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# A COMPARISON OF ORGANIZATION THEORIES

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It is the purpose of this note to suggest a framework that permits a comparison of certain theories of organization that appear in the literature of economics with those in the literature of administration. The relations between the economist's theory of the firm and what is usually called in administrative writings the "theory of organizational equilibrium" have never been made explicit, and writers in the one field often appear unaware of the possible implications for their work of the investigations that have been carried on in the other.

Part I of this paper will set forth brief descriptions of the theories in question. In Part II a framework will be proposed capable of encompassing both theories as special cases. In Part III the general framework will be interpreted in terms of each of the special theories.

## I

The theory of the firm (F-theory) and the theory of organization (O-theory) are both concerned with the behavior of a person, or people, trying to gain certain ends by the manipulation of variables at their disposal (strategic variables). The problem of "optimal," "rational," or "efficient" behavior with respect to these ends can be formulated as a problem of finding the maximum (with respect to the strategic variables) of some function that is taken as a measure of success in attaining these ends (e.g., in the theory of the firm, finding the output that maximizes profit). Theories of organization have been concerned not only with optimal solutions, but with the whole set of viable solutions--that is, solutions that permit the survival of the organization (e.g., in the theory of the firm, outputs that yield a positive profit).

Theory of the Firm

In its usual form, the problem of optimality in the theory of the firm is to maximize profit--the difference between value of product and cost of production. The firm may produce a number of different products, and employ a number of different factors of production. The product prices may be given, or, more generally, may be functions of the quantities produced. Similarly, the factor prices may be given or may be functions of the quantities of factors employed.

The problem of optimality may be divided into a problem of technological optimality and a problem of economic optimality. Optimality in the latter sense implies optimality in the former. Outputs of product are connected with inputs of factors by certain relations or constraints, so that, for example, if all the inputs are given and all but one of the outputs, the remaining output will have a definite upper bound. These constraints define the "possibilities of production." A set of product outputs and factor inputs may be regarded as technologically optimal if no single output can be increased or input decreased without decreasing at least one other output or increasing at least one input. The set of technologically optimal outputs and inputs corresponds to the economist's usual notion of the production function. If there are n outputs and m inputs, the set of optimal output-input combinations may have as many as (n+m-1) degrees of freedom.

By introducing fixed prices for outputs and inputs, or by introducing equations of demand and supply, respectively, for them, the point can be found on the production function which corresponds to economic optimality--i.e., to profit maximization. We include here both the cases of perfect and imperfect competition, but not the case of oligopoly, where these prices are regarded as functions also of the quantities produced and consumed by other firms.

Let us consider further the case where the output of the firm is a single product. In this case we may first find all technologically optimal inputs for each input, and then determine which of these is economically optimal in terms of given prices or supply curves for the factors--i.e., which gives the lowest cost of product. In this way we derive the familiar cost curve which, when combined with the demand curve, permits us to determine the output that will maximize profit.

Theory of Organization

In the F-theory, a single participant, the entrepreneur, is explicitly treated as a rational individual. The other participants--employees, customers, suppliers--enter into the theory only implicitly and only as passive "conditions" to which the entrepreneur adjusts in finding the solution that is optimal to him.[1] One such condition is the price of the factor "labor," another is the demand schedule, or the total revenue schedule which describes the behaviors of customers.

In the O-theory the participants are generally treated in a more symmetrical fashion.[2] Each participant is offered an inducement

1 It is on this point that oligopoly theory departs fundamentally from the older theories of the firm.

2 The theory we are describing here is essentially that proposed by Chester I. Barnard in The Functions of the Executive (Cambridge:

Harvard University Press, 1936). See also, Herbert A. Simon, Administrative Behavior (New York, Macmillan, 1947); H. A. Simon, D. W. Smithburg, and V. A. Thompson, Public Administration, ch. 18, 23.

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for his participation in the organization. Through his participation, he makes a contribution to the organization. The participant's contributions may be regarded as "factors," the inducements offered to him as "products." Thus the organization transforms its members' contributions into inducements which it, in turn, distributes to these members.

As a simple example, consider an organization with an entrepreneur, one employee, and one customer. The system of inducements and contributions may then be represented thus:[3]

| Participant | Inducements | Contributions |
|---|---|---|
| Entrepreneur | Revenue From Sales | Costs of Production |
| Employee | Wage | Labor |
| Customer | Goods | Purchase Price |

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In order to simplify this example as far as possible, the table includes only the obvious tangible inducements and contributions. In the general development of the theory this restriction is neither essential nor desirable.

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The customer's contribution of the purchase price is used to provide inducements to the entrepreneur in the form of revenue. The entrepreneur's contribution provides the employee's wages. The employee's contribution is transformed into goods that provide the customer's inducement. This last transformation corresponds to the usual production function. However, we also have another "production function" that constrains the money flows--these flows must obey the basic accounting equation.

We might now proceed, as in the F-theory, to prescribe criteria of optimality (in addition to the technological criteria we have already incorporated in the system) and to derive optimum values for the inducements and contributions. However, O-theory has generally been concerned not so much with optimality as with the conditions necessary for organizational survival, that is, the conditions under which the participants will continue to participate.

It may be postulated that each participant will remain in the organization if the satisfaction (or utility) he derives from the net balance of inducements over contributions (measured in terms of their utility to him) is greater than the satisfaction he could obtain if he withdrew. The zero point in such a "satisfaction function" is defined, therefore, in terms of the opportunity cost of participation. In general, the survival criterion will not yield a unique solution to the values of inducements and contributions. The solutions that are compatible with survival we have previously referred to as viable solutions.

To restrict further the set of viable solutions, a weak optimality condition can be imposed that does not involve any assumption of interpersonal comparison of satisfactions or utilities.[4] A viable solution is regarded as optimal if no further increase could be made in the net satisfaction of any one participant without decreasing the satisfaction of at least one other participant. Imposition of this condition still leaves us, in general, without a unique solution — a unique set of values of the inducements and contributions.

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4 There is an obvious analogy of this condition with the weak optimality condition used in modern theories of welfare economics.

II

We proceed now to a more rigorous formulation of the theory. It will be convenient to adopt a number of notational conventions. By $\vec{x}$ is meant the vector with components $(x_1, x_2, \ldots, x_n)$. The scalar product of two vectors is defined by:

(1) $$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i$$

By $\vec{x} \geq \vec{y}$, we mean that $x_i \geq y_i$ for all i, and $x_k > y_k$ for at least one k. By $\vec{x} \geqq \vec{y}$, we mean that $\vec{x} \geq \vec{y}$ or $\vec{x} = \vec{y}$. By $\vec{x} \not\geq \vec{y}$ we mean the negation of $\vec{x} \geq \vec{y}$, i.e., $\vec{x} = \vec{y}$ or $y_k > x_k$ for at least one k. All the functions that appear will be assumed to satisfy appropriate conditions of continuity and differentiability.

We postulate a set of functions, $S_i$, where:

(2) $$S_i = S_i(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m) \quad (i = 1, \ldots, p)$$

and a set of constraints

(3) $$H_j(\vec{x}, \vec{y}) = 0 \qquad (j = 1, \ldots, k)$$

The point set, $\Gamma$, in the $(x_1, \ldots, x_n, y_1, \ldots, y_m)$ - space that satisfies (3) we call the set of <u>achievable</u> points in that space. We may also speak of the achievable points, $\Sigma$, in the $\vec{S}$-space, i.e., the $\vec{S}$ that satisfy (2) for $(\vec{x}, \vec{y})$ in $\Gamma$. We assume that $\Sigma$ is bounded in the positive orthant, i.e. that $\vec{S} \leqq \vec{K}$ for some $\vec{K} \geq 0$.

If (3) can be solved for k of the variables in terms of the remaining $r = m + n - k$, and the values are substituted in (2) we get:

(4) $$S_i = S_i(Z_1, \ldots, Z_r) \qquad (i = 1, \ldots, p)$$

where the $Z_j$ are the remaining independent variables. Alternatively we may consider the $Z_j$ to be parameters such that:

(5) $\quad x_k = x_k(\vec{Z}),\ y_l = y_l(\vec{Z}) \qquad (k = 1, \ldots, n; l = 1, \ldots, m)$

and the achievable point set, $\Sigma$, as the transform in the $(\vec{x}, \vec{y})$-space of the entire Z-space (or in some cases, the positive orthant of the Z-space). In the cases with which we are concerned, $k \leq m$, and hence $r = n + m - k \geq n$. Where $k = m$, we may regard (3) as a transformation of $\vec{x}$ into $\vec{y}$.

If the set $\Sigma$ includes at least one point in the positive orthant, that is, a point for which

(6) $\quad \vec{S} \geq 0$

we say that the system (2) - (3) is <u>viable</u>.

Next, we define the weak criterion of optimality. A solution, $\vec{S}$, of (4) is <u>optimal</u> if there does not exist another solution, $\vec{S}'$, such that $\vec{S}' \geq \vec{S}$. If $\Sigma$ is closed and compact as well as bounded, then optimal solutions exist.

The optimal solutions can be found by the method of Lagrangian multipliers. We consider

(7) $\quad T = \sum_{i=1}^{p} \lambda_i S_i = \vec{\lambda} \cdot \vec{S}$

where the $\lambda_i$ are constants to be determined. The optimal solutions are then the solutions that maximize T, regarded as a function of $\vec{Z}$.

Necessary conditions for a maximum of T are:

(8) $\quad \sum_{i=1}^{p} \lambda_i \frac{\partial S_i}{\partial Z_j} = 0 \qquad (j = 1, \ldots, r)$

These equations give r conditions to determine $\vec{Z}$ as a function of $\vec{\lambda}$. (Since the equations (8) are homogeneous of the first degree in the $\lambda_i$, $\lambda$ can be defined only up to multiplication by an arbitrary scalar, hence has (p-1) degrees of freedom.) Generally, we can determine $\vec{Z}$

uniquely by imposing (p-1) auxiliary conditions, for example, the conditions

(9) $S_i = L_i$, a constant $(i \neq m)$.

where the components of $L_i$ are chosen so as to lie in the projection of $\Sigma$ upon the (p-1)-space orthogonal to the axis of $S_m$. The solution that corresponds to $L_i = 0$ $(i \neq m)$, we will call the solution <u>optimal to the $m^{th}$ participant</u>. It will not always be possible to solve (8) for $\vec{Z}$ as a function of $\vec{\lambda}$. In particular cases, (8) may determine certain of the $\lambda_i$ and of the $Z_i$ uniquely, and leave completely indeterminate others of the $\vec{Z}$. We shall examine such a special case later.

## III

We will now, by giving particular interpretations to the elements of the formal system just presented, show the relations between the two theories described in Part I.

Starting with the O-theory, we assume that $S_i$ takes the special form:

(10) $S_i = \phi_i(y_i) - \psi_i(x_i)$

where $S_i$ is the net satisfaction of the $i^{th}$ participant, $y_i$ his inducement and $x_i$ his contribution, and where $\phi_i' = d\phi_i/dy_i > 0$, $\psi_i' > 0$. The x's and y's are subject to one or more relations of the form (3).

In terms of our earlier example, involving three participants, the variables may be interpreted as follows: $S_1$ is profit, $y_1$ the entrepreneur's revenue, $x_1$ his cost of production; $S_2$ is the employee's satisfaction, $y_2$ his total wage, and $x_2$ the quantity of labor he supplies; $S_3$ is the customer's satisfaction, $y_3$ the quantity of goods he buys, and $x_3$ the amount he pays for them. In this example the production relations, (3) become:

(11a) $y_1 = x_3$ , (11b) $y_2 = x_1$ , (11c) $y_3 = \zeta(x_2)$

Equation (11c) is the ordinary production function, assumed already to incorporate the conditions of technological optimality. The first equation in (10) takes the special form:

(12) $S_1 = y_1 - x_1 = x_3 - y_2$

The right-hand equality in (12) may be regarded as the accounting equation, or in the form $(y_1 + y_2) = (x_1 + x_3)$ as a sort of "production function" for money.

Substituting (11) in (10) we get relations of the form:

(13) $S_i = S_i \ (x_1, x_2, x_3)$

For the system to be viable we require $S_i \geq 0$ $(i = 1, 2, 3)$ optimal solutions (with the weak criterion) then correspond to the maximum of:

(14) $T = \lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3$

Where $S_1$ has the special form (12), our equations (8) become:

(15a) $\dfrac{\partial T}{\partial x_1} = -\lambda_1 + \lambda_2 \phi_2' = 0$

(15b) $\dfrac{\partial T}{\partial x_2} = -\lambda_2 \phi_2' + \lambda_3 \phi_3' \zeta' = 0$

(15c) $\dfrac{\partial T}{\partial x_3} = \lambda_1 - \lambda_3 \phi_3' = 0$

From this system we obtain $x_1$, $x_2$, and $x_3$ as functions of $\lambda_1/\lambda_2$ and $\lambda/\lambda_3$. Hence we have a 2-parameter family of optimal solutions.

By setting $S_2 = 0$, $S_3 = 0$ we get the particular solution that maximizes $S_1$ subject to the condition that the system be viable. This solution is closely analagous to the ordinary solution in the F-theory. For, using the relations (11) and (12) we may write:

(16a) $\quad S_1 = x_3 - x_1$

(16b) $\quad S_2 = S_2(y_2, x_2) = S_2(x_1, \varrho(y_3)) = 0$

(16c) $\quad S_3 = S_3(y_3, x_3) = 0$

where $\varrho$ is the function inverse to $\varphi$. Solving (16b) for $x_1$ and (16c) for $x_2$, we get:

(17) $\quad S_1 = x_3(y_3) - x_1(y_3)$

Maximizing (17) with respect to $y_3$, we find the solution that maximizes the entrepreneur's profit. Equation (16b) takes the place of the usual production cost curve; (16c) defines the total revenue curve--both on the assumption that the employee and the customer each receive an inducement just sufficient to keep them in the system.

Before we accept the analogy with the F-theory as complete, we must look more closely at the "cost curve" and "revenue curve" that we have derived. To examine this question, we may carry through the usual derivation of the solution for the F-theory. We do this first for the case of imperfect competition, using a special form of the relations (16).

Consider the system:

(17) $\quad S_1 = y_1 - x_1; \; S_2 = y_2 - \psi_2(x_2); \; S_3 = \phi_3(y_3) - x_3.$

We define the wage rate (w) and the product price (p) by the identities

(18) $\quad w = y_2/x_2; \; p = x_3/y_3.$

We assume that the entrepreneur fixes P, w, and $x_2$ so as to maximize his profit for a given demand curve for the product, and supply curve for labor.

To derive the demand curve we assume that the customer maximizes $S_3$ with respect to $y_3$ for fixed p:

(19) $$S_3 = \phi_3(y_3) - y_3 p \,,$$

(20) $$\frac{\partial S_3}{\partial y_3} = \phi_3' - p = 0$$

Similarly, to derive the supply curve we assume the employer maximizes $S_2$ with respect to $x_2$ for fixed w:

(21) $$S_2 = x_2 w - \psi_2(x_2)$$

(22) $$\frac{\partial S_2}{\partial x_2} = w - \psi_2' = 0$$

Substituting (22) and (20) in (17), we get a function of $x_2$:

(23) $$S_1 = \xi(x_2)\phi_3' - x_2\psi_2'$$

Maximizing this with respect to $x_2$, we find:

(24) $$\psi_2' = \xi'\phi_3' + \xi\phi_3''\xi' - x_2\psi_2''$$

If we make the plausible assumptions that

$\phi_3'' \leq 0,\ \psi_2'' \geq 0$ it follows that:

(25) $$\psi_2' \leq \xi'\phi_3'$$

On the other hand, if we follow the procedure of equations (13) -(15) to find the optimal solutions for the theory of organization, we get:

(26) $$\psi_2' = \xi'\phi_3'$$

We conclude that the solution of the F-theory is an optimal solution in the sense of the O-theory only in case $\phi_3'' = 0,\ \psi_2'' = 0$. But in this case, $\phi_3'$ and $\psi_2'$ are constants, say $p^*$ and $w^*$, respectively and equations (20) and (22) reduce to:

(27) $p = p^*$ $w = w^*$

This is the case where at a given price the entrepreneur can sell any quantity of his product, and at a given wage employ any quantity of labor. It is the case, in short, of perfect competition.

We may summarize the results of the foregoing paragraphs in two propositions:

1. In the case of perfect competition, the F-theory gives us the particular solution in the set of solutions for the O-theory that is optimal to the entrepreneur.

2. In the case of imperfect competition, the solution of the F-theory is not an optimal solution of the O-theory.

Let $(S^*_1, S^*_2, S^*_3)$ be the satisfactions corresponding to the solution of the F-theory under imperfect competition; and $(S^+_1, S^+_2, S^+_3)$ the satisfactions corresponding to the solution that is optimal to the entrepreneur in the O-theory. Then, since $S^+_2 = S^+_3 = 0$, either $S^*_1 < S^+_1$, or one of the components $S^*_2$, $S^*_3$ must be negative. Hence if the former solution is viable, it yields the entrepreneur a smaller profit than the latter.

We see also that equations (16b) and (16c) are not identical with the cost curve of the F-theory. From (17) we have for the former:

(28) $y_2 = \psi_2(x_2)$; $x_3 = \phi_3(y_3)$

while from (20) and (22), we have for the latter:

(29) $y_2 = x_2\psi_2'(x_2)$; $x_3 = y_3\phi_3'(y_3)$

If we define $w^+$ as $y_2/x_2$ where $y_2$ is given by (28), we have $w^+ = \psi_2/x_2$, while $w = \psi_2'$. Similarly $p^+ = \phi_3/y_3$, while $p = \phi_3'$. For this particular solution in the O-theory, wages and prices are propositional to the <u>average</u> utilities of labor to the employee and of the product to the customer, respectively; while in the theory of the firm, wages and prices are proportional to marginal utilities.

Since the two theories give different answers to this basic question, which is to be regarded as correct? The answer to this question depends, in turn, on two considerations:

a. Are the participants other than the entrepreneur to be regarded as passive? That is, are we to restrict ourselves to the solution that is optimal to the entrepreneur? The F-theory answers the question in the affirmative; the O-theory leaves it open.

b. Do the participants bargain in terms of increments of contribution and inducement, or is the bargaining an all-or-none question of participation or non-participation. The F-theory assumes the former alternative; at least one solution of the O-theory takes the latter.

These are obviously questions to be answered by empirical observation of the actual institutional processes rather than by a priori postulation. We shall not attempt to answer them here.

Two final comments are in order here. Enlargment of the F-theory to include situations of oligopoly and bilateral monopoly appears to point in the direction of a theory of organization similar to that developed here. Second, the optimal solutions of the O-theory bear an obvious resemblance to the solutions of the non-zero sum game.