COWLES COMMISSION DISCUSSION PAPER: Economics No. 2030

The Shape of Engel Curves

(Outline)

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January 17, 1952

One of the difficulties in confronting classical utility theory with empirical data is the fact that the usual calculus approach does not show whether at a maximum all quantities bought are nonnegative. Thus, Allen and Bowley in "Family Expenditure" considered a preference scale, effectively equivalent to a quadratic utility function, which they thought would yield linear Engel curves. In that case expenditures on some commodities would be negative over some part of the income range. To avoid this difficulty the problem has to be reformulated as follows. Take again a quadratic utility function

$$\frac{1}{2} x' A x + a' x = u(x) \qquad (1)$$

where x and a are n-vectors, and A an nxn-matrix. This is to be maximized subject to

$$p' x = M \qquad (2)$$

where p is the price vector and M money income, and

$$x \geq 0. \qquad (3)$$

Condition (3) will only be satisfied by the usual Lagrange solution if some of the quantities x are excluded and put equal to zero. The problem

then becomes how to choose a subset S of the x_i so that (1) is at a maximum relative to all possible subsets that produce nonnegative solutions. These optimal subsets can be found by a method somewhat analogous to "complete induction," at any rate if prices are given.

Immediately to the right of the point M = 0 the value of u will be determined mainly by the linear terms, and S will then consist of the x_i with the largest corresponding a_i. By a continuity argument one can then gradually extend (or sometimes reduce) S for increasing M, passing from one changing point to another. Between changing points the Engel curves will still be linear, but at these points they will have kinks. The slopes change there in accordance with the theory of rationing (cf. Tobin and Houthakker in Review of Economic Studies, 1950-1); for an individual good they will usually decrease because room has to be made for goods newly introduced into the budget. Some conjectures on the Engel curves for groups of commodities (as they are found in practice) can also be made.