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A Note on Markowitz's Theory of Investment Companies

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1. In dealing with the higher moments of the distribution of profits of an investment company, you made the remark (in your Cowles Commission Discussion Paper as well as in your address to the Econometric Society) that this is easily tackled by merely adding more dimensions to the efficient set. But is this so? To construct the efficient set in the space (x_1, \dots, x_n) , one has to know which of the x 's are "inputs" (or "bads," like labor) and which are "outputs" (or "goods," like consumers' goods). Granted for a moment that profit expectation is always a desirable thing, and that variance is always undesirable, what do we know about skewness, kurtosis, etc.? Are they desirable (or undesirable) over the whole space of the x ?
2. I think these doubts make it necessary to look deeper into the utility function of sure profits, in the case of a decision maker who is not a consumer but the agent of a company. With the latter, the form of the utility function of sure profits is determined, not as much by "psychological" predilections (as in the case of Mosteller's national guards who acted different from his Harvard students) as by legal provisions regarding bankruptcy, and by the facts of competition with other investment outlets for the public. Disregarding the competition facts till 3. below, we may say that the investment company's dislike of high variances of its profit is due to the cost that would occur in the case of bankruptcy. If y is profit, the utility function of the

company is

$$u(y) = y \text{ for } y \geq K$$
$$u(y) = -A \text{ for } y < K,$$

where K and A are two positive constants: K depends on the firm's cash reserves, and A expresses the penalty (cost of liquidation, etc.) resulting from bankruptcy. The expected utility $E u(y)$ will depend on (a) the probability that profit exceeds K , and on (b) the "truncated" mean profit (averaged from K to infinity). These two parameters of the y distribution, and not its mean and variance, are the "natural" ones to describe the investment company's preferences between alternative probability distributions. However, the conditions under which the parameters (a) and (b) can be, with reasonable approximation, replaced by the mean and the variance of y , can be easily worked out, if, for reasons of mathematical convenience, you prefer to work with the mean and the variance. Note, however, that such approximation may not be a good one, and that this may force you to introduce higher moments: but, then, you will be able to state whether, say, the third, and, for that matter, even the second moment is a "good" or a "bad" thing at all points of a specified region of the moment-space, or whether it is good in some points, and bad in others. You can also find whether different results are observed if the function $u(y)$ is defined differently, for example, by making the liquidation penalty depend on the size of the difference $K-y$ (whenever this difference is positive).

3. On a still deeper level, you may amplify legal considerations (bankruptcy threat) by game-theoretical ones. A game is played between the company management and its shareholders. The shareholders are consumers, and an assumption is made as to their utility function of sure profits. If you assume that this utility function has a negative second derivative everywhere, then each shareholder considers the variance to be undesirable. (More generally, there may exist a frequency distribution of various types of shareholders.) The company management's revenue depends on how

many shareholders subscribe. (In fact, the enormous and costly advertising and selling effort of the open-end investment companies suggests how important it is for them to recruit shareholders.) Hence the company management will take account of the subscribers' utility functions in addition to its own desire to avoid bankruptcy.

To sum up: Unless you can show, for each moment, that this moment is either desirable or undesirable in a specified region of the moment space, you cannot construct the opportunity surface (the efficient set). Accordingly, I suggest you submit the utility function of the company management to a deeper analysis. This may be needed even to justify your treatment of the second moment of the profit distribution; and it is certainly needed with respect to the higher moments.