COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 285

Amoroso's Theory of Interest

by Jacob Marschak

June 6, 1950

NOTE: The following review, written for Mathematics Reviews, may be of interest to staff members working on assets, micro-dynamic models, etc. The review is "streamlined" in that only the most general of Amoroso's models is given explicitly. (This made it necessary to generalize the equation that corresponds in the original to (D) below).

Amoroso, Luigi. Revisione della teoria matematica dell' interesse. Atti Istituto Nazionale Assicurazioni. 14, pp. 83-102 (1942).

Denote by $y(t)$, $t_0 \leq t \leq t_1$, an individual's income per time unit, at time t. Write $\eta(t) = y(t)/P(t)$ where $P(t)$ is the price level. Let his investment opportunities, aside from borrowing and lending, be expressed by the integral equation in $\eta(t)$,

$$\text{(A)} \qquad \int_{t_0}^{t_1} F[\eta(t), \eta'(t)]\, dt = 0,$$

where $\eta'(t) = d\eta/dt$. Denote by $x(s,t)$ his money receipts (payments if negative) per time unit at time t, in consequence of loans contracted at time s. Write $\xi(s,t) = x(s,t)/P(t)$. Then, for every s, $t_0 \leq s \leq t \leq t_1$,

$$\text{(B)} \qquad \int_{s}^{t_1} P(t)\, \xi(s,t) e^{-t \cdot i(s)}\, dt = 0,$$

where $i(u)$ is the interest rate at time u. The individual chooses the functions η, ξ so as to maximize, subject to (A) and (B) and to certain boundary conditions, the expression

(C) $$\int_{t_0}^{t_1}\left[\int_{t_0}^{t}\Phi(\xi+\eta,\ \xi_1,\ \xi_2+\eta')\,ds\right]dt,$$

say, where $\xi_1 = \partial\xi/\partial s$, $\xi_2 = \partial\xi/\partial t$. Denote the optimal functions for the a-th individual by $\xi^{(a)}$, $\eta^{(a)}$. They are seen to depend upon the interest rate i(t); this function of time is common to all individuals and is determined by the condition that, per unit of time, the excess of all receipts over all payments equal the amount of money M(t) created (or destroyed) by the banks:

(D) $$\sum_a \int_{t_0}^{t_1} \xi^{(a)}(t,s)\ P(s)\ ds = M(t).$$

The model is reduced to the more special one of Irving Fisher (The Theory of Interest, New York 1930), (1) by disregarding the distinction, implicit in (B), between long-and short-term loans; (2) by assuming that the opportunity and utility functions corresponding, respectively, to F in (A) and Φ in (B), are independent of the derivatives of η and ξ; (3) by regarding time as a discrete variable; (4) by assuming the price level P constant over time; (5) by regarding banks as ordinary utility-maximizing lenders rather than as creators of money led by purposes of price policy. The general model is solved as a problem in the calculus of variations. by Amoroso