Quantitative Description of Technological Change (Abstract)

by Gerard Debreu

December 8, 1950

1. Description of the Model.

A commodity is defined by its physical nature, its location (we suppose that there is a finite number of such pairs) and the time interval it refers to (all successive time intervals are supposed equal); it is characterized by the subscript h.

Each consumption-unit is characterized by a subscript i; its activity is described by a consumption vector x_i of the commodity space $\mathbb{R}$ whose components are quantities of commodities consumed or negatives of quantities produced. Its preference ordering permits the construction of a numerical satisfaction function $s_i(x_i)$. The s_i are considered as the components of a vector s of the satisfaction space $\mathbb{R}^*$.

The production activity of the economic system is described by an input-vector $y \in \mathbb{R}$ whose components are net quantities of commodities consumed by the whole production sector or negatives of net quantities produced. Constraints such as the limitation of technological knowledge determine the set $\mathbb{Y}$ of possible y. A family of sets $\mathbb{Y}_j$ is a decomposition of $\mathbb{Y}$ if $\mathbb{Y} = \sum_j \mathbb{Y}_j$. $\mathbb{Y}_j$ in this case is the set of possibilities of the jth production-unit whose input-vector is y_j.

Finally let us define $x = \sum_i x_i$, $z = x + y$; one has necessarily $z \leq z^0$ where z^0 is the utilizable physical resources vector.

2. Discussion of the Model.

No uncertainty is introduced. The constraint of given stocks at the end of the last time interval gives rise to several difficulties: they can be overcome by the absence of such a constraint or by the time extension of the economic activity to infinity.

The problem of change can be conceived in two different ways:

1) as a study of a structural change in the data $s_i(x_i)$, $\mathbb{Y}$, z^o

2) as a study of the dynamic evolution of the system: One compares two different time-intervals.

The second problem is the more difficult one: the satisfaction functions change from t to t +1 but not in an arbitrary way, and similarly for $\mathbb{Y}$.

Definitions of $\mathbb{Y}$ and $\mathbb{Y}_j$ range from abstract ones such as the set of possibilities which can be imagined on the basis of some pooling of all existing knowledge to concrete ones taking into account the innumerable obstacles which prevent it.

3. Technological Change.

The definition of technological change as a change in y is much too wide since it can result from a change in $s_i(x_i)$, $\mathbb{Y}$, z^o, or in economic organization. We therefore define it as a change of $\mathbb{Y}$. This change comes as well from invention as from the diffusion of technological knowledge.

The changes of $\mathbb{Y}$, $s_i(x_i)$, etc. are considered as data; their little known and all important interrelations are not studied.

4. Measurement.

If one makes a number correspond to a given technological change, its optimal properties result from the use to which it is put in prediction or action. In the absence of specific problems of this kind a quantitative description may be sought.

5. Change of Technological Knowledge.

The quantitative description of a change in $\mathbb{Y}$ seems to raise insuperable conceptual difficulties, so that this change will be measured in connection with other characteristics of the economic system; however as few as possible of these characteristics will be included so that the measure obtained will still be representative of the basic change.

6. Production Potential.

The constraints

$$y \in \mathbb{Y}^0 \qquad\qquad z \leqq z^0$$

determine the set $\mathbb{S}^0$ of attainable s; this set seems to be, in the most general sense, the production potential of the economic system. If $\mathbb{Y}^0$ is replaced by $\mathbb{Y}^1 \supset \mathbb{Y}^0$, $\mathbb{S}^0$ is replaced by $\mathbb{S}^1 \supset \mathbb{S}^0$. A quantitative description of this change will be later suggested.

7. Measure of Technological Change in Connection with Given z^0 and s^0.

Let $s^0 \in \mathbb{S}^{0\,\mathrm{Max}}$ be the observed standard of living and $\mathbb{Z}^1$ be the set of z defined by

$$y \in \mathbb{Y}^1, \qquad\qquad s \geqq s^0 .$$

The relative position of z^0 and $\mathbb{Z}^1$ describes the gain originating from the replacement of $\mathbb{Y}^0$ by $\mathbb{Y}^1$. The problem of description of this relative position leads to the introduction of a coefficient of resource-utilization ρ defined by

$$z^M = \rho\, z^0 \qquad\qquad z^M \in \mathbb{Z}^{1\,\mathrm{Min}} \;\; (1)$$

(see Economics No. 297, Section 9). The gain originating from the technological change is thus described by $1 - \rho$, or $z^0(1 - \rho)$, or its value $p^0 \cdot z^0(1 - \rho)$. This definition can be extended to nonoptimal situations.

If s^0 is allowed to vary in $\mathbb{S}^{0\,\mathrm{Max}}$ Max ρ and Min ρ afford a quantitative description of the effect of the technological change on the production potential in connection with z^0.

---

(1) In a vectorial space ordered by the relation $u \leq v$ ($u_i \leqq v_i$ for every component, and $u \neq v$) a minimal element of a set $\mathbb{U}$ is a vector $u \in \mathbb{U}$ such that there is no $v \leq u$ in $\mathbb{U}$. $\mathbb{U}^{\mathrm{Min}}$ is the set of minimal elements of $\mathbb{U}$. A maximal element of $\mathbb{U}$ and $\mathbb{U}^{\mathrm{Max}}$ are defined in a symmetrical way.

Alternatively the gain attributable to technological change can be defined as the minimum expansion of z^1 which permits the achievement of s^1 with $\mathcal{Y}^o$.

This definition allows for changes in population, tastes, resources, introduction of new commodities.

8. Other Definitions.

The variation of real national income $p \cdot (x^1 - x^o)$, could be thought of as another measure of the gain due to technological change. This quantity can be negative; while this would happen only in exceptional cases, it throws some doubt on the general reliability of this measure. Moreover this number depends not only on $\mathcal{Y}$, z^o, $s_i(x_i)$ but on the economic organization of the system.

Another definition, the variation of $-\frac{p \circ y^-}{p \circ y^+}$ (y^- is formed with the negative components of y, y^+ with the positive components) does not seem justifiable either.

The choice has to be made between a definition conceptually acceptable which does not lead to an easy numerical evaluation but for which approximation processes can be devised and a definition leading to an easy evaluation but conceptually unsatisfactory.

9. Technological Change Over a Long Period of Time.

The direct comparison of two distant time intervals is meaningless. A chain process has to be devised. The multiplication of the percentages $\rho^o, \rho^1, \ldots, \rho^t$ or the addition of $1 - \rho^o, 1 - \rho^1, \ldots, 1 - \rho^t$ are inacceptable for they are not percentages of the same thing (they are respectively percentages of $z^o, z^1, \ldots, z^t$). The dollar values of the gains, if they are made comparable, afford a simple and better basis.

10. Technological Change in the Small.

We assume that $\mathcal{Y}_j^{Min}$ is represented by the equation

$$e_j(y_j, \alpha_j) = 0$$

where α_j is a numerical parameter. The equations determining ρ are

$$s_i(x_i) = s_i^0 \qquad e_j(y_j, \alpha_j) = 0$$

$$\frac{\text{grad } s_i}{p} = \sigma_i \qquad \frac{\text{grad } e_j}{p} = \varepsilon_j \quad (2)$$

$$\sum_i x_i + \sum_j y_j = \rho z^0 .$$

A variation of the α_j and the corresponding variation of ρ are related by

$$d\rho \quad p \cdot z^0 = -\sum_j \frac{1}{\varepsilon_j} \quad \frac{\partial e_j}{\partial \alpha_j} \; d\alpha_j .$$

The second member depends only on technological data for each industry.

If the standard of living, the technological possibilities, the physical resources and the degree of efficiency of the economy all vary, the relation

$$\sum_i \frac{ds_i^0}{\sigma_i} - \sum_j \frac{1}{\varepsilon_j} \; \frac{\partial e_j}{\partial \alpha_j} d\alpha_j = d\rho \; p \cdot z^0 + \rho \; p \cdot d\, z^0$$

makes a synthesis of the different factors of economic evolution which is not possible in the large.

---

(2) p is the intrinsic price-vector associated with z^M (see Economics No. 297, Section 9), σ_i, ε_j are numerical proportionality coefficients.