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An Investigation of the Demand for a Rationed Commodity

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I. This paper contains a summary of some parts of an investigation of the demand for liquor in Sweden with special consideration to the fact that liquor there is a rationed commodity ([2] , [4]). In fact, every person wishing to buy liquor (or wines) other than for immediate consumption on a licensed premises must first apply for a ration book. The size of the ration varies according to the applicants age, marital status and income.

The liquor consumption is supervised by the Swedish Board of Control, which keep a thorough data concerning consumption and ration book holders, thus providing excellent material for statistical investigations.

In what follows, the consumption of liquor refers, if not explicitly mentioned otherwise, to the amount of liquor sold by bottle stores.

II. For a preliminary analysis of the liquor consumption based on time series for the period 1923 -39, the economic model is supposed to be of the (linear) process analytical type ([2]). In the notations of ([3]) such a model is expressed by

$$B\, y'(t) + \Gamma z'(t) = u'(t)$$

where $y'(t)$, $z'(t)$ and $u'(t)$ are (column) vectors with the endogenous variables, the exogenous variables and the disturbances as elements. The matrix B is supposed to be triangular, and the moment matrix of disturbances is diagonal. Such a model is partitionable into uniequational models ([2] , of [3] , 2.4.10.). The consumption per ration book holders is supposed to be explained by the price of liquor, consumers income, average size of ration and cost of living index. The average size of the ration is not supposed to be explained by the consumption. An eventual price formation relation cannot contain the consumption or the average size of ration as explaining factors. Thus, the matrix B is considered to be triangular.

The logarithmic values of the variables have been used. Accordingly, the coefficients can be regarded as elasticities. The price elasticity of demand turned out to be about - 0.3.

III. A. The significance of the different elasticities will now be examined. Firstly, the price elasticity of demand for a rationed commodity will be considered. It is supposed, that the commodity in question is rationed in such a way that a consumer is entitled to purchase a maximum quantity A during a ration period. It is further assumed, that the rationing does not affect the quantity purchased by those consumers who would have bought a quantity less than A had there been no rationing and also that the remaining consumers purchase quantity A. Suppose now, that, in absence of rationing, the individual elasticities are constant and are all equal to, say, E_p. Let the market elasticity for the rationed commodity be E_p^r. Further, let F denote the relative number of consumers who take out their full rations and U denote the ratio between the average amount purchased, Q, and the ration A. U can be named the purchasing frequency.

We then have the following fundamental relationship

$$E_P^r = E_P \left[1 - \frac{F}{U}\right] \text{ or } \frac{E_P^r}{E_P} = 1 - \frac{FA}{Q} = \frac{Q-AF}{Q}$$

Thus, E_P^r depends not only on E_P but also on the distribution of consumption.

The above relation can be expressed in words as follows: The ratio of the (market) elasticity for a rationed commodity and the individual elasticity is equal to the ratio of the consumption of those consumers who do not take out their full rations and the total consumption. Now, allowing a change in the size of ration, we can speak of the elasticity of demand with respect to size of ration, or briefly, the ration elasticity, E_A which is found to be

$$E_A = \frac{F}{U}$$

and thus

$$E_P^r = E_P \left[1 - E_A\right]$$

We always have $F \leq U$ and thus $0 \leq E_A \leq 1$; $\left|E_P^r\right| \leq \left|E_P\right|$.

B. Let $Q(X_1, P)$ and $Q(X_2, P)$ be the demand functions of two consumers, who at the price P_0 consumes the quantities X_1 and X_2 respectively, where $X_1 < X_2$. If then the relation $Q(X_1, P) < Q(X_2, P)$ holds for every consumer and for a certain price interval, then we will say that the supposition regarding consumption ranking is fulfilled in the price interval in question.

Now, let the market consumption of the rationed commodity at a certain price P_0 be Q_0 and the corresponding purchasing frequency be U_0, while at the price P_1 the consumption is Q_1 and the purchasing frequency is U.

Now, calculating an elasticity $\overline{E_P^r}$ according to the familiar relation

$$\overline{E_p^r} = \frac{\log Q_1 - \log Q_o}{\log P_1 - \log P_o} ,$$

we have, if the supposition regarding consumption ranking is fulfilled

$$(1) \quad \overline{E_p^r} = M\left[E\right] \int_{U_o}^{U_1} d \log U \Bigg/ \int_{U_o}^{U_1} \frac{d \log U}{1 - \frac{F}{U}}$$

where $M\left[E\right]$ is a certain weighted average of the individual elasticities of those consumers, who do not take out their full ration.

In relation (1) the variable F is considered to be function of U or the purchasing frequency. It can be shown, that this is an one-valued function with respect to changes in price, income and size of ration if, for example, the following conditions are fulfilled. The individual price elasticities are equal, so the individual income elasticities and moreover, the incomes of the consumers vary in the same proportion. Now, knowing pairs of values for F and U, and choosing a function representing them, the elasticities $M\left[E\right]$ can be evaluated.

When dealing with the distribution of consumption, it is natural to take into account the fact that the quantity bought should be regarded as a stochastic variable. If the distribution function for the purchases made by a person during a rationing period fulfills certain conditions, one will obtain relationships analogous to the above.

IV. Next, let us consider the market income elasticity. Generally, if the individual income elasticities are all constant and equal, the market elasticity will only in certain cases agree with the individual elasticity. In fact, a levelling out of incomes implies in most cases a market elasticity closer to the value 1 than the individual elasticity.

For a rationed commodity the market income elasticity is the product of three factors:

1. The market elasticity for those consumers who do not take out their full ration;
2. The ratio between the purchases of those consumers and the total consumption;
3. The elasticity for the incomes of those consumers with respect to total income.

V. Finally, we have to consider the significance of the elasticity with respect to avarage size of ration, E_A^t say, when this elasticity is calculated from time series. It is then supposed, that the commodity in question is differentially rationed. This means that the consumers are divided into several ration groups with different sizes of ration.

Changes in the average ration may arise from different causes. The rations within one or several rationing groups can be changed on certain occasions, resulting in sudden changes in the average ration. The elasticity in question is then of the same kind as the ration elasticity E_A mentioned above and as such dependent on the distribution of consumption. However, changes in the size of the average ration can also, for example, be caused by the original group of consumers being increased by the addition of new consumers with a lower demand, thus introducing a stratification effect (cf [5]) in total consumption. If the commodity is differentially rationed, the elasticity with respect to the average ration, E_A^t, then also depends on the rationing policy. Let it be supposed, that the new consumers are distributed among the different ration groups in such a way that the purchasing frequencies in these groups are the same as before, other factors being constant. We then have

$$E_A^t = \frac{M_{W'}\left[U_{\gamma}\right]}{M_{W''}\left[U_{\gamma}\right]},$$

where $M_{W'}$ and $M_{W''}$ are two weighted averages of the purchasing frequencies U_{γ}. If further the rationing policy is carried out in such a way, that the purchasing frequencies U_{γ} are equal, we have

$$E_A^t = 1.$$

In this case, the stratification effect is removed by fixing the elasticity in question to 1. Dealing with a demand relation, which is linear in the logarithms of the variables this meant that the (total) purchasing frequency can be taken as the dependent variable.

For the example chosen, a tightening up of rationing policy would be indicated by an elasticity E_A^t less than 1. A more liberal rationing policy is indicated by an E_A^t greater than 1.

References

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