COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 277

# SOME RECENT LITERATURE ON EXPECTATIONS IN ECONOMICS†

by Wm. C. Hood

February 1, 1950

## OUTLINE

INTRODUCTION

I. SHACKLE'S CRITICISM OF RECENT LITERATURE

II. SHACKLE'S HYPOTHESIS

- Summary
- The Potential Surprise Function*
- The Stimulation Function*
- The Determination of Focus Values*
- The Utility Function and the Gambler Indifference Map*
- Suggested Modifications of the Hypothesis
  - Change 1a
  - Change 1b
  - Change 2

III. SHACKLE'S TREATMENT OF SELECTED PROBLEMS COMPARED WITH THAT OF SOME OTHER AUTHORS

- Introduction
- Shackle on the Friedman-Savage Problem
  - The Problem
  - The F. - S. Hypothesis*
  - The "Shackle" Hypothesis.
- Shackle's Approach and the Principle of Minimaxing Regret
  - Introduction
  - Shackle's Cases and Marschak's Classification of Degrees of Information
  - Shackle's Principle of Choice and the Principle of Minimaxing Regret.

IV. APPRAISAL.

---

† The writer gratefully acknowledges advice received in discussions with Professors Brownlee, Friedman and Marschak, and regrets that blame cannot be assigned to them for not correcting his mistakes.

* Material in these sections is not presented below; reference is made to original work of Shackle, Friedman and Savage. It will be presented orally.

## INTRODUCTION

In the present paper an attempt is made to relate Shackle's recent book Expectation in Economics[1] to the literature of its subject. To this end Shackle's appraisal of the gap in the literature which he wishes to fill is studied first; an outline of the concepts and constructions offered to fill this gap follows; then a comparison of the treatment of certain specific problems by Shackle and other selected authors is made and in conclusion the present writer's appraisal of the volume is stated.

## I. Shackle's Criticism of Recent Literature

The main and only substantial criticism of existing literature is of the way in which the probability distribution attaching a degree of belief to each possible outcome of a given action is used in analyzing an individual's choice of actions in a particular class of situations.

There is no fundamental objection to the use of probability distributions to describe the set of degrees of belief associated by an individual with each conceivable outcome of an action, though there is (what I consider) a superficial objection to such use and in fact, Shackle does not use the probability distribution in this descriptive role. The fundamental objection is to the particular analytical use of the distribution made in the analysis of a given group of problems. The objection is to the characterization of such a distribution by its mean, and variance, and other parameters, the calculation of which involves combining by some algebraic formula each pair of values comprising the entire distribution. The assumption that such parameters are in fact the characteristics of the distribution utilized by the individual in weighing the merits of the action whose possible outcomes are described by the distribution is the main bone of contention.

Let it be emphasized however, that Shackle does not take issue with this

---

1. Cambridge, 1949.

particular analytical use of the distribution in all problems, but only in that class of problems in which the individual is contemplating a decision which he can only make once or twice in his lifetime. Examples of such decisions are decisions to make large investments, to enter a given profession, or to act under circumstances which will never be reproduced. It was the particular use typically made of probability distributions in the analysis of such decisions to which Shackle objected.

The uniqueness of a decision, or the infrequent necessity or opportunity for a decision implies:

a. that the decision-maker is interested only in the result of one action, and not a repetition of actions; and/or

b. that no opportunity in fact exists for repeated actions under similar conditions.

Thus when a decision is unique, the conditions are lacking under which application of actuarial principles would be permissible.

It seemed to Shackle therefore, that there was a gap in the literature on the treatment of the problem of unique decisions under uncertainty and it is this gap which he attempted to fill.

In what follows we shall be concerned with a discussion of actions which will lead to the gain or loss of money income.

## II. Shackle's Hypothesis

### Summary

Our procedure here will be to outline Shackle's approach quite uncritically first, and then to suggest two modifications which might be made to his model to simplify it and yet retain its essential spirit.

Shackle's first assumption as to the motivation of an individual contemplating a once-for-all decision is not different from that of most writers on this problem.

"The real incentive for embarking on some given venture, whose objective results will not develop and their character become known until some date in the future, is the immediate mental experience which the decision to embark on this course will give us, namely, the enjoyment by anticipation of a high level of success." (pp. 15-16)

It is the anticipation of gain from a venture which stimulates an individual to decide upon it. However, enjoyment of an anticipated gain is tempered (a) by the extent to which the individual considers such an outcome of the venture as unlikely, and (b) by the danger that losses of varying degrees of likelihood may occur.

In considering a given venture, an individual is confronted with a set of possible gains and losses each of which is considered to be more or less likely, each of which differs in its capacities to raise his hopes or his fears for the project. At this point Shackle introduces a postulate which is the foundation of his entire theory; without accepting it, one cannot be sympathetic with the spirit of the hypothesis. Tis postulated that:

"The power of mutually exclusive hypotheses of success to afford enjoyment by imagination is not additive, and that therefore the power of the entire set of hypotheses of success associated with any course of action to afford enjoyment by imagination is simply that of one alone amongst these hypotheses, whichever has this power in higher degree than any of the others; and similarly that the power of the entire set of hypotheses of misfortune associated with this course of action to cause distress by imagination is simply that of the most powerful amongst these hypotheses." (p. 17)

In other words, it is contended that an individual is stimulated by promise of gain from a venture, and deterred by promise of loss, but in order to economize on mental effort, or to reach some sort of equilibrium in his mental debate on the merits of a particular venture, he is thought to concentrate his attention on that particular promise of gain which most stimulates him and on that loss

which most distresses him. It is this gain and loss called the focus gain and the focus loss respectively which epitomize the merits of the particular venture for him and which are crucial in comparing the merits of this proposed venture with the merits of all others. If fact Shackle would argue that in these problems involving once-for-all decisions, it is rational to select the gains which maximize stimulation and the losses which maximize distress for exclusive consideration.

In Shackle's presentation of his theory there are three principal functions. The first is called the potential surprise function, the second, the stimulation function, and the third the utility function, the contours of which are used in the book under the title gambler indifference map. Let us consider the nature and use of each of these in turn.

The potential surprise function $p = p(x)$

The stimulation function $s = s(x, p$

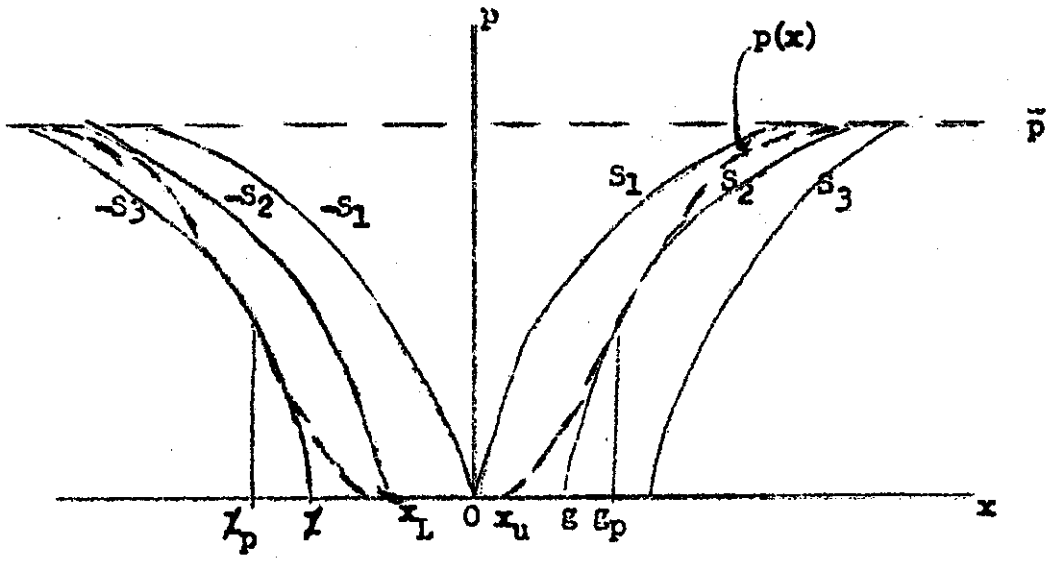
The determination of focus values.

The Utility function $u = u(g, \chi)$ and the gambler indifference map.

Suggested modifications of the hypothesis.

This model is on the whole rather complicated and unorthodox; it would be helpful if some simplification and reversions to orthodoxy could be introduced without abandoning the fundamental postulate concerning the role of focus values. Two such changes will be suggested here.

Change 1a. It is claimed above that the rejection of the use of the probability distribution as a means of describing degrees of belief in the outcomes of a project was not crucial to the argument. I would recommend this reversion to orthodoxy, for so far as I can see it would require merely a restatement of the argument without any alteration of basic content and import. Since the probability distribution gives degrees of belief, rather than disbelief, in outcomes of a venture it would be necessary to redefine the stimulation function in terms of probabilities rather than potential surprises, and this would mean that




This figure to be discussed orally.

stimulation would vary directly with both arguments of the function. But this is no fundamental change; it is still entirely possible to derive focus values in a quite analogous way and to use these to epitomize the venture. Moreover there would be the advantage that the laws of the probability calculus relating to joint, conditional, and marginal probabilities could be used directly in problems requiring this, thus avaoiding the need for using another calculus, the calculus of potential surprise, as Shackle has done.

Change 1b. If one wished to retain the diagrammatic treatment that Shackle uses, one could express degrees of disbelief according to the axion of the probability calculus which would require that the area under the function be unity. Again analogues of the laws of probability would then be applicable.

Change 2. It would seem that Shackle's requirements could be satisfied if the stimulation function itself were called the utility function and if it were postulated that the utility attached to a given project were the utility (stimulation) associated with its focus gain less the disutility (distress) attached to its focus loss. It would still of course be entirely possible to draw indifference curves showing combinations of standardized focus gains and losses possessing equal degrees of utility (or disutility) and it might be useful for some purposes to do this. Shackle, however, nowhere suggests that this is his procedure. Stimulation seems to be a category apart from utility (he uses the word "attractiveness" rather then "utility" — the word "utility" never appears). It might be remarked that the identification of stimulation with utility would obviate the necessity for calculating the standardized focus gains and losses except for drawing a two dimensional indifference map.

## III. Shackle's Treatment of Selected Problems Compared With That of Some Other Authors

### Introduction

Shackle's own applications of his structure cover a wide variety of topics. Considerable space is devoted to the theory of the single investment by a firm.

One would probably be intrigued by the theory of clarification of expectations evolved in the book and the prediction on the basis of it that the downturn of investment and employment would be more rapid after a boom than their upturn after a slump. There is an application of the model to the problem of the speculator who, faced with an uncertain situation, is to decide what assets to hold for capital gain. The treatment is ingenious, a bit strained in places, but worth study. Also, a theory of the bargaining process is worked out, but in my view, the author claims too much for this theory. Other important matters receive scantier attention in this slender book; for example, Hart's plea for models which permit of flexibility of plans does not go unheeded and Shackle claims to have such a model. (On this particular point see p. 118)

However, we are not able to go into these matters thoroughly here, and since they are easily read I propose to: (1) apply Shackle's model to a problem recently treated in the literature, and (2) apply to Shackle's model a criterion of rational behavior that has received an increasing amount of attention of late. The problem to which I want to apply Shackle's model is that treated by Friedman and Savage in their article entitles The Utility Analysis of Choices Involving Risk (J. P. E. Aug. 1948, pp. 279-304). The criterion of rational behavior which I want to use within (part at least) of Shackle's framework is that of minimizing maximum regret, first discussed in economic literature, I believe, by Prof. Marschak in Role of Liquidity Under Complete and Incomplete Information (AER, May 1949, pp. 182-195).

Shackle on the Friedman-Savage Problem.

The Problem: Friedman and Savage held that the purpose of their paper was "to suggest that an important class of reactions of individuals to risk can be rationalized by a rather simple extension of orthodox utility analysis" (op. cit. p. 279) This extension of the utility analysis resulted in the formulation of an operational hypothesis to explain the following observed facts:

"a. low-income consumer units buy or are willing to buy insurance;

b. low-income consumer units buy or are willing to buy lottery tickets;

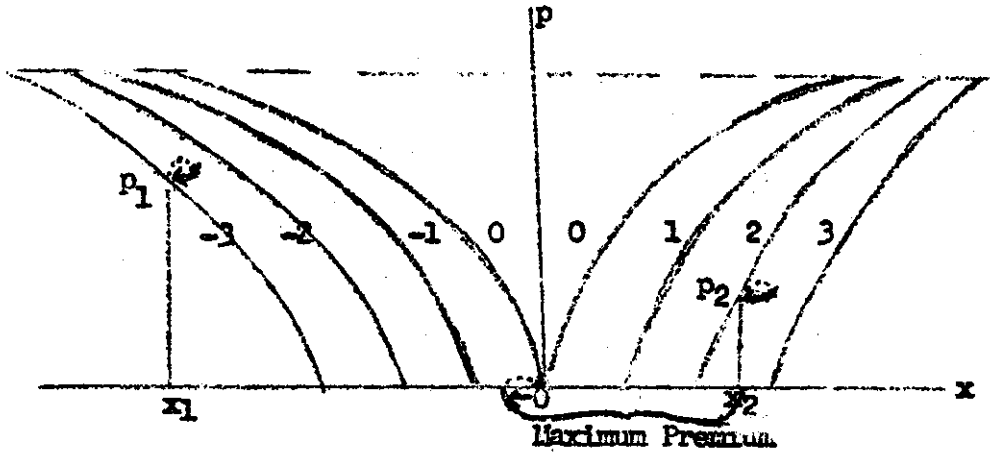
c. many consumer units buy or are willing to buy both insurance and lottery tickets

d. lotteries typically have more than one prize" (pp. 303-4).

Our purpose here is to show that Shackle's model will also account for the first three facts — I haven't tested it for the fourth. Whereas F. - S. would claim that consumer units behave as if their set of postulates (including maximization of expected utility) were obeyed, I would claim that Shackle is able to argue that consumer units behave as if his set of postulates were obeyed.

The "Shackle" Hypothesis: To formulate this problem for Shackle we may proceed as follows. First let us adopt the simplification suggested above of regarding the stimulation function as a utility function. Since we will be concerned with two situations in one of which certainty is given up in favor of uncertainty and in the other of which the reverse is true, it will be convenient (though not imperative) to regard the utility of the certain venture in each case as zero. It will also be at the origin of the gain-loss scale and have zero potential surprise. The gain of the uncertain venture to be compared with it we may define to be the difference between the higher of the two uncertain outcomes and the certain outcome; the loss may be defined as the certain outcome less the lower of the two uncertain outcomes.

The insurance case: In the insurance case, a certain venture is preferred to an uncertain one, the uncertain venture being that of not buying insurance.



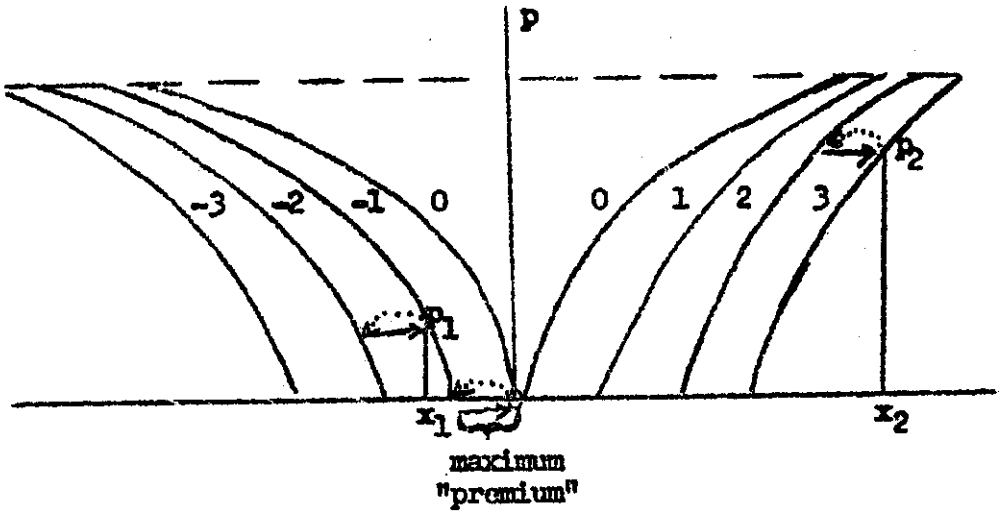
Suppose the individual is comparing a certain income with an income, his present income, which is $x_2$ higher and a possible lower income which is $x_1$ lower. Let the potential surprise associated with each be $p_2x_2$ and $p_1x_1$ respectively. If his stimulation function is of the form represented by the contours in the above diagram, then, since the disutility of the loss exceeds the utility of the gain, the net utility of the uncertain venture is less than that of buying insurance and paying $x_2$ as a premium. We may note that for the gain and loss indicated, the certain venture is the more preferred the lower is the potential surprise associated with the loss and the higher is the potential surprise associated with the gain.

The question may legitimately be asked — what is the maximum premium the individual would pay for insurance against the risks here indicated. This can conveniently be shown on the diagram provided we assume with Shackle that the rate of change of utility (stimulation) with respect to gain or loss is equal to a given constant irrespective of the value of p. We must ask the question: what certain income would have utility equal to the utility of the uncertain venture, and what would be the difference between this income and the maximum gain of the uncertain venture here interpreted as the individual's present income?

This can be interpreted by imagining the contours to be shifted uniformly to the left along the x axis, (or the x-axis to be shifted to the right) until $p_2$ falls on a contour which is equal to, but opposite in sign to that on which $p_1$ falls. Equivalently we may imagine the origin shifted to the left until situated on a contour whose value is equal to 1/2 the difference between the value of the contour on which $p_2$ was situated and the value of the contour on which $p_1$ was situated in the original situation. The difference between $x_2$ and this relocated origin is the maximum premium the individual would be prepared to pay for insurance against the gain or loss assumed in this example.

The Gambling Case: In the gambling case an uncertain venture is preferred to a certain one, the certain venture being that of not gambling.

Suppose the individual is comparing a present and certain income with income $\Delta x_2$ higher and income $\Delta x_1$ lower, these representing gain and loss respectively to which a particular gamble might lead.




Let the potential surprise associated with each be $p_2x_2$ and $p_1x_1$ respectively. If the stimulation function is of the form represented by the contours in the above diagram, then since the utility of the gain exceeds the utility of the loss, the net utility of the uncertain venture is greater than that of the certain income and hence the individual will prefer the gamble. We may note that for the gain and loss indicated, the preference for the gamble will be the greater the greater is the surprise associated with the loss, and the less is the surprise associated with the gain. Again we may ask: how much is the individual willing to pay for the privilege of gambling in this case? If we impose the same restrictions on the utility function as before we can again conveniently answer this question on the graph. We must formulate the matter this way: When would the utility of the gamble just equal the utility of certain income, and what would be the difference between the gain in the case illustrated and the case now supposed? In this case we suppose the contours to be shifted uniformly to the <u>right</u> (or the x-axis to the left) until $p_2$ falls on a contour of value equal to

but opposite in sign to the value of the contour on which $p_1$ falls. Equivalently we may imagine a translation of the origin to the left by such an amount as would place it on a contour whose absolutevalue is equal to 1/2 of the difference between the value of the contour on which $p_2$ was situated and the value of the contour on which $p_1$ was situated in the original situation. The distance (measured in $) over which the origin is translated is the maximum amount the individual would pay for this gamble.

The above represents such an interpretation of the Shackle model as enables it to yield an hypothesis concerning individual behavior which is consistent with the facts that consumers will buy both insurance and lottery tickets. There are certain features of the F. - S. hypothesis not included in the above formulation as for example distinctions between low and high income consumer units. However, I am not concerned to press for a substitution of this hypothesis for that of F. - S. I am only concerned to suggest that it may be regarded as an alternative. It does not involve in particular, the assumption that an individual maximizes the expected value of utility in the sense in which Bernoulli, F. and S. use this term.

Shackle's Approach and the Principle of Minimaxing Regret.

Introduction: Recently there has appeared in statistical and economic literature a principle of rational choice among a set of contemplated ventures whereby the individual is supposed to minimize the maximum regret which he expects could result from his choice whatever the future situation might turn out to be.*

---

* Cf. J. Marschak: "Role of Liquidity..." op. cit. References to statistical writings by Wald, Arrow, Blackwell, and Girshick will be found in this article as well as to Savage who orally proposed the principle of minimaxing regret rather than maximining gain in the case of games played with nature.

It is proposed: (a) to consider Marschak's classification of degrees of information about future events and to relate the cases treated by Shackle to this classification; and (b) to suggest a definition of "regret" for Shackle's model and apply this so as to show that Shackle's principle of choosing that action which maximizes the utility of the focus outcomes of a venture does not in general lead to the same choice of action as would be indicated by the principle of minimaxing regret thus defined.

Shackle's cases and Marschak's classification of degrees of information. Marschak has distinguished (op. cit. p. 183) four degrees of information which an individual may have about the future of — in the type of case we are considering — about the outcome of a venture. Let us denote the probability distribution of outcomes by pr(x). The cases are:

1. The individual thinks he knows pr(x) and every element of pr(x) is 0 or 1. — certainty
2. The individual thinks he knows pr(x).

} complete information

3. The individual does not think he knows pr(x) but has data on which to estimate pr(x).
4. The individual does not think he knows pr(x) and has no data. — ignorance

} incomplete information

Underlying the Marschak classification is a notion which is foreign to Shackle's view of the world. It is the following: in any given state of nature some or all events are random, and the state of nature determines the probability distribution of random events, rather than the particular event itself. In the cases of complete information, the individual is thought of as knowing the state

of nature, that is the probability distribution of random events, and in the case of incomplete information he does not think he knows the state of nature but rather, conceives several states of nature to each of which he may or may not be able, on the basis of data available to him, to assign a degree of probability.

As I interpret Shackle the state of nature is viewed as giving rise to a perfectly definite result -- if the result is different the state of nature is different. I would guess that his view is completely deterministic in the sense that every cause has its unique effect and vice versa. The individual, in expressing his lack of surprise at the possibility of a given outcome of a venture, is expressing his belief that some unique state of nature will exist in the future and give rise to this unique outcome.

Hence when we interpret this classification for Shackle we associate a state of nature with a given result and not with a given distribution of results.

Interpreting Shackle this way it is apparent that he has considered no cases of complete information, i.e. cases in which the individual thinks he knows what the state of nature will be, for in none of Shackle's cases is the individual presumed to think he knows precisely what the exact outcome of a venture will be. There is no real analogue of the ignorance case in the Shackle model, that is there is no consideration of a case in which the individual not only does not know what the states of nature will be but is not able to assign surprise to them. Thus Shackle is really concerned with the analogue of Marschak's third case in which the individual does not know what the state of nature will be but is able to form an estimate of its likelihood and hence to assign to it a degree of potential surprise. Like Marschak, Shackle recognizes that information which it is thought may become available to the individual will influence his decision and he gives some attention to cases in which this is a factor.

Shackle's Principle of Choice and the Principle of Minimaxing Regret

If the degree of information is that of Marschak's case two, Marschak proposes as a rule of rational behavior the choice of action which maximizes expected utility or income, but I believe that it is a general proposition, that this leads to the same optimum action as the minimaxing regret principle when regret is defined appropriately. In cases three and four, minimaxing (expected) regret is proposed as the rational principle of choice; in general this involves minimaxing a utility function.

Shackle has proposed for his analogue of case three, the maximization of the utility of focus outcomes. It will be convenient to illustrate his analysis in a slightly different way which will facilitate development of and comparison with the use of the minimaxing regret principle. It will be noted that again we "force" Shackle to regard his stimulation function as a utility function. Let us consider three alternative ventures which the individual may launch, and let us suppose that four possible states of nature are anticipated each of which would lead to a unique outcome for any one of the ventures. We may construct a table of the utilities and disutilities attached by the individual to each outcome of each venture for each anticipated state of nature, taking account of course of the potential surprise associated by the individual with each state of nature. From this we may compute the utility of the focus loss — $u(\gamma_p)$ — and the utility of the focus gain — $u(g_p)$ — and the utility — u — of the venture. Denote states of nature by $N_i$ and ventures by I, II, and III.

| | $N_1$ | $N_2$ | $N_3$ | $N_4$ | $u(\gamma_p)$ | $u(g_p)$ | u | |
|---|---|---|---|---|---|---|---|---|
| I | -10 | -2 | 12 | 16 | -10 | 16 | 6 | |
| II | -20 | -18 | -2 | 10 | 20 | 10 | -10 | |
| III | -6 | -5 | 4 | 13 | -6 | 13 | 7 | ⟸ |

Venture III would be selected according to Shackle's principle.

Now, define <u>regret</u> associated with a given N and a given venture as the difference between the maximum utility expected for that N with any venture, and the expected utility associated with that N and the venture in question. The regret matrix then becomes the following:

| | $N_1$ | $N_2$ | $N_3$ | $N_4$ | Row Maximum | |
|---|---|---|---|---|---|---|
| I | 4 | 0 | 0 | 0 | 4 | <== |
| II | 14 | 16 | 14 | 6 | 16 | |
| III | 0 | 3 | 8 | 3 | 8 | |

According to the principle of minimizing maximum regret venture I would be chosen. This one example suffices to show that in general these two principles of behavior do not lead to the same choice of action. It may be noted that if the utility under $N_4$ for venture I had been 18 rather than 16 both principles would have led to the same result.

There is not opportunity here to examine this question further. It is known that certain arguments can be brought to bear against the minimaxing regret principle. Again, the object at present is not to decide finally between the two principles but to study some of the implications of each.

## IV. Appraisal

As a method of approaching the analysis of individual decision-making under uncertainty, Shackle's book appears to the present writer to have one main defect and one principle merit. The objection to the use of a probability distribution to describe the individual's view of an uncertain situation seems ill-founded. The concept of focus values as opposed to mathematical expectations and variances is in my view a contribution to the analysis of decisions made once-for-all.

Shackle's rejection of the notion of a probability distribution is perhaps most emphatic when he condemns those who think of an individual as conceiving a probability distribution of probability distributions. To Shackle this represents a multiplication of nonsense. To him it implies that an individual summarizes a situation in his mind, and then, not being sure that his summary is correct, prepares a set of such summaries, and then a master summary of the summaries. Shackle asks:

> "What purpose is served by building up this elaborate description of the uncertainty situation (the summary not the summary of the summaries - II) which exists in an individual's mind if, after all, it fails to describe his uncertainty in exact and unequivocal terms and is declared to be itself, not a certainly correct, but a possibly incorrect description? If it merely describes his state of mind before the event, how will knowledge of the actual event tell him whether that description was correct or not?
>
> "Let us ask the same question of our own approach: What actual ex post, outcome of a single venture will show the answer, which the enterpriser gave to himself ex ante to the question 'What will the outcome be?' to have been correct, and what will show it to have been in some degree a misjudgement? It is fully justified if the actual outcome falls inside his inner range, and it is wrong in increasing degree according as the potential surprise he attached ex ante to what has ex post to be the truth was higher." (p. 116)

To this argument several replies must be made:

1. In the first place, he does not appear to recognize that a state of nature may be conceived as a probability distribution and hence anticipations

of states of nature be conceived as a distribution of such distributions.

2. One might argue that on his view of nature one could declare a probability distribution to have been correct if the actual outcome of a policy lay at the mode of the distribution, though this would not be sufficient for the view of nature suggested in 1. above.

3. How could Shackle judge the "correctness" of potential surprise functions related to ventures which were rejected and whose outcomes will never be known?

4. Since Shackle's focus outcomes do not ordinarily fall in the inner range of the p(x) function, he is arguing that an individual bases his decision on an outcome (or outcomes) which if it (they) occurred would show his p(x) function to have been "incorrect".

5. Finally, Shackle himself makes use of his own analogue of a distribution of distributions, namely, a potential surprise function of potential surprise functions in his analysis of the problem of clarification of expectations.

In short, I do not believe Shackle should have attempted to throw out the probability distribution as the method whereby the individual describes uncertain outcomes.

This does not mean however that I have no sympathy with the focus-value hypothesis and the crucial postulate underlying it. In particular, if it produces theories not contradicted by observations it has the merit of bringing to the fore an important economizing activity of the individual, namely economizing of e fort involved in making decisions. The psychic and monetary cost of engaging in "calculating" is an important one. I would like to see it even more specifically introduced in our models than Shackle and those who talk of the "irrational passion for dispassionate rationality" have done. If we adopt the suggestion of Arrow, Blackwell and Girschik in their article on Bayes and Minimax Solutions of Sequential Decision Problems (Econometrica, 1949, pp 213-244 esp. p. 215.) that

the statistical theory of sequential sampling may be reformulated to account for individual behavior under uncertainty, perhaps we will have a framework within which this can be accomplished.

In final brief summary, we may state our position in this way: Shackle went too far in rejecting the use of probability distributions in their descriptive role; for the class of problems he considered, however, his hypothesis built on the focus-values postulate as an alternative to that based on the usual analytical role of probability distributions has considerable merit, and makes his book a significant contribution.