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A NOTE ON MODELS AND CLEARING OF THE MARKET[1]

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1. In order to fit demand equations and supply equations it is necessary to have data on quantities demanded and supplied. For a wide class of commodities, data is available only for the quantity actually sold[2] (which is, of course, also the quantity bought). Therefore it appears impossible to fit a demand curve unless quantity demanded is equal to quantity sold. The same holds for supply curves.

2. Four possible ways of meeting this kind of situation suggest themselves, in the form of alternative assumptions underlying the economic model:

(1) That the market is always cleared and in equilibrium, i.e., that by the operation of some sort of adjustment process the quantities supplied and demanded are always kept exactly equal, though not necessarily constant from period to period. In this case both are observable, because both are equal to amount sold.

(2) That the quantity sold is always equal to the quantity demanded, the supply function having no effect on output (perhaps because of the existence of unused resources or for any other reason). In this case quantity demand-

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1. I have profited from discussing this subject with J. Bronfenbrenner, K. J. Arrow and S. G. Allen.

2. Exceptions are the labor and dwelling-space markets in periods when amounts supplied exceed amount demanded.

ed is observable; quantity supplied is not but it has no influence anyway. (The roles of demand and supply might be interchanged here, of course.)

(3) That sellers set a price at which buyers buy whatever their demand schedules tell them to buy at that price (given the other conditions such as income, cash balances, etc.). This situation is a kind of special case of assumption (2); here sellers can excercise some control over amount sold by varying the price.

In this case adjustment to disequilibrium takes place via accumulation (or decumulation) of inventories as a result of the excess of output over quantity demanded (or vice versa), rather than via a freely fluctuating price as in assumption (1). Hence this is the monopoly case.

(4) That one of the pair (quantity supplied, quantity demanded) is always observable[3], and that the nonobservable one is equal to some explicit function of the other <u>observable</u> variables in the system.[4] Then wherever the non-observable one occurs in any equation it can be replaced by this function of observable variables, so that the system will contain only observable variables. Thus one of the pair <u>is</u> observable, and the other is obtainable by substituting values of observables into a function (but this function cannot be tested directly to see whether or not it is in fact a good approximation of the nonobservable variable which it replaces; see Cowles Commission Discussion Paper: Economics No. 241, pages 15-17, for a discussion of this point.)

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3. Not necessarily always the same one; for example, the observable one might be the minimum of the two, i.e., (given our legal institutions) the amount sold.

4. Such a function might express demand for inventories, for example, as a linear combination of lagged values of income, income change, price, and price change. The function might be assumed directly, or derived theoretically from other assumptions.

(A generalization of assumption (4) is to assume that neither quantity demanded nor quantity supplied is observable, but that each of these two nonobservable quantities is equal to some explicit function of observable variables—this is applying the idea of assumption (4) to both demand and supply instead of to just one.)

3. The Walrasian general-equilibrium theory makes use of assumption (1). For each commodity there are three equations: demand, supply, and equilibrium condition stating that the quantities demanded and supplied are equal. Tinbergen's model for the U. S. for 1919-32 also uses assumption (1) for some markets[5] though he usually combines the supply equation with the equilibrium condition to eliminate quantity supplied and obtain an equation for the price.

4. Assumption (2) is implicit in the following simple Keynesian model:

$$C = \alpha_0 + \alpha_1 Y + u$$
$$Y + T = C + I + G$$

where I + G - T is exogenous. The consumption function is meant to be a demand equation (it is derived from the theory of consumer behavior) and the other equation is an ex post identity—therefore the variable C must represent demand in one equation and actual consumption in the other, which is to say that assumption (2) is being made.[6]

Assumption (2) is also implicit in two recent models. One is the Klein-Marx model[7], and the other is the Colin Clark model[8] for the United States, 1921-1941.

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5. J. Tinbergen, Statistical Testing of Business Cycle Theories, II (Geneva, League of Nations, 1939), 52-71.

6. Klein's models I and II in "The Use of Econometric Models as a Guide to Economic Policy," Econometrica, 15 (1947) p. 111-51, are essentially like this simple Keynesian model.

7. Klein, Economic Fluctuations in the U. S. 1921-41, Cowles Commission Monograph 11, pp. 63-6.

8. Colin Clark in Econometrica, 17 (April 1949).

5. Tinbergen's model uses assumption (3) for some equations, namely those for markets in which he believes there are administered prices.[9]

6. Klein's model III[10] is similar to the simple Keynesian model mentioned earlier with refinements, but I believe it is best understood if it is regarded as a model based on assumption (4). Stripped of its money and labor equations (which can be dropped harmlessly because each takes with it one variable which appears nowhere else in the system) and of its separate treatment of the housing market, the Klein model III reduces to these five equations:

$$(1)\quad C = \alpha_0 + \alpha_1 Y + \alpha_2 t + u$$

$$(2)\quad I = \beta_0 + \beta_1\left(\frac{pY - E}{q}\right) + \beta_2\left(\frac{pY - E}{q}\right)_{-1} + \beta_3 K_{-1} + \beta_4 t + u_2$$

$$(3)\quad \Delta H = \gamma_0 + \gamma_1 (Y - \Delta H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t + u_3$$

$$(4)\quad Y + T = (Y + T)_{-1} + \delta_0 + \delta_1 (u_3)_{-1} + \delta_2 . p + u_4$$

$$(5)\quad Y + T = C + I + \Delta H + G$$

where the endogenous variables are C, I, ΔH (change in real inventories), Y (disposable income) and p (general price level), and the predetermined variables are:

E = excise taxes

q = investment-goods prices

K_{-1} = lagged year-end stock of capital

H_{-1} = " " " " " inventories

T = GNP - Y

G = government expenditures

$(u_3)_{-1}$ = lagged disturbance to inventory equation

p_{-1} = lagged general price level.

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9. Tinbergen, loc. cit.

10. This model appears in Economic Fluctuations, pp. 102-5, and in the cited article in Econometrica, 15 (1947), pp. 125-9.

All quantities are measured in deflated dollars, except prices and excise taxes.

7. Now consider the following model, based on assumption (4).

$$(6)\quad C = \alpha_0 + \alpha_1 Y + \alpha_2 t + u_1$$

$$(7)\quad I = \beta_0 + \beta_1 \left(\frac{pY - E}{q}\right) + \beta_2 \left(\frac{pY - E}{q}\right)_{-1} + \beta_3 K_{-1} + \beta_4 t + u_2$$

$$(8)\quad \Delta H = \Delta H^D + u_3$$

$$(9)\quad (Y + T)^S = (Y + T)^S_{-1} + \delta_0 + \delta_1 (u_3)_{-1} + \delta_2 \Delta p + u_4$$

$$(10)\quad (Y + T)^S = (Y + T)^D + u_3$$

$$(11)\quad \Delta H^D = \gamma_0 + \gamma_1 (Y - \quad H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t$$

$$(12)\quad (Y + T)^D = C + I + \Delta H^D + G$$

where now ΔH^D is demand for inventory-change, and $(Y + T)^S$ and $(Y + T)^D$ are aggregate supply and demand. This model contains seven endogenous variables: C, I, ΔH^D, ΔH, $(Y + T)^S$, $(Y + T)^D$, p. It contains an aggregate supply equation (9); an aggregate demand equation (12) made up of the sum of separate demand equations (6), (7), (11) and government demand; an adjustment equation (10) stating that quantities supplied and demanded are equal except for a random disturbance u_3; and an additional equation (8) stating that the disturbance u_3 in the adjustment process is all concentrated in imperfect satisfaction of the demand for inventories.[11] The meaning of this equation (8) is that the actual amount sold is always exactly equal to the amount supplied, that demand for goods by consumers, investors and government is exactly satisfied, and that demand for inventories (and hence aggregate demand) is satisfied except for a random disturbance.[11]

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11. Klein in Economic Fluctuations, p. 102, says: "According to equation [(8')], all inventories held are demanded as a result of a definite behavior pattern except for the amount u_3, which is the random disturbance. We call u_3 undesired or excess inventories which the entrepreneur holds because he misjudged the market. We shall assume, for the economy as a whole, that supply and demand balance except for a random disturbance."

8. The following point, though apparently involving hair-splitting, is important: admittedly, it is easy to insist that the aggregate supply equation and the demand equations for consumption and investment be exactly satisfied, because each of these equations has in it a disturbance which can always be called upon to make the quantity supplied or demanded exactly equal to the observed quantity sold. The reason for so insisting, even though it apparently makes no difference whether the disturbance is said to be (a) entirely within the supply (or demand) equation (the market being cleared exactly) or (b) partly in the imperfection of the market adjustment process, is that in fact we observe only amounts sold; and if we are to claim that we <u>observe</u> any amount supplied or demanded, we <u>must</u> assume that it is precisely equal to an amount sold, which means that the corresponding supply or demand equation is exactly satisfied. The demand for inventories is treated differently, i.e., assumed to be satisfied with a random disturbance instead of exactly, because it is assumed that the market is cleared in the aggregate except for a random disturbance which is completely absorbed by inventory changes.[11]

9. We will show that the simplified Klein Model III given by equations (1)-(5) can be easily derived from equations (6)-(12). There are two unobservable variables: ΔH^D and $(Y + T)^D$. Equation (12) expresses $(Y + T)^D$ as a function of ΔH^D and observable variables; equation (11) expresses ΔH^D as a function of observable variables only. Therefore if we substitute (11) for ΔH^D wherever ΔH^D occurs, and then (12) for $(Y + T)^D$ wherever $(Y + T)^D$ occurs, we will have reduced the seven equations to five, and all remaining variables will be observable. Equations (6), (7) and (9) will be as before, while (8) and (10) will look like this:

$$(8') \quad \Delta H = \gamma_0 + \gamma_1 (Y - \Delta H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t + u_3$$

$$(10') \quad (Y + T)^S = C + I + [\, \gamma_0 + \gamma_1 (Y + \Delta H) + \ldots + \gamma_5 t + u_3 \,] + G$$

Now if we substitute ΔH from (8') for the expression in the square bracket in (10'), and then relabel the variable $(Y + T)^S$ as $Y + T$, the equations (6)-(10) finally become identical with (1)-(5).

Thus we have explained the Klein model III, minus its refinements, by showing that it makes use of assumption (4). In particular, it assumes that the market is cleared except for a random disturbance u_3 which is completely absorbed by (undemanded) inventory changes, i.e., that other demand equations and all supply equations are exactly satisfied.[12] Therefore observed values of aggregate output can be called "supply" and observed values of consumption and investment can be called "demand." Observed values of inventory change are a sum of demanded inventory change and the disturbance u_3; an untestable figure for demanded inventory change is obtainable by calculation from equation (11) using observed values of the variables appearing on its right side.

10. A different interpretation could be put upon the Klein Model III if it were not for his explicit statement quoted in footnote 11 above. We could say that Klein's model uses assumption (1), i.e., that all markets are cleared exactly. Then the inventory disturbance would be regarded as no different from the consumption, investment, and supply disturbances. The only trouble with this interpretation is that it makes difficult the explanation of the presence of $(u_3)_{-1}$ in the supply equation (9). But still it is clear that assuming the market cleared except for a random disturbance, while formally an instance of assumption (4), is not essentially different from assuming the market cleared exactly (assumption (1)). Other examples of assumption (4) can be found which are essentially different from assumption (1); see sections 11-12 below.

11. The following model is based on assumption (1) regarding the aggregate goods market, and on assumption (4) regarding the labor market. The labor market appears

12. Of course, this kind of a theory breaks down if demand ever exceeds supply by more than the existing amount of inventories.

here explicitly because the supply function has been replaced by a group of more autonomous equations from which it is derivable: the production function (17), equations (18)-(21) to determine the input of labor, and equation (22) to determine the input of capital.

(13) $C = \alpha_0 + \alpha_1 Y + \alpha_2 t + u_1$

(14) $I = \beta_0 + \beta_1 \left(\frac{p\,Y - E}{q}\right) + \beta_2 \left(\frac{p\,Y - E}{q}\right)_{-1} + \beta_3 K_{-1} + \beta_4 t + u_2$

(15) $\Delta H = \gamma_0 + \gamma_1 (Y - \Delta H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t + u_3$

(16) $Y + T = C + I + \Delta H + G$

(17) $Y + T = \epsilon_0 + \epsilon_1 n + \epsilon_2 K^* + \epsilon_3 t + u_5$

(18) $n =$ minimum of (n^D, n^S)

(19) $n^D = \phi_0 + \phi_1 w + \phi_2 p + \phi_3 p\,Y + u_6$

(20) $n^S =$ exogenous variable

(21) $\Delta w = \theta_0 + \theta_1 w + \theta_2 p_{-1} + \theta_3 p_{-2} + \theta_4 (n^S - n^D) + u_8$

(22) $K^* = \ldots$

(23) $w = \Delta w + w_{-1}$

Endogenous variables are C, I, H, p, and Y (defined as before), and

n = employment

n^D = demand for labor

n^S = supply of labor (which might be treated as exogenous)

Δw = change in wage rate

K^* = measure of capital input (for which K_{-1}, lagged end-of-year capital stock, might be used)

w = wage rate

Exogenous variables are E (excise taxes), q (capital-goods prices), T, G, and t. All quantities are measured in deflated dollars except prices, wages, and quantities of labor.

12. In this model all quantities are observable at all times, except for n^D which is only observable when $n^D = n$, i.e., when $n^D \leq n^S$ and there is unemploy-

ment. This means that for all slack years when there is unemployment, n is substituted for n^D in (19), and equation (19) is fitted; while for all boom years when there are jobs available and unfilled at the going wage, n^S is substituted for n in (17), and equation (19) is not fitted. All other equations (except (18) which is dropped) are fitted from all years, both slack and boom.

In order for this model to be fitted in this way, it is necessary that each year be classified as either slack ($n^D \leq n^S$) or boom ($n^D \geq n^S$) <u>before</u> equation (19) is fitted, because this classification determines which years belong to the samples from which (19) is fitted.[13, 14]

Note that (19) (as obtained by fitting in slack years) must be used to obtain calculated values of n^D for boom years, so that the quantity ($n^S - n^D$) in (21) can be evaluated--and note further that these calculated values of n^D for boom years cannot be tested against observation because n^D is not observable in boom years. Thus, the unobservable variable here which is obtainable as a function of observable variables, according to assumption (4), is n^D (in boom years only).

The aggregate goods market is cleared exactly, i.e., assumption (1) is made here. This is seen from the fact that total quantity sold Y + T is given both by the aggregate demand equation (16) and by the supply equation derivable from the production function (17).

13. The models presented here are not intended to be ready to use for prediction purposes; there are various improvements and refinements which should be introduced first, particularly in connection with the money market and the price of capital goods. These models are merely intended to illustrate the various methods of dealing with the fact that is usually only quantity sold which is observed, and not quantities supplied or demanded per se.

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13. This might be done arbitrarily on the basis of unemployment figures--any year when unemployment is less than, say, a million might be declared a boom year on the assumption that frictional unemployment is a million.

14. Or it might be assumed that $n^D \leq n^S$ always, in which case (18) would be dropped and n would be substituted for n^D in (19) and (21); then both n and n^S would be observable and equation (19) would be fitted for the sample of all years. This pro-

(Footnote 14, continued from page 9)

cedure would be statistically (degrees-of-freedom-wise) more satisfactory, but it assumes away the possibility that demand for labor might exceed supply as must have been the case during World War II when jobs were unfilled at the going wage rate. It also removes the model from the province of assumption (4) by making all variables observable and thus removing the problem with which this note began; cf. footnote 2, page 1.