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PRODUCTION RELATIONS IN THE RAILROAD INDUSTRY

by

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My purpose in writing this paper is to outline a program of research. The goal of the project is the estimation of a production function for the railroad industry. I have divided the task into two main parts:

The first part concerns an analysis of the technology of railroad operations. For reasons which will be discussed below, I have found it useful to apply linear models of production in this work.

The second part deals with the specification of a set of production relations, and their estimation. The nature of the information available on rail operations will make necessary the use of statistical methods to estimate the parameters of these relations.

Students of the industry have recognized that transport service is produced by the simultaneous operation of distinct processes. A list of such processes appeared in a recent rail-cost study.

"The basic Transportation services briefly consist of the following:

a. "Running Service (all line-haul or road operations; but excluding train switching)."

* An alternative method of constructing estimates of production relations was presented at the December meeting of The Econometric Society, in a paper by Anne H. Gross. Liss Gross makes use of engineering and technical data to construct the coefficients of the processes in her model. It might be argued that the relations she estimated are not "technological" relations, because most of the entrepreneurial decisions she took as engineering data were really the result of a maximization process in which prices entered. That Liss Gross has observed is a point on the efficient point set. A different set of prices would have induced the operation of different activities and the existence of other facets on the efficient point set.
b. "Switching service (all switching operations whether yard or train switching )

c. "Station Platform service (all platform work, car-load and less-car-load )

d. "Station clerical service (all freight and station operations, excluding only platform work and such train work as dispatching, etc.)

e. "Special Services (cleaning cars, furnishing grain-doors, bedding livestock, etc.)

f. "less car-load pick up and delivery service.

g. "Loss and Damage "

Once we conceive of production as the combination of distinct processes, it would seem fruitful to use a linear model in order to investigate the relations between the input and output variables.

The set of processes which I shall use for analysis will be different from those enumerated above. I shall discuss the following processes:

a) Loading
b) Yard Switching
c) Line haul operation
d) "Yard Train Operation and Train Switching
e) Repair.

A. Loading

The Loading Process deals with the relations between the bulk and tonnage of cargo, the capacity of cargo cars, and the cargo car requirements. The problem facing the firm which operates the loading process is to minimize the cost of transport. If the cost of operating all types of cars is the same,

the problem reduces to that of minimizing the number of cars required to transport the cargo.

Suppose the cargo were a single commodity of uniform density. (By density will be meant the number of tons per cubic foot of cargo. By bulk will be the number of cubic feet per ton of cargo.) Suppose further that cargo cars were of uniform weight and space capacity, such that when fully loaded with this commodity, both capacities were simultaneously exhausted. The problem facing the loading firm would admit a simple solution. Specification of the tonnage of this commodity to be hauled would immediately determine the number of cargo cars required. However, this example only serves to reveal the actual complexity existing in loading operations. The cubic foot displacement per ton of cargo varies from commodity to commodity. Furthermore, weight and space capacity differs between types of cargo cars, and each cargo car must carry a variety of commodities. The problem facing the shipper is to arrange the variety of commodities entering each car in order to minimize the cost of using cargo cars. Of course, each shipper has a restricted range of products to load, so that the variations in cubic feet per ton which may be affected is limited, compared to the possible variation between all commodities. However, this does not change the problem facing the shipper. It does mean that some shippers may waste both weight and space capacity because of the weight and bulk of the units of cargo.

The model of the loading process shows how the shipper would act to minimize cost. In this model, there are two types of cargo and two types of cargo cars. This model could be extended to include more commodities and more types of cargo cars.

The commodities are:

\[ Y_{11} \text{ tons of commodity } C_1 \]
\[ Y_{21} \text{ cubic feet } \quad C_1 \]
\[ \begin{align*}
Y_{31} & \text{ tons of commodity } C_2 \\
Y_{41} & \text{ cubic foot } C_2 \\
I_{11} & \text{ weight capacity of car } A, \text{in tons} \\
I_{21} & \text{ space } C_1, \text{in cubic foot} \\
I_{31} & \text{ weight } C_2, \text{in tons} \\
I_{41} & \text{ space } C_2, \text{in cubic foot} \\
Z_{11} & = \text{ number of } A \text{ cars} \\
Z_{21} & = \text{ number of } B \text{ cars} \\
L & = \text{ number of loaded cars} \\
\end{align*} \]

<table>
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The relations between the commodities and the activity levels are the following.

\[ \begin{align*}
X_j & \text{ are the activity levels.} \\
(i) & \quad Y_{11} = X_2 + X_4 \\
(ii) & \quad Y_{21} = \delta_1 X_3 + \delta_1 X_4 \\
\end{align*} \]
\[ (1.12) \quad \gamma_1 = \gamma_5 + \gamma_6 \]
\[ (1.13) \quad \gamma_1 = \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 \]
\[ (x) \quad 0 = \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 \]
\[ (x) \quad 0 = \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5 \]
\[ (x) \quad 0 = \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5 - \gamma_6 \]

Restrictions (in) \[ (x) \] mean that all cars must be loaded up to the point where neither space nor weight capacity are wasted, or where only one of the capacities is wasted. \[ L_1 \] is an independent output with respect to all the processes concerned. It should be viewed as an output of the leading process. In this model, the \[ L_1 \] are given as the objective. The problem is to find the minimum quantities of cars \( A \) and \( B \) which will carry the required output. The solution to the minimization problem gives the efficient point set in the space of cargo cars. Then the relative costs of operating the cards are given, the quantity of \( A \) and \( B \) with which minimum cost is determined. If we know that it costs the same to operate both cards, the problem becomes much simpler. Then we would try to find cargo combinations which will utilized both cars with the total capacity of each car. The quantities of \( A \) and \( B \) required will depend on the relation between \( \gamma_1 \) and \( \gamma_2 \) of the output, and the relative quantities of \( \gamma_1 \) and \( \gamma_2 \) which we should like to have. If two lines can be drawn, having a positive slope, then the two lines are tangent to each other.
cost of operating A might be different from the cost of operating B, there is always the possibility that it would be profitable to use only one type of car, wasting certain capacities, because of the higher costs of using the other type. For example, suppose that in the loading process, \( \frac{B_a}{B_b} = \frac{1}{3} \); \( \frac{A_a}{A_b} = \frac{1}{2} \). If the cost of using A were three times that of using B, it would clearly pay to use only B, although some tonnage capacity may be wasted.

It is possible that there is no actual substitution of products between cargo cars. One might be tempted to call a commodity \( C_1 \) loaded in Car A a different commodity from \( C_1 \) going in Car B. The model would then be much simplified, showing a number of commodities being loaded into only one type of car. There would be a separate list of goods for each type of car. However, it seems reasonable that the decision to restrict certain commodities to certain types of cars is made only after all possibilities of shifting between cars is considered. A model like the one presented above therefore seems to be the most relevant.

Under present institutional arrangements, only a small fraction of freight is loaded by the railroads themselves. The majority of freight traffic is carload freight, consisting of fully loaded, sealed cars, which are loaded by the consignor. It is nevertheless necessary to deal explicitly with the loading

Suppose \( \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_a \succ \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_b \)

Case (i) \( \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_a \succ \begin{pmatrix} Y_{11} \\ Y_{31} \end{pmatrix} \succ \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_b \). In this case, there are positive values of \( A \) and \( B \), which will carry \( Y_{11} \) and \( Y_{31} \).

and satisfy the condition that neither capacity be wasted.

Case (ii) \( \begin{pmatrix} Y_{11} \\ Y_{31} \end{pmatrix} \succ \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_a \)( \begin{pmatrix} Y_{11} \\ Y_{31} \end{pmatrix} \prec \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}_b \). In this case, there are no positive values of \( A \) and \( B \), which will satisfy the restrictions of no waste of either capacity.
process. The reason for this lies in the nature of the pricing of railroad services. If the roads were to charge the shipper for transporting a loaded car, regardless of its contents, the road would have no interest in the minimizing task described here. But rail charges are in ton-miles, not car-miles. Further the ton-mile rate differs between commodities. If the rates are to bear any relation to cost, the road must know what are the carload car requirements of different commodities.

B. Switching

Loaded Cars is a final output of the Loading Process. It is, however, an intermediate product, from the point of view of the combined operation of all processes. Loaded cars are an input into the next process, that of yard switching. Yard switching is performed by the railroad in almost all cases. It consists of picking up loaded cars from consignor's tracks in the vicinity of main yards. The cars are switched into the yards, spotted, and assembled into trains. Usually there are four yard switching operations performed on each loaded car between the time it leaves the consignor and arrives at the consignee:

one switch to take car from shipper's track to yard
one switch to assemble car in train
one switch to take car out of train at destination
one switch to deliver car to consignee track or to assemble car in way-train.

If the car must be hauled over the tracks of a second road before it reaches its destination, a fifth operation, namely inter-train switching, is performed on the car. Another type of switching, which will be considered later is train switching. This consists of switching cars by the engine of the train; this type of switching takes place when the origin or destination of cars is not
located near yard facilities.

The following commodities enter the switching process:

- $I_{12}$ loaded cars
- $I_{22}$ loaded and switched cars (processed cars)
- $Z_{12}$ labor
- $Z_{22}$ fuel
- $Y_{22}$ number of switches
- $I_{32}$ tractive effort in pounds from locomotive A
- $Z_{32}$ switching locomotive A
- $Z_{52}$ switching locomotive B
- $I_{42}$ tractive effort provided by locomotive B.

The switching process would have the following activities:

<table>
<thead>
<tr>
<th></th>
<th>Provision of Locomotive A</th>
<th>Provision of Locomotive B</th>
<th>Switching with Locomotive A</th>
<th>Switching with Locomotive B</th>
<th>Disposal of tractive effort</th>
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<tr>
<td>$I_{22}$</td>
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<td>$I_{32}$</td>
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</table>

The relations between the commodities and the activity levels are the following:

(i) $I_{12} = -x_3 - x_4$
(ii) $I_{22} = x_3 + x_4$
\( i = 0 = \alpha_3 x_1 + \alpha_1 x_3 - x_5 \)
\( iv = 0 = \alpha_4 x_2 + \alpha_2 x_4 - x_6 \)
\( v = \beta_1 x_3 - \beta_2 x_4 \)
\( vi = \gamma_1 x_3 - \gamma_2 x_4 \)
\( vii = \delta_1 x_3 - \delta_2 x_4 \)
\( viii = \theta_2 x_2 \)
\( ix = \eta_2 x_3 + \eta_2 x_4 \)

The number of loaded cars to be switched and the number of switches per car is given to the firm which operates the switching process. The problem is to find the minimum number of locomotives A and B which will do the required switching. When the relative costs of operating A and B are known, the number of A and B which will minimize cost are determined. Note that it was necessary to specify as final outputs of this process both \( y_{22} \), the number of switches performed on each loaded car, and \( l_{22} \), the number of switched cars. It is possible that there would be more activities showing switching with either locomotive type. These activities would show how the required tractive effort, labor and fuel, varied with the number of switches per loaded car. It is conceivable that tractive effort depends only on the number of cars to be switched.

I have provided an activity which makes available tractive effort, and another activity to dispose of unused tractive effort. This takes into account the provision of tractive effort in units larger than needed to perform one switch on one car. The relation between number of switches performed on each car, the number of cars to be switched, and the number of locomotives required seems to be non-linear, in the range where the capacity of a switch engine is not fully utilized.

It might be as well to deal directly with the non-linear relation existing between the two outputs and the other inputs, e.g.,

\[ y_{22} = f (a_{12}, z_{22}, p_{22}, z_{22}, l_{12}) \]

We would still be able to analyze the problem of combining distinct processes,
though the relations within the process might be non-linear.

C. **Line Haul Operation**

The third process which I shall discuss is Line Haul Operation. The Line Haul Process consists of taking switched, loaded cars as an input, and producing two outputs, switched, loaded, transported cars, and car-miles. The switched, loaded transported cars then form an input into the other processes engaged in switching the car to its destination.

Other studies of rail production have used as an overall measure of output the number of ton-miles or car-miles produced. This measure is convenient because it is an average of the haul of different tons of different commodities over different distances. It is inconvenient as an overall measure of output because it masks those input elements which vary with the number of cars loaded and switched, and those which vary with the length of haul. The reason for this lies in the fact that 100 car-miles may mean 100 cars transported one mile, or one car transported 100 miles. This difficulty has been avoided in the Line Haul Process, because I have normalized on the number of switched, loaded cars expressed received, i.e., the output car-miles is/for one car. To double the number of car-miles produced with one car it is necessary to operate some other activity.

There is again in this process the same element of non-linearity which appeared in switching. This arises from the provision of tractive effort in indivisible units; it has been handled in the same manner as in the Switching Process.

Another output of the Line Haul Activity which must be considered is the addition to, or subtraction from balance in the movement of loaded cars from West to East and from East to West. In normal years in this country, there is a greater flow of loaded traffic from West to East than return. This raises
a problem of routing empty cars back to the west in the shortest time. A network of optimal routing would minimize the cost of returning empty cars.∗

Under present institutional arrangements, empty cars are routed back to owner roads along the routes by which they were originally transported. In this way, each road pays the cost of its own contribution to the lack of balance. Although the total cost is larger than it would be under an optimal routing plan, it is nevertheless true that the firm bears the consequences of its actions. Under an optimal routing plan, certain roads might pay a disproportionate share of the cost of returning empties. It would then be necessary to work out compensation arrangements between different management, so that those roads which contributed to lack of balance would pay the cost.

Each firm regards the lack of balance as something beyond its control. It then adds to the cost of transporting an extra loaded car from west to east, the cost of sending it back empty. It subtracts from the cost of sending an extra loaded car from east to west the cost of sending one less empty car in that direction. This will be taken into account in the Transport Process by regarding empty car-miles as a specific output variable.

A model of the transport process would include the following commodities:

\[ Y_{13} \text{ loaded car-miles} \]
\[ Y_{23} \text{ empty car-miles} \]
\[ L_{23} \text{ loaded, switched cars} \]
\[ Y_{33} \text{ loaded, switched, transported cars} \]
\[ L_{13} \text{ tractive effort in pounds} \]

∗ The problem of routing empty cars can be handled in the same fashion as the problem of routing empty tramp shipping. An optimal route for shipping was discussed in T.C. Koopmans, "Optimum Utilization of a Transport System," LFC 101.
<table>
<thead>
<tr>
<th>Provision of tractive effort</th>
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<th>Test to West 2</th>
<th>Last to West 4</th>
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The relations between the commodities and the activity levels are the following:

(i) \( y_{13} = \alpha_1 x_2 + \alpha_2 x_3 + \alpha_3 x_4 + \alpha_4 x_5 \)
(ii) \( y_{23} = \beta_1 x_2 + \beta_2 x_3 + \beta_3 x_4 + \beta_4 x_5 \)
(iii) \( I_{23} = -x_2 - x_3 - x_4 - x_5 \)
(iv) \( y_{33} = x_2 + x_3 + x_4 + x_5 \)
(v) \( 0 = I_{13} = \delta_0 x_1 - \delta_1 x_2 - \delta_2 x_3 - \delta_3 x_4 - \delta_4 x_5 - x_6 \)
(vi) \( z_{13} = -x_1 x_2 - x_3 - x_4 - x_5 \)
(vii) \( z_{23} = -x_1 x_2 - x_3 - x_4 - x_5 \)
(viii) \( z_{33} = -x_1 \)

For the sake of simplicity, I have included only one type of locomotive furnishing tractive effort. The problems raised by having more than one kind of
locomotive are illustrated in the switching process. More important here is the relation between the two outputs, loaded car-miles; and loaded, switched transported cars; and the other input variables. As the activities are constructed, twice the quantity of tractive effort is required to haul twice the number of cars — whether in fact the work required changes in direct proportion with cars, I don’t know. It is not specified how or whether tractive effort changes with changes in mileage. It is possible that only fuel and labor requirements vary with mileage, while tractive effort requirements depend only on the weight of the haul. If we were to express this as a non-linear relation we might have the following:

\[ Y_{13} = f (Y_{23}, L_{33}, Z_{13}, Z_{23}, Z_{33}) \]

It is never possible to observe the tractive effort actually used — we know only the quantity made available by the number of locomotives in use.

D. Other Processes

Two other processes which might be investigated in detail will only be mentioned. The three processes analyzed so far will serve to bring out the relevant points.

Repair constitutes a fourth process. The inputs used by the repair process would be labor, materials, and machinery. As an input intermediate to all processes, the repair process would receive damaged equipment produced by the first three processes. As an output intermediate to all processes, repair would produce renovated equipment.

A more interesting process is the operation of way trains and of train switching. In many localities, the volume of freight traffic does not justify the use of yard switching facilities. Delivery and pick-up of loaded cars is made in “way” or local freight trains. The loaded cars are then switched by
the locomotive of the train. Train switching represents about 13 percent of all switching activity. It should be noted that this is a case which the traditional linear model cannot handle. If the use of yard facilities is to be preferred at one level of activity, it is to be preferred at all levels, under a linear set-up. This is due to the postulate that all commodities are divisible. However, the main unit of input in switching is the locomotive, not the tractive effort it provides. (We of course assume that when a solution specifies 1.5 locomotives, 2 must be used.) We must recognize that "by Train operation and Train Switching are a fifth process which is operated when too great a proportion of yard locomotive capacity would be wasted by the provision of yard facilities.

E. Combination of Processes

Some idea of how the processes are combined is given if we view the progress of a car from the time it is loaded to the time it arrives at its destination. This will be shown by an enumeration of the processes which are operated. First take a car which is loaded at and destined for localities near large yards. The processes operated are:

(a) loading
(b) yard switching
(c) transportation
(d) yard switching.

If the car is loaded at a point where there are no yard facilities and destined for a point without yard facilities, the processes combined are:

(a) loading
(b) train switching
(c) way train operation
(d) yard switching
(e) line-haul transport
(f) yard switching
(g) way train operation
(h) train switching.
Let us now examine the three processes combined. There are a number of commodities which are intermediate to all the processes. They are

1) Loaded Cars \[ L_{11} + L_{12} = 0 \]
2) Loaded and Switched Cars \[ L_{22} + L_{23} = 0 \]

The final outputs are

(1) \( Y_{11} \) Tons of different commodities
(2) \( Y_{21} \) Cubic Feet of different commodities
(3) \( Y_{22} \) number of switches
(4) \( Y_{13} \) loaded car-miles
(5) \( Y_{23} \) empty car-miles
(6) \( Y_{33} \) loaded, switched, transported cars.

If we look at the whole technology matrix, there are only a few commodities entering more than one process. These are labor, fuel, Loaded Cars, and Loaded, Switched Cars. If we specify the types of labor used in Processes 3 and 6 to be different commodities, and assume that fuel is not a limiting factor, then the only variables relating these activities are Loaded Cars, and Loaded, Switched Cars. These two commodities signify that the output of one Process is the input of the next.

The transformation function will be a polyhedral cone in 5 dimensions, showing, for certain restrictions on the input variables, the maximum value of any one input, for given values of the other four. I believe that this transformation function can be partitioned into three functions, because the only relations between the Processes are the Intermediate Products \( L_{11}, L_{12}, L_{22}, L_{23} \).

(1) \[ Y_{11} = f_1 (Z_1, Y_{21}) \]
(2) \[ Y_{22} = f_2 (Z_j, L_{22}) \] \( i \neq j \neq k \)
(3) \[ Y_{13} = f_3 (Z_k, Y_{23}, Y_{33}) \]

It is in this form that it may be possible to use the production relations for
purposes of statistical estimation. If we were to estimate the three relations as shown, we would get some idea of the changes in inputs corresponding to variation in specific output variables.

This problem was recognized by Klein in a study where he attempted to estimate a single production function for the railroad industry.* The function Klein estimated contained as output, \( X \) = ton-miles of a group of commodities. The inputs were:

\[
\begin{align*}
v_1 & \quad \text{quantity of labor, in man-hours} \\
v_2 & \quad \text{quantity of fuel, in equated tons} \\
v_3 & \quad \text{use of capital equipment, measured in train-miles.}
\end{align*}
\]

He attempted to account for the lack of homogeneity of the measure, ton-miles, by introducing further variables into the production function. These were:

\[
\begin{align*}
z_1 & \quad \text{average length of the haul} \\
z_2 & \quad \text{percentage of freight in the form of products of mines.}
\end{align*}
\]

Alternative to using \( z_2 \), he tried:

\[
z_3 \quad \text{average weight of cargo per loaded freight car.}
\]

This approach recognizes that inputs may vary with the different dimensions of the output variable, but it doesn't go far enough in distinguishing outputs.

\[F. \text{ Available Data}\]

The observations on input and output variables listed below show what might be done in estimating a set of relations, as opposed to one relation.

The data are yearly observations on individual Class I Railroads.

---

I. Loading:

Output

a. tons of revenue freight, originated, and number of carloads originating on firm's line by commodities within the following commodity groups:

(i) Products of Agriculture

(ii) Animals and Products

(iii) Products of Mines

(iv) Products of Forests

(v) Manufactures and Miscellaneous

(vi) Forwarder Traffic

Input

a. total number of cars loaded (originating)

b. total number of cars available for loading

II. Switching

Output

a. a direct measure of number of switches per car is not available

b. number of switched cars - this equals the number of cars received plus cars originated, plus cars terminated plus cars delivered to connecting carriers.

Input

a. Yard Switching Locomotive Hours - Freight

   (i) % coal burning steam

   (ii) % oil burning steam

   (iii) % Diesel locomotives

b. Yard and Train Switching Locomotive miles - per 100 loaded freight cars.

c. Labor - men and man-hours of all employees engaged in yard operations.

c. Repairs in Dollars
   (i) Yard Switching Tracks
   (ii) Yard - Steam and other Locomotives

d. Stocks of Equipment - Switching Locomotives
   (i) Diesel Electric
   (ii) Steam
   (iii) Electric

III. Line Haul Transport - Freight

Output

a. Car-Miles
   (i) Loaded
   (ii) Empty
   (iii) % eastbound of total loaded freight car-miles

b. Number of loaded cars transported - this may be found by totalling the number of cars received and originated, or by a more roundabout method:
   (i) take an estimate of tons per car by dividing ton-miles of revenue freight by car-miles.
   (ii) divide this estimate of the number of tons per car by the total number of tons of revenue freight hauled.

Input

a. Number of Freight Locomotives
   (i) Steam
   (ii) Diesel Electric
   (iii) Electric
b. Gross ton-miles of locomotives and tender

c. Freight train locomotive-miles broken down into:
   (i) Steam
   (ii) Diesel-Electric
   (iii) Electric
   (iv) Total

d. Labor - men and man-hours; all employees engaged in Line Haul Freight

e. Equated net tons of fuel and power consumed Road Freight

f. Repairs in dollars
   (i) Freight train cars
   (ii) Locomotives
   (iii) Running Tracks

IV. The following data is available on way and local train operation:

a. Train switching locomotive miles - Freight

b. % train switching hours at way stations of total train hours,

c. Men and man-hours of transportation employees engaged in local and way train operation.