\partial M} < 0$$

(5) $$\frac{\partial x_j}{\partial a_i} + x_i \frac{\partial x_j}{\partial M} = \frac{\partial x_i}{\partial a_j} + x_j \frac{\partial x_i}{\partial M}$$

which show (if we substitute p_i for a_i) that the classical problem is a special case of the one considered here. We also find

(6) $$\left(\frac{\partial}{\partial b_i} + x_i v_i \frac{\partial}{\partial M}\right) x_i v_i < 0$$

from which it follows that quality does not necessarily decrease as a result of an income-compensated change in the quality price. A more definite conclusion is

$$\frac{\partial v_i}{\partial b_i} + v_i \frac{\partial v_i}{\partial a_i} < 0 \tag{7}$$

i.e., if the quality price of a good increases and its quantity price is adjusted so as to keep the cost of the original purchases unchanged, then the quality bought of that good will decrease. A disguised symmetry condition says that

$$\frac{\partial x_i}{\partial b_i} = \frac{\partial (x_i v_i)}{\partial a_i} + x_i^2 \frac{\partial v_i}{\partial M} \tag{8}$$

thus relating price effects on qualities and on qualities.